Archimedes’ Principle

Purpose

In this lab we will use two methods to experimentally determine the buoyant force \( F_b \) acting on a cylinder as it is submerged to different depths within a fluid.

Introduction

It can be observed that an object submerged in a fluid has an apparent weight less than when the object is outside the fluid. This is a result of the fact that as the force of gravity is acting downward on the object, there is also a buoyant force acting upward on the object while it is immersed in the fluid. The buoyant force is due to a difference in pressures between top and bottom of an object within the fluid. There exists a pressure difference because the pressure increases as the depth increases. Archimedes discovered that the buoyant force on an object has the same magnitude as the weight of the fluid displaced by that object.

![Figure 1: Forces acting on objects submerged within a fluid. The length of vectors are proportional to the magnitudes of the forces. There are forces acting on all sides of an object submerged in a fluid, however the horizontal components at any particular depth cancel.](image)

To show this, first consider the vertical forces acting on a metal cylinder held in static equilibrium supported from above by a string as shown in Figure 2. When the cylinder is partially submerged in water, the tension \( F_t \) and buoyant force \( F_b \) act upward and the force of gravity \( F_g \) acts downward.

\[
\begin{align*}
\Sigma F_y &= F_{vy} + F_{hy} + F_{ty} \\
0 \text{ N} &= (F_g) + (F_b) + (F_t) \\
F_g - F_t &= F_b
\end{align*}
\]

The magnitude of the tension pulling up on the cylinder is the same as the magnitude of the tension pulling down on the scale. The scale will report a mass less than the actual mass when the cylinder is submerged. This is the apparent mass \( m' \). The tension will be the apparent weight.

\[
F_t = m'g
\]

Thus the buoyant force is equivalent to the difference between the actual weight and the apparent weight of the cylinder.

\[
F_b = F_g - F_t = mg - m'g = (m - m')g
\]

Next, consider a column of the fluid in static equilibrium, equivalent in volume to the submerged part of the cylinder. The force of gravity on the column of fluid acts downward, but the fluid below it pushes upward on the column. Since the mass of the column of fluid can’t be easily measured, we can express it in terms of the density \( \rho_f \) of the fluid and volume \( V_{disp} \) of the column.

\[
\begin{align*}
\Sigma F_{dispy} &= F_{c,dispy} + F_{b,dispy} \\
0 \text{ N} &= (-F_{c,dispy}) + (F_{b,dispy}) \\
F_{c,disp} &= F_{b,disp} \\
m_{disp}g &= F_{b,disp} \\
\rho_fV_{disp}g &= F_{b,disp}
\end{align*}
\]
If the actual cylinder is next to the column of fluid, then the pressures at the bottom of the cylinder and at the bottom of the column of fluid are the same. And if the column of fluid and submerged part of the cylinder are the same shape, the buoyant forces on them must consequently also be equal (because $A_{\text{cylinder}} = A_{\text{disp}}$).

$$P_{\text{cylinder}} = P_{\text{disp}}$$ (11)

$$\frac{F_{b}}{A_{\text{cylinder}}} = \frac{F_{b,\text{disp}}}{A_{\text{disp}}}$$ (12)

$$F_{b} = F_{b,\text{disp}}$$ (13)

Thus the buoyant force on the cylinder is equivalent to the weight of the fluid it displaces, and is expressible in terms of the volume of fluid it displaces.

$$F_{b} = \rho_{f} V_{\text{disp}} g$$ (14)

In this laboratory we will directly measure the density of the fluid and the volume of the cylinder at different submerged heights to calculate the buoyant force we expect when we submerge the cylinder to different heights and compare those results to values of the buoyant force when we actually submerge the cylinder to the corresponding depths.

For this experiment, you will be using an aluminum cylinder. The cylinder has score marks dividing it into approximately one-quarter sections. For ease of reference, the boundaries on either side of each quarter section of the cylinder will be called levels within this document. See Figure 3 for a visual representation.
Procedure—Part I

1. Measure the mass $m_{\text{beaker}}$ of the empty beaker.
2. Determine the uncertainty $\delta m_{\text{beaker}}$ of the mass of the empty beaker.
3. Determine the fractional uncertainty $\delta m_{\text{beaker}}/m_{\text{beaker}}$ of the mass measurement.
4. Fill the beaker with approximately 325 mL of water.
5. Record the volume $V_f$ of the water.
6. Determine the uncertainty $\delta V_f$ of the volume of water.
7. Determine the fractional uncertainty $\delta V_f/V_f$ for the volume measurement.
8. Convert the mass measurement of the water from mL to m$^3$, using the fact that 1 mL = $10^{-6}$ m$^3$.
9. Measure the mass $m_{\text{both}}$ of the beaker with the water in it.
10. Determine the uncertainty $\delta m_{\text{both}}$ in the total mass measurement.
11. Determine the fractional uncertainty $\delta m_{\text{both}}/m_{\text{both}}$ for the total mass measurement.
12. Calculate the mass $m_f$ of the water without the beaker.

$$m_f = m_{\text{both}} - m_{\text{beaker}} \quad (15)$$

13. Calculate the density $\rho_f$ of the water.

$$\rho_f = \frac{m_f}{V_f} \quad (16)$$

14. Measure the diameter $d$ of the aluminum cylinder.
15. Determine the uncertainty $\delta d$ of the diameter measurement.
16. Determine the fractional uncertainty $\delta d/d$ for the diameter measurement.
17. Calculate the radius $r$ of the aluminum cylinder.
18. Do the following steps 4 times. Start with level 1 of the cylinder and increment to the next level for each successive trial.

(a) Measure the height $h$ of the level from the bottom base of the cylinder (end without the hook).
(b) Determine the uncertainty $\delta h$ of the height measurement.
(c) Determine the fractional uncertainty $\delta h/h$ for the height measurement.
(d) Calculate the volume $V_{\text{below}}$ of the part of the cylinder below the current level.

$$V_{\text{below}} = \pi r^2 h \quad (17)$$

(e) Calculate the weight $F_{c,\text{disp}}$ of an amount of water that has the same volume as the part of the cylinder below the current level.

$$F_{c,\text{disp}} = \rho_f g V_{\text{below}} \quad (18)$$

19. You should now have four force values ($F_{c,\text{disp}1}$, $F_{c,\text{disp}2}$, $F_{c,\text{disp}3}$, $F_{c,\text{disp}4}$).
Procedure—Part II

1. Set up the apparatus similar to that displayed in Figure 2 by hanging the cylinder from the bottom of the electronic scale.

2. Below the cylinder, place the beaker of water on the lab jack.

3. Record the mass $m_0$ of the aluminum cylinder. This is the measured mass when zero quarters of the cylinder is submerged in water.

4. Determine the uncertainty $\delta m_0$ of the mass of the aluminum cylinder.

5. Determine the fractional uncertainty $\delta m_0/m_0$ of the mass measurement.

6. Calculate the weight of the cylinder $F_g$. Use $g = 9.80665 \text{ m/s}^2$ for this calculation.

7. Do the following steps 4 times.
   (a) Raise the lab jack until the water is at the next level.
   (b) Record the apparent mass $m'$.
   (c) Determine the uncertainty $\delta m'$ of the apparent mass measurement.
   (d) Determine the fractional uncertainty $\delta m'/m'$ of the apparent mass measurement.
   (e) Calculate the magnitude of the tension $F_t$ in the string. The apparent mass $m'$ multiplied by the magnitude of the gravitational acceleration ($g = 9.80665 \text{ m/s}^2$) will provide us with the tension $F_t$ in the string. The tension is equivalent to the apparent weight of the cylinder.
   (f) Calculate the magnitude of the buoyant force $F_b$.

   \[ F_b = F_g - F_t \]  

8. You should now have four force values ($F_{b1}$, $F_{b2}$, $F_{b3}$, $F_{b4}$).

Determining Uncertainties in Your Final Values

In the results section of your notebook, state the results of both parts of your experiment in the form $F_{g,\text{disp}} \pm \delta F_{g,\text{disp}}$ and $F_{b} \pm \delta F_{b}$. You should have 8 values For Part I, $\delta F_{g,\text{disp}}$ should be equal to the largest fractional uncertainty from your values of diameter, mass of the empty beaker, mass of the filled beaker, the volume of the fluid, or the height multiplied by your value for $F_{g,\text{disp}}$. Note, $\delta F_{b}$ in Part II should be equal to the largest fractional uncertainty from your values of actual mass or apparent mass multiplied by your value of $F_{b}$.

\[
\delta F_{g,\text{disp}} = F_{g,\text{disp}} \ast \max \left( \frac{\delta m_{\text{beaker}}}{m_{\text{beaker}}}, \frac{\delta m_{\text{both}}}{m_{\text{both}}}, \frac{\delta V}{V_f}, \frac{\delta d}{d}, \frac{\delta h_i}{h_i} \right) \text{ where } i = 1, 2, 3, 4 \quad (20)
\]

\[
\delta F_{bi} = F_{bi} \ast \max \left( \frac{\delta m_0}{m_0}, \frac{\delta m'_i}{m'_i} \right) \text{ where } i = 1, 2, 3, 4 \quad (21)
\]

You should also address the following question:

1. Do your results for the buoyant forces $F_b$ and the weights $F_{g,\text{disp}}$ of the displaced water agree within their uncertainties? Be sure to clearly state the quantitative values you are comparing. If there are any large discrepancies, quantitatively comment on their possible origin.