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Book Review of Creativity in Mathematics and the Education of Gifted Students

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1. Opening Remarks

A clarion call such as "Creativity for all" sounds hauntingly similar to calls such as "Algebra for all", "Quantitative Literacy for all", "Democracy for all" etc, heard from bodies such as the National Council of Teachers of Mathematics (NCTM), policy makers and politicians respectively. The implication of such calls is that very little actually gets accomplished when forces that dictate reality factor into the implementation of such ideals. This preamble sets the stage for the critical review of the book in question, namely "*Creativity in Mathematics and the Education of Gifted Students"*

The title suggests that the book addresses two topics, namely (1) construct of creativity, (2) gifted education. The conjunction "and" implies that the two topics will be interwoven together. The back cover of the book states that the book breaks through in the sense that it stitches together the research agendas of psychologists working in creativity and giftedness with those of mathematics educators interested in the same topics. It remains for the reader to go through the book and evaluate whether it delivers on the promise of a "breakthrough". The book consists of 23 chapters organized in different parts which are (I) Perspectives on creativity and giftedness in mathematics; (II) Psychological aspects of creativity and giftedness and educational policy, and (III) Developing mathematical creativity and giftedness in different educational contexts. The lead editor Leikin writes a comprehensive synthesis in the fourth part which is erroneously printed as Part III in the table of contents. The chapters are developed versions of papers that were presented at the 5th International Conference on Mathematical Creativity and the Education of Gifted Students as well as an international workshop on creativity, both held in Haifa, Israel in 2008. Our motivation for compiling this review was familiarity as long term researchers with the field of mathematics gifted education and having organized a similar topics study meeting under the auspices of the first joint meeting of the Korean and American mathematical societies in Seoul last year.

2. Creativity and Giftedness in mathematics education: Polya's enduring legacy

A simple literature search within the mathematics education research journals will indicate a scarcity in articles that tackle giftedness or creativity. In fact, in ESM, there are 6 articles that report on studies related to giftedness and creativity in the last 40 years. Reliance on the writings of eminent mathematicians as research literature has long been the norm for those interested in mathematical creativity. Several prominent 20th century mathematicians like Jacques Hadamard,

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George Polya and Garrett Birkhoff attempted to demystify the mathematician's craft and explain the mystery of "mathematical" creation. Hadamard (1945), influenced by Gestalt psychology of his time described the creative process as that of *preparation-incubation-illumination and verification*. Hadamard, like Poincaré (1948) attributed a large part of the creative process to unconscious drives that occurred during the incubatory period before any insight (or the Aha! moment) occurred. This description is generic in a sense and does explain the Gestalt or the whole of the creative process in any field per se but is vague because it offers no insight specifically into the mathematician's mind. However a number of studies since have examined the role of an incubation period in creative problem solving. For instance Sio & Ormerod (2007) conducted a meta-analytic[2] review of empirical studies that have investigated incubation effects on problem solving. They noted that one theoretical reason for studying incubation is because it is closely associated with insightful thinking. According to this report and others similar to it (Vul & Pashler, 2008), understanding the role of incubation period may also allow us to make use of it more efficiently to foster creativity in problem solving, classroom learning, and working environments. Educators try to incorporate incubation periods in classroom activity in temporal pauses during classroom discourse (Barnes, 2000) or extended time periods for project related learning (Sriraman 2003), and positive incubation results in positive effects in promoting students' creativity (Sriraman 2004, Sriraman 2005a) and evident for the mathematicians (Kaufman & Sternberg, 2006). Incubation should not be neglected in the classroom. Students should be encouraged to engage in challenging problems and experience this aspect of problem solving, till a flash of insight results in the "Eureka" or "Aha!" moment and the solution is born (Sriraman & Yaftian, 2010, in press).This affective theme is touched upon in numerous chapters by Alexander Karp (chp2), Movshovitz-Hadar & Kleiner (chp3) and Liljedahl (chp4) that report on personal qualities, historical cases of creative acts and creative attributes of individuals.

The astute reader familiar with the research on problem solving starting with the heuristic methods of Polya (1945, 1954) will know that his work is often viewed as prescriptive as opposed to generative. This misperception is due to the proportion of textbooks that appropriated Polya's four stage model of probem solving while ignoring his more substantive work in creative and generative problem solving. The book under review does the field a major favor by correcting this wrong and numerous chapters use Polya's framework of heuristic and plausible reasoning. The chapters by Taylor (chp 5), Leikin, Koichu & Berman (chp 8), Leikin (chp9), utilize interesting and mathematically challenging tasks and examine strategies, solutions and pitfalls in a true *Polyaesque* fashion. Polya's (1954) algorithmic and logical approach described the creative process of looking for solutions and "discovering" new theorems via heuristics. Polya's (1954) detailed heuristics offer valuable insights into a systematized approach to prove new results with the caveat that the mathematician has some intuition a priori about the truth or falsity of a new proposition. Thus, heuristics can be viewed as a decision-making mechanism, which lead the mathematician down a certain path, the outcome of which may or may not be fruitful (Sriraman, 2009). At this juncture, it is useful to spell out that heuristic reasoning as approached by many researchers in mathematics education is a far cry from the generative intentions of the mathematician's writing about it. Although research on the use of heuristics was

[1] -There were 117 studies included in this meta-analysis that most of them support the existence of incubation effects on problem solving.

a major focus of problem solving research in the 1970's, little attention has been given to the use of heuristics in a creative process (English & Sriraman, 2010. In fact, when given a specific set of heuristics, the program AM (Automated Mathematician) made several interesting discoveries such as DeMorgan's Laws, Goldbach's conjecture and the Unique Factorization theorem (Lenat, 1976).

On the other hand, Birkhoff (1956, 1969) like the English mathematician Hardy (1940) compared the creative mathematician to an artist, a weaver of patterns of ideas, and his writings address the limitations of theorem proving machines. The underlying themes that emerge from a careful examination of the writings of these eminent mathematicians are (1) the mathematicians' ability to somehow choose and/or construct a viable combination of thought patterns that produces new results, and (2) the aesthetic appeal that mathematicians have for their craft (Sriraman, 2005a). Both Poincaré and Hadamard believed that aesthetics was a central (major) factor contributing towards the fruitful combinations that occurred in creative process (Van der Waerden, 1953). The second theme has been the subject of renewed interest as seen in some recent articles (Brinkmann & Sriraman, 2009). Some chapters in Leikin, Berman & Koichu's book look at creativity and giftedness anecdotally using the themes spelled out above without substantial empirical work on actual ways to foster their development. This is a limitation of the book which is more than compensated by actual models of talent development tried with students (chps 14 and 20) and teachers (chp 19) along with an entire part of the book devoted to the psychological aspects of creativity and giftedness (Part 11). This attention to psychology could be regarded as an important contribution of the book to the field of mathematics education for the many reasons, some of which are discussed in the next section. Again our belief in the discursive and intertextual nature of theorizing in the field of mathematics education suggests we anchor this argument on empirically and historically grounded facts (Sriraman, 2010).

3. Psychology, Eminence and Creativity

Mainstream psychology has maintained a continuous albeit waxing and waning interest in the study of creativity. One of the major hurdles has been the difficulty of constructing instruments that can somehow operationalize and empirically measure a construct as fuzzy as creativity. The literature in mainstream psychology reveals a sporadic pattern on the study of creativity from the 1940's until the 1980's. Since then a renewed interest in studying this vague and elusive construct is evidenced in the explosion of the literature on creativity prominent among which are the massive works of Mihaly Csikzentmihalyi, Dean Simonton and Robert Sternberg among many others (see Csikzentmihalyi, 1988, 2000; Simonton, 1984; Sternberg, 2000). Several general theories on human creativity grounded in historical and empirical data are now available. However as Leikin points out periodically in the book, there is little that has actually been applied or used within mathematics education. There was a considerable research gap for decades for mathematics education researchers interested in the phenomenon of mathematical creativity and how mathematicians invoke and adjust learning processes in research. Burton's (2004) *Mathematicians as Enquirers* created a foundational base out of which such research could progress, as opposed to relying on the testimonies of eminent mathematicians as we have in the past. The qualitative research undertaken by Burton complemented as well as illuminated the prodigious work of researchers in psychology. In one of the reviews (Sriraman, 2005b), it was suggested that the "ultimate" intellectual payoff of such research might be a shared and acceptable epistemology of the phenomenon of creativity in mathematics education and psychology. The gap between psychology and mathematics education is one that is repeatedly alluded to in Leikin, Berman & Koichu's collection of chapters. Even though the book under review suggests that definitional problems of creativity abound, in fact the concept of creativity is well-defined in literature as paradoxical as it may be to study this construct because in many ways it is self defining. In other words, we are able to engage or judge acts of everyday creativity such as improvising on a recipe (Craft, 2002), use a tool in a way it wasn't intended, or intuit emotions and intended meanings from gestures and body language in day-to-day communication (Sriraman, 2009). Children are particularly adept at engaging in creative acts such as imaginary role playing or using toys and other objects in imaginative ways. "Aha!" experiences occur not only in individuals working on scientific problems but also in day-to-day problems such as realizing a person's name or relational identity after having forgotten it. However it is important to distinguish between everyday creativity and domain specific or paradigm shifting creativity. Domain specific creativity or "extraordinary creativity" causes paradigm shifts in a specific body of knowledge and it is generally accepted within that works of "extraordinary creativity" can be judged only by experts within a specific domain of knowledge. This distinction is brought about more clearly in the book in Part III of the book by using specific tasks (out-of school activities, statistics enrichment modules) and specific populations (elementary education, teacher education, undergraduate education) to explore what is meant by domain specific creativity in mathematics. In reading through the seven chapters that constitute this section, we attempted to discern if there was a classification or taxonomy of thinking styles in gifted students anchored to prior literature on the topic. Although Krutetskii's (1976) seminal work on the classification of abilities in mathematically gifted children was repeatedly cited in the book, this was difficult to find within the chapters per se in the book suggesting that each part of the book could have used a summative chapter that chunked together the major themes and findings in that particular section for the sake of better theoretical anchoring and intertextuality (Sriraman, 2010).

4. Closing Remarks

As stated earlier, mathematics giftedness is a topic that has existed on the fringes of mathematics education research and most of the prior research on the topic is found within psychology (e.g., Sriraman 2005a, 2008). The same applies for the research on mathematical creativity. For instance many of the theories in the field of creativity has been generated from eminent samples of mathematicians, artists, scientists, etc., which should make us skeptical about its applicability to "normal" samples. However substantial problem solving and problem posing frameworks do exist within mathematics education along with efforts from various groups of researchers to study these constructs. The book under review represents a coordinated effort to organize the efforts of such researchers into one cohesive collection. More grass roots efforts such as these are needed if the field of mathematics education were to have a voice in the ongoing debates of creativity and giftedness occurring in the field of psychology. The book paves the way for doing so. However there is not much treatment of the notion of fallibility in mathematical creativity in the spirit of Lakatos (1976). Although creative mathematical activity is fallible, this probability of existing fallibility leads to major success achievements of humans. For instance, the various proofs of the fifth postulate of Euclid presented with flaws, resulted in the discovery of non-Euclidean geometry. The other example is the Poincaré Conjecture which mathematicians made efforts to prove for over 100+ years and in 2003 it was proved by Russian mathematician Grigory Perelman. We dare say that Perelman's proof was the result of creative attempts of group of mathematicians for about 100 years, though their proofs consisted of flaws and deficiencies (Sriraman & Yaftian, 2010). The book under review is analogously a creative attempt to move the research on creativity and giftedness in mathematics education forward.

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