Ballistic Pendulum - Theory

First, we determine the potential energy of the pendulum at the top of its swing.

Referring to Figure 1, if we let $v$ be the speed of the ball with mass $m$ immediately before impact with the pendulum bob and $V$ be the speed of the ball and pendulum with mass $m + M$ immediately after the collision, then we can determine an equation for the momentum of the collision. The initial momentum at that instant before impact is $mv$ and from the law of conservation of linear momentum this quantity must equal the final momentum of the entire system immediately after impact. Hence $mv = (M + m)V$ and can be solved for the speed $v$.

$$v = \left(\frac{M + m}{m}\right)V$$  

After impact, the ball is raised from the initial equilibrium position of the pendulum to a height above that position. The kinetic energy of the system (ball and pendulum) at the instant following impact is $\frac{1}{2}(M + m)V^2$. When the center of gravity reaches its highest position, the potential energy of the system has been increased by an amount $(M + m)g\Delta h$ and the kinetic energy has become zero. If the small frictional loss in energy is neglected, the loss in kinetic energy equals the gain in potential energy, so $\frac{1}{2}(M + m)V^2 = (M + m)g\Delta h$. Again, this can be solved for the speed $V$.

$$V = \sqrt{2g\Delta h}$$  

According to Figure 1 if we measure the distance $R$ from the pivot point to the center of mass of the pendulum and measure the angle $\theta$ which the pendulum sweeps out from equilibrium, then we can determine the change in its vertical position $\Delta h$.

$$\Delta h = R(1 - \cos \theta)$$

Using equations one through three we can determine the initial velocity of the projectile from measured quantities. You should write out this equation for $v$ in terms of $R$, $\theta$, $M$, and $m$. 

\[ \text{Figure 1: Ballistic Pendulum.} \]
Introduction — Part II

The second method in this lab involves an independent measurement of the horizontal distance a projectile travels while falling from a measured vertical height. If we launch a ball horizontally when there is no pendulum to catch it, the ball undergoes projectile motion. If the air resistance is negligible, then the horizontal acceleration vanishes. Vertically, the acceleration is due to gravity.

If we choose a coordinate systems shown in Figure 2, where the $x$-axis is horizontal and positive in the direction of launch and the $y$-axis is vertical with the positive direction upward, then we can write down a list of the kinematic variables for this coordinate system.

\begin{align*}
\Delta x &=? \quad \Delta y =? \\
v_{ix} &=? \quad v_{iy} = 0 \text{ m/s} \\
v_{fx} &=? \quad v_{fy} =? \\
a_x &= 0 \text{ m/s}^2 \quad a_y = -g \\
\Delta t &=?
\end{align*}

![Figure 2: Projectile motion of a horizontally launched ball. The values of the kinematic variables have been listed with a question mark if they are initially unknown.]

Although you will not be able to directly measure the time of flight, or any of the velocities, you can still measure the displacements of the ball. After you have found an average value for your displacements over several trials, you can take advantage of a couple of the kinematic equations.

\begin{align*}
\Delta x &= v_{ix} \Delta t + \frac{1}{2} a_x \Delta t^2 \quad \rightarrow \quad \Delta x = v_{ix} \Delta t \quad \rightarrow \quad v_{ix} = \frac{\Delta x}{\Delta t} \quad (4) \\
\Delta y &= v_{iy} \Delta t + \frac{1}{2} a_y \Delta t^2 \quad \rightarrow \quad \Delta y = \frac{1}{2} a_y \Delta t^2 \quad \rightarrow \quad \Delta t = \sqrt{\frac{2\Delta y}{a_y}} \quad (5)
\end{align*}

Because the initial velocity is horizontal, the magnitude of its $x$-component is equivalent to the launch speed of the ball.

\[ v_i = \sqrt{v_{ix}^2 + v_{iy}^2} = \sqrt{v_{ix}^2 + 0^2} = \sqrt{v_{ix}^2} = |v_{ix}| = v \quad (6) \]

Procedure — Part I

1. You will need to detached the pendulum by unscrewing its axle. Set the axle aside.
2. Determine the mass of the ball $m$ and the mass of the pendulum $M$.
3. Determine the uncertainties of the masses $\delta m$ and $\delta M$ from the precision of the scale.
4. Calculate the fractional uncertainties $\delta m/m$ and $\delta M/M$ for these measurements.
5. Load the ball into the pendulum.
6. Determine the distance $R$ from the center-of-mass of the loaded pendulum to its pivot point. Place the loaded pendulum within the hanging loop of string. The location of the center-of-mass will be at the location of the string when the loaded pendulum is balanced. The pivot point is the hole where the axle fits.

Why is it important to find the center-of-mass when the pendulum is loaded with the ball? Why is the center-of-mass important?

7. Determine and record $\delta R$ based on the precision of the meter stick.

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8. Calculate the fractional uncertainty $\delta R/R$ for this measurement.

9. Retrieve the axle and reattach the pendulum. Make sure the pendulum is facing the correct direction to catch any ball launched from the spring gun.

10. Determine the angle offset $\theta_{\text{offset}}$ of the angle indicator. You can do this when the pendulum is freely hanging in front of the spring launcher. Move the angle indicator until it is resting firmly against the pendulum. Make sure you also determine if your $\theta_{\text{offset}}$ is positive or negative.

11. Take note of how many brass disks are attached to your pendulum. You will use this to determine which launcher setting you should use for all parts of this experiment.

<table>
<thead>
<tr>
<th>number of disks</th>
<th>range setting</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>short</td>
</tr>
<tr>
<td>1</td>
<td>medium</td>
</tr>
<tr>
<td>2</td>
<td>long</td>
</tr>
</tbody>
</table>

12. Do the following steps 10 times.
   (a) Remove the ball from the pendulum.
   (b) Load the ball into your launcher. Use the black cylindrical plunger to push the ball until the yellow range indicator is at the appropriate setting. You will temporarily need to lift the pendulum.
   (c) Reset the angle indicator. It should rest firmly against the pendulum.
   (d) Fire the projectile by gently and firmly pulling the string straight up.
   (e) Record the angle $\theta'$ the pendulum reaches.

13. Average your values of $\theta'$.

14. Subtract the angle offset from your averaged angle value.

   $$\theta = \theta' - \theta_{\text{offset}}$$

   [1] This is where the sign of your offset angle is important. If the offset is positive, then $\theta$ will be less than $\theta'$. If the offset is negative, then $\theta$ will be greater than $\theta'$.

15. Determine $\delta \theta$ from the precision of the scale attached to the apparatus.

16. Calculate the fractional uncertainty $\delta \theta/\theta$ for this measurement.

17. Calculate $v$ using Equations (1), (2), and (3).

   [1] When using a spreadsheet program, be careful with trigonometric functions in your spreadsheet formulas. Many require the argument of a trigonometric function to be expressed in radians, not in degrees.

   \[ \text{[radians]} = \text{[degrees]} \times \left( \frac{\pi}{180} \right) \]

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**Procedure — Part II**

1. Lift and latch the pendulum at $90^\circ$ so it is out of the way.

2. Open and place the folding target board on the floor so a ball launched from the spring gun will strike near the center of the horizontal surface. You will need to fire the ball a couple times until the board is positioned appropriately.

3. Tape a blank piece of paper over the location where the ball will strike the board.

4. Secure a piece of carbon paper over the blank piece of paper.
Be sure that the side that is face down is the side that can leave a mark on the paper.

From this point forward, do not allow the board to move until after your horizontal measurements have been made. If it moves, you will need to start over.

5. Do the following steps 10 times.
   (a) Load the ball into the launcher.
   
   What range setting should you be using?
   
   (b) Fire the ball from the launcher.

6. Remove the carbon paper. Do not remove the paper that now has ten carbon marks on it.

7. Determine the vertical displacement $\Delta y$. You will need to measure from the cross hairs marked on the spring gun to the point on the floor directly below the cross hairs.

8. From the point on the floor you just found, determine the horizontal displacement $\Delta x$ for each of the ten carbon marks.

9. Average your values of $\Delta x$.

10. To determine $\delta \Delta x$ use the value of the longest distance between impact points on your paper. If you are consistent with your procedure, this should be rather small ($< 4$ cm).

   $$\delta \Delta x = \Delta x_{\text{max}} - \Delta x_{\text{min}}$$

   Why do you suppose the uncertainty is not the smallest increment on the measuring stick?

11. Calculate the fractional uncertainty $\delta \Delta x / \Delta x$ for this measurement.

12. Determine $\delta \Delta y$ based on the precision of the meter stick.

13. Calculate the fractional uncertainty $\delta \Delta y / \Delta y$ for this measurement.

14. Calculate the launch speed of the ball $v$ from Equations (4), (5), and (6).

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**Determining Uncertainties in Your Final Values**

In the results section of your notebook, state the results of both parts of your experiment in the form $v \pm \delta v$. Note, $\delta v$ in Part I should be equal to the largest fractional uncertainty from your values of mass ($m$ or $M$), angle $\theta$, or distance $R$ multiplied by your value of $v$ from Part I. For Part II, $\delta v$ should be equal to the largest fractional uncertainty from your values of horizontal displacement $\Delta x$ or vertical displacement $\Delta y$ multiplied by your value of $v$ from Part II.

$$\delta v = v \times \left( \frac{\delta m}{m}, \frac{\delta M}{M}, \frac{\delta \theta}{\theta}, \frac{\delta R}{R} \right)$$

(7)

$$\delta v = v \times \left( \frac{\delta \Delta x}{\Delta x}, \frac{\delta \Delta y}{\Delta y} \right)$$

(8)

You should also address the following question:

1. Do your results for $v$ in the two parts agree within their uncertainties? Be sure to clearly state the quantitative values you are comparing. If there are any large discrepancies, quantitatively comment on their possible origin.