Introduction

The purpose of this experiment is to familiarize the student with some of the instruments used in making measurements in the physics laboratory, to provide practice in graphing techniques, and to show how errors in laboratory measurements are analyzed. In science, error does not mean a mistake. Error in a scientific measurement means the inevitable uncertainty that is present in all measurements. Errors cannot be eliminated. The best you can hope for is to minimize errors and reliably estimate how large they are. Study the entire discussion and instrument descriptions carefully before starting to make measurements.

Experimental Error

Every measurement is subject to errors. In the case of measuring the distance between two points, a number of measurements will usually give slightly different results. These differences may be due to inaccuracy in estimating the fraction of a division. The error in physical measurements becomes of crucial importance when comparing experimental results with theory. The basis of the scientific method is to test our hypotheses against experimental data. Unless you take the experimental errors into account in the physics labs, you could quickly find that you have disproved all the laws of physics! It is therefore very important that you learn the techniques for estimating the effect that measurement errors have on your laboratory results.

Random Errors

Random errors (often called statistical uncertainties) are those produced by unknown and unpredictable variations in the experimental situation. Many kinds of random errors can be reduce by repeating a measurement many times and determining an average value for that measurement.

Systematic Errors

Systematic errors are errors associated with a particular instrument or experimental technique. Systematic errors cannot be reduced by repeated measurements, as the same error occurs in every measurement.

Comparing Random Errors and Systematic Errors

The distinction between random errors and systematic errors is perhaps best illustrated with an example from an outdoor thermometer. Figure 1 illustrates how four different thermometers might displace some temperature measurements when the actual temperature is 22°C.

Taking several measurements of this outside temperature with a temperature sensor would yield a value each time of 22°C if there were no errors of any kind and the outside temperature is held constant. Referencing the data in Figure 1 and Table 1 we see that random errors cause the sensor temperature to scatter about the outside temperature with more values closer to the outside temperature, and fewer values farther from the outside temperature. Systematic errors cause the sensor values to be away from the outside temperature, but always higher (or always lower), hence systematically shifted away from the actual value. The random errors may be due to variable cloud cover, varying sensor placement during individual measurements, etc. The systematic errors may be due to a consistent cloud cover, sensor housing, or a consistent bias with sensor placement.

Note that the effects of random errors can be minimized by taking many measurements and averaging the results. Systematic errors cannot be minimized in this way. On the other hand, if the value of a systematic error can be calculated or measured independently, its effect on the result may be minimized by subtracting it from the result. Random errors cannot be minimized in this way. In most experiments, a combination of random and systematic errors are present at the same time.

The difference between random errors and systematic errors is often expressed using two words that are usually confused. The words are precision and accuracy. Although they both relate to experimental uncertainty, their meanings are quite distinct. Precision refers to how large the uncertainty in an experimental quantity is, compared to the size of the quantity itself. Accuracy refers to how close an experimental result is to the value of the experimental quantity measured (the accuracy of an experimental result may not be known at the time the experiment is performed).
Figure 1: Examples of systematic and random errors and their relationship to precision and accuracy. Multiple measurements are displayed in each thermometer. The actual temperature is 22°C, and is indicated by the arrow.

<table>
<thead>
<tr>
<th>Systematic Error</th>
<th>Random Error</th>
<th>Temperature Measurements (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Large</td>
<td>Large</td>
<td>20 13 16 21 15 14 18 18 19 14</td>
</tr>
<tr>
<td></td>
<td></td>
<td>This distribution shows no tendency toward a particular value (lack of precision) and does not acceptably match the actual temperature (lack of accuracy).</td>
</tr>
<tr>
<td>Large</td>
<td>Small</td>
<td>18 17 16 17 16 18 17 17 18 18</td>
</tr>
<tr>
<td></td>
<td></td>
<td>This distribution shows a tendency toward a particular value (high precision) but every measurement is well off from the actual temperature (low accuracy).</td>
</tr>
<tr>
<td>Small</td>
<td>Large</td>
<td>21 18 25 23 20 19 26 23 19 23</td>
</tr>
<tr>
<td></td>
<td></td>
<td>This distribution shows no impressive tendency toward a particular value (lack of precision) but each value does come close to the actual temperature (high accuracy).</td>
</tr>
<tr>
<td>Small</td>
<td>Small</td>
<td>22 23 21 22 22 23 21 22 22 23</td>
</tr>
<tr>
<td></td>
<td></td>
<td>This distribution shows a tendency toward a particular value (high precision) and is very near the actual temperature each time (high accuracy).</td>
</tr>
</tbody>
</table>

Table 1: Thermometer data correlating to the images in Figure 1. The actual temperature is 22°C.

**Personal Error**

A *personal error*, sometimes referred to as a blunder, is an error that arises because of an error made by the experimenter in reading the instrument or in recording the value of the measurement. Blunders can usually be detected by having more than one person take the measurement or by double-checking the measurement. By taking these previous steps you have not improved your accuracy or precision (i.e. you have not minimized systematic or random error). You have just avoided a blunder.
Measurement Uncertainties

Estimating Direct Measurement Uncertainty

The correct way to state the result of a measurement is to give the best estimate of the quantity and the range within which you are confident the quantity lies. In most of the experiments you will be asked to estimate the uncertainties in the quantities you measure, such as a distance or time interval. This requires some practice, so let us explore an example. In reading a ruler or a meter scale you can usually interpolate between the divisions on the scales. For example, if you are measuring the distance $x$ between two points on a meter stick whose smallest division is 0.001 m (or 1 mm, we call this $\delta x$) and $x = 0.100$ m then in quoting an uncertainty, there is a standard format which we will follow.

$$x \pm \delta x = (0.100 \pm 0.001) \text{ m}$$

Meaning $x \pm \delta x$ can have a value between 0.101 m and 0.099 m. For many (but not all) of the measuring devices used in this lab, we will take the uncertainty to be the resolution of the instrument (i.e., the smallest unit it can measure).

Fractional Uncertainty and Precision

The uncertainty $\delta x$ by itself does not tell the whole story. An uncertainty of one inch over one mile would indicate an unusually precise measurement, whereas an uncertainty of one inch over three inches would indicate a rather crude estimate, hence their precision are very different. The quality of a measurement is indicated not just by the uncertainty $\delta x$, but by the ratio of $\delta x$ to $x$, which leads us to define the fractional uncertainty as

$$\text{[fractional uncertainty]} = \frac{\delta x}{|x|}$$

A numerical example:

$$\frac{\delta x}{|x|} = \frac{0.001 \text{ m}}{0.100 \text{ m}} = 0.01$$

The fractional uncertainty is also called the relative uncertainty or the precision, while $\delta x$ is sometimes called the absolute uncertainty to avoid confusion with the fractional uncertainty. In most serious measurements, $\delta x$ is much smaller than the measured value $x$. Note that fractional uncertainty is dimensionless.

The overall result of an experiment is usually calculated from a formula combining several measured values. When a measurement involves two steps, the estimation of uncertainty involves two steps. The first step is to determine the uncertainties measured directly and the second step is determine how these uncertainties propagate through the calculations to produce an uncertainty in the final answer. In order to properly propagate uncertainty, a student should have a basis in calculus which is outside the scope of this course. Therefore we will consider the uncertainty in the final answer to be the uncertainty which gives the largest fractional uncertainty derived from direct measurements since that quantity will ultimately have the largest impact on the final answer.

Reporting Uncertainties: An Example

Suppose you measure the time it takes a toy car to travel over a known distance; and you want to determine how fast it was traveling. First, you record the measurements of the distance and time along with their uncertainties.

$$x = 10.000 \text{ m} \quad t = 4.0 \text{ s}$$

$$\delta x = 0.001 \text{ m} \quad \delta t = 0.2 \text{ s}$$

The uncertainties would be based off the precision of the meter stick and your reaction time on the stopwatch. The speed can be calculated from the constant speed equation.

$$v = \frac{x}{t} = \frac{10.000 \text{ m}}{4.0 \text{ s}} = 2.5 \text{ m/s}$$
To determine the uncertainty in this calculation, we need to look at the fractional uncertainties of the measured values.

\[ \frac{\delta x}{|x|} = \frac{0.001 \text{ m}}{10.000 \text{ m}} = 0.0001 \quad \frac{\delta t}{|t|} = \frac{0.2 \text{ s}}{4.0 \text{ s}} = 0.05 \]

The largest fractional uncertainty will tell us which measurement has the largest affect on the final value of the speed. At the end of most of the laboratory handouts in this course you will see a maximum function, which tells you to select the largest of the listed fractional uncertainties. For this example the largest fractional uncertainty comes from the time measurement.

\[ \max \left( \frac{\delta x}{|x|}, \frac{\delta t}{|t|} \right) = \frac{\delta t}{|t|} \]

In general, you will find the uncertainty in a calculated value by the product of that calculated value and the largest fractional uncertainty of the measurements used to calculate that value. So, for this example, the uncertainty in the speed \( \delta v \) will be found from the product of the speed \( v \) and the largest fractional uncertainty.

\[ \delta v = v \times \max \left( \frac{\delta x}{|x|}, \frac{\delta t}{|t|} \right) = v \times \frac{\delta t}{|t|} = \left( 2.5 \frac{\text{m}}{\text{s}} \right) \times (0.05) = 0.125 \frac{\text{m}}{\text{s}} = 0.1 \frac{\text{m}}{\text{s}} \] (rounded)

The result should be reported clearly as the sum or difference between the central value \( v \) and its uncertainty \( \delta v \).

\[ v \pm \delta v = (2.5 \pm 0.1) \frac{\text{m}}{\text{s}} \]

The ± means that the value of the speed most likely falls between 2.4 m/s and 2.6 m/s (even if it is not known to the experimenter what the theoretical or accepted value should be).

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**Significant Figures**

The significant figures in a number are all figures that are obtained directly from the measuring process and exclude those zeros, which are included solely for the purpose of locating the decimal point. Precision refers to the reproducibility of a measurement; accuracy to its correctness, in terms of some criteria such as other, more careful measurements. The precision of an experimental result is implied by the number of digits recorded in the result. Because the quantity \( \delta x \) is an estimate of uncertainty, it should not be stated with too much precision. For our purposes we can state the following rules:

1. Experimental uncertainties should almost always be rounded to one significant figure.

2. The last significant figure in any stated measurement should usually be of the same order of magnitude (in the same decimal position) as the uncertainty.

An important qualification to the rules above is as follows: To reduce inaccuracies caused by rounding, any numbers to be used in subsequent calculations should normally retain at one least significant figure more than is finally justified. Note that the uncertainty in any measured quantity has the same dimensions (represented by units) as the measured quantity itself.

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**Graphing**

Graphs are frequently used to present and analyze data in science. Graphs are useful when we are trying to minimize random error through repeated measurements of a quantity. There exist several rules of plotting that lead to clear, useful graphs. Some of the points on which we grade are summarized here:

1. A graph should be labeled so that its meaning is clear. The quantity plotted along each axis should be indicated, along with the units in which the plotted data are expressed. An appropriate title should also be on the page.
2. Scales should be chosen for ease of use. This is best accomplished by choosing one division equal to a multiple of 1, 2, 5, or 10. The scales used on the two axes need not be the same, but should be chosen so that the data fill the page.

3. In this course we will plot linear equations, meaning the formulas used will take the form of a line.

\[ y = mx + b \]

The value of \( m \) is the slope of the line, and \( b \) is the \( y \)-intercept. Therefore we will plot \( y \) vs. \( x \) with the corresponding \( y \) variable plotted along the vertical axis and the corresponding \( x \) variable along the horizontal axis. Often the \( y \)-intercept will be zero in the formula used in the lab, but it may not be on your graph if you are truly fitting your data points as expected.

4. The individual points should not be connected directly together with a line, but instead express a trend. This trend can be seen directly with the addition of a regression line, a statistical tool used to mathematically express a trend in the data. Due to measurement errors, the line may not pass through all data points or the origin. Because we are interested in the trend of the data and not the fluctuation between neighboring data points, do not play connect-the-dots with the data points.

![Horizontal Velocity vs Elapsed Time](image)

**Figure 2: Plot of Linear Data.**

**Note:** If graphing on a computer, some programs (e.g. Microsoft Excel) may automatically decide on the format of the values on axis and the limits of the axis. This may lead to poor graphs and confusion. You can and should choose your own limits and format of values.

For many purposes a graph is used only to provide a compact display of data. We will want to take an additional step and use the graph to quantitatively analyze the data. For example we will want to find a constant value in many of our experiments. We can do this by taking the slope of the line.

\[
\text{slope} = \frac{y_2 - y_1}{x_2 - x_1}
\]

You should note that the most accurate calculation of the slope is by selecting two points on your best-fit line as far apart as possible. Since the slope is a ratio of the quantity plotted vertically to the quantity plotted horizontally, it carries the units of that ratio. You are encouraged to graph on a computer. Please note the method of statistical analysis used by your chosen software program as different methods can give different results.
Laboratory Notebook Guide

Included below are some general guidelines for writing laboratory notebooks. Some of the sections may not be applicable to a given lab, but this general format, if followed, will provide you with clear, concise, and organized data that will facilitate your ability to do well on the laboratory quizzes. Preparing parts of your notebook ahead of time will also make your laboratory period more efficient and give you the ability to inquire about misunderstandings or quickly recognize blunders that could affect your final results.

**Title**  Title of the experiment, date performed, your name, name of all your lab partners.

**Objective**  A simple one sentence statement of the purpose of the experiment.

**Theory**  State the theoretical basis for the experiment (i.e., the relationship between the measured quantities and the desired quantity). This part is best done before you come to class as a way to make sure you understand the experiment to be performed.

**Apparatus**  A labeled drawing or sketch of the most important parts of the experiment is the best way to describe the apparatus.

**Procedure**  Describe the steps you took to obtain the data or you can just include your printed laboratory instructions. Include a brief explanation of how you estimated the errors involved in each measurement.

**Data and Computations**  Present your data clearly and neatly in columns or tables with clear headings and units included. Be sure to pay attention to significant figures. You can re-copy your data to make it neater, but include your original data sheet. It is often appropriate to include the results of any computations, error analysis, and precision calculations in this same table. Also show any formulae you used for calculations which were not included in the Theory section.

**Graphs**  Graphs should be large enough to allow the viewer to visualize the data easily. If the slope of the graph is to be measured directly from the graph then the data should stretch across the full page. If the slope is calculated from a computer then the graph can be a bit smaller. As a general rule, make all graphs fill at least half the page.

**Results**  State your results (qualitative and quantitative) clearly and concisely. Your statement of results should address the objective section of the lab. This is the appropriate place to include possible “sources of error”, but be careful that you can back-up your claims with numerical estimates. Stating, for example, that “the equipment was bad” is too general and not an acceptable answer. Remember, the point of the experiment is to do the best you can with the equipment available, and to make a reasonable quantitative assessment of the uncertainties in your results. Check to see if your final values overlap within their uncertainties (i.e. look for overlap in the range of possible final values). If there is no overlap could there be a systematic error or were the uncertainties underestimated? If you think the uncertainties were underestimated, by what numerical values must they be increased for there to be agreement? Do those larger values seem reasonable? These are the types of questions you should consider in your results section.