Electric Fields and Electric Potential

Purpose:
To determine the quantitative relationship between the electric field and radius of two concentric circles.

Introduction:
When a charged particle is placed in an electric field it experiences a force. The electric field \( \vec{E} \) at a point is defined to be the force per unit charge acting on a small positive test charge which has been placed at that point. Algebraically this is written as

\[
\vec{E} = \frac{\vec{F}}{q_o}
\]  

where \( q_o \) is a small positive test charge. The magnitude of the electric field is measured in newtons/coulomb.

The electric potential difference, \( \Delta V \), between two points is defined to be the work done per unit charge (against electrical forces) in moving a small positive test charge slowly between two points. If one of the points is taken to be a reference point with zero potential, then the electric potential at any point can be defined in terms of the work done per unit charge in moving a small positive test charge from that reference point to the point at which the potential is to be determined. For isolated charges, the reference point is usually taken to be infinity. The potential is measured in volts. (1 volt = 1 joule/coulomb)

Of the two quantities that have been defined, it is the electric potential difference that is the easiest to measure. If one knows the electric potential at every point in space, it is possible to obtain the electric field. The rate of change of electrical potential in any direction is equal to the negative of the component of the electric field in that direction. For example, the \( x \) component of the electric field is equal to the negative of the rate of change of the electric potential in the \( x \) direction. Algebraically this would be expressed as

\[
E_x = -\frac{\Delta V}{\Delta x}
\]

Comparing equation 1 and equation 2 we can see that a newton/coulomb is equivalent to a volt/meter.

You have seen in lecture that in a three dimensional space a point charge generates a field described by the following equation:

\[
E = \frac{1}{4\pi\epsilon_o} \frac{Q}{r^2}
\]

Therefore we say that the electric field of a point charge is proportional to \( r^{-2} \). We can then write this in a general form:

\[
E = k \left( \frac{1}{r^n} \right)
\]

In this experiment, you are to quantitatively determine the dependence of the electric field to the radius between two concentric circles of aluminum on a piece of low-conductivity paper. What we want to find is the value of \( n \) for this particular geometry. Notice that \( n \) is an exponent, signifying that the formula is not linear. Therefore we can use the natural logarithm to put the equation into the familiar form for a linear function, \( y = mx + b \). There is an appendix in your text book devoted to logarithms if you need to review. Before we can apply the natural log function we must make the equation dimensionless. To do this, we can first divide both sides of Equation 4 by \( r_o^n = (1m)^n \).

\[
\frac{E}{r_o^n} = \frac{k}{r_o^n} \left( \frac{1}{r^n} \right)
\]

If we call \( k/r_o^n = \hat{E} \) and we multiply both sides by \( r_o^n \) we get

\[
E = \hat{E} \left( \frac{r_o^n}{r^n} \right)
\]

Now we divide through by \( E_o = 1 V/m \)
\[
\frac{E}{E_o} = \frac{E'}{E_o} \left( \frac{r_o}{r} \right)^n
\] (7)

Now that the equation is dimensionless, we take the natural logarithm of both sides (be sure to notice the algebraic step regarding the radius term)

\[
\ln \left( \frac{E}{E_o} \right) = \ln \left( \frac{E'}{E_o} \left( \frac{r_o}{r} \right)^{-n} \right)
\] (8)

Using an identity for logarithms gives

\[
\ln \left( \frac{E}{E_o} \right) = (-n) \ln \left( \frac{r}{r_o} \right) + \ln \left( \frac{E'}{E_o} \right)
\] (9)

In this form we can graph \( \ln(E/E_o) \) on the y-axis and \( \ln(r/r_o) \) on the x-axis to find the value of \( n \) which is equal to the slope of the line.

In this lab, we directly measure the potential difference between the two concentric aluminum circles shown on the apparatus in Figure 1. The apparatus is set up by connecting a DC power supply directly to the aluminum, establishing an electric field within the circles and in the plane of the paper that depends on the distance from the center of the circles, \( r \). The \( r \)-dependence of the electric field can be determined by measuring the potential at a number of points between the two circles and then using the rate of change of the electrical potential in the \( r \) direction to get a numerical value for the \( r \) component of the electric field. A positive value of \( E \) indicates that the electric field will be radially outward.

**Figure 1: Apparatus.**

**Laboratory Procedure:**

**Part I - Taking Direct Measurements**

1. Make a table of values to be measured in your laboratory notebook.
2. Before connecting to the source of DC power make sure to connect the positive terminal to the outer circle and the negative terminal to the inner circle.
3. The ground lead on the digital multimeter (COM port) should be connected to the point holding the negative lead to the inner circle.

4. The probe is used to measure the potential at various points on the paper. Make certain that the digital multimeter is set to measure DC voltages. Once this set up is verified (you may want to check with your lab instructor), plug in the leads to the DC power supply.

5. Check to make sure there is contact between the aluminum and the banana jack by placing the probe on the outer ring and verifying its potential difference is 12 V. If you get some other voltage please see your lab instructor. Confirm the zero of the centimeter scale along the measurement channels is placed at the center of the two circles.

6. Choose one of the three channels and starting at the outer ring, measure and record the potential $V$ at intervals of 0.50 cm. The first point will be $r = 7.0$ cm from the middle.

7. Determine $\delta r$ from the precision of the scale attached to the apparatus.

8. Determine the fractional uncertainty ($\delta r / r$) for this measurement and record this in your data table.

9. Determine $\delta V$ from the precision of the voltmeter.

10. Determine the fractional uncertainty ($\delta V / V$) for this measurement and record this in your data table.

11. Repeat steps 6–10 at intervals of 0.50 cm for points between the two rings until you get a reading for the last point at 1 cm from the inner ring.

12. The potential should depend only upon $r$. To help verify this, repeat steps 6–11 for the two remaining channels.

13. Average your values of $V$ which you have obtained for each value of $r$ from the three channels to get a single value of $V_{avg}$ for each value of radius.

14. Calculate the value of the electric field $E(r)$ over each 0.50 cm interval. This can be found by evaluating the quantity

$$E(r) = -\frac{\Delta V}{\Delta r}$$

$\Delta V$ is the potential difference between consecutive measurements of $V_{avg}$ and $\Delta r$ is the distance between consecutive measurement, 0.50 cm.

$$\Delta V = V_j - V_i$$

Example: $\Delta V_1 = V_2 - V_1$, $\Delta V_2 = V_3 - V_2$, $\Delta V_3 = V_4 - V_3$, etc.

15. It is reasonable to assume that this value of $E(r)$ is at the midpoint of the interval, so we must calculate that value which is just the average radius of two consecutive measurements $r_{mid}$.

$$r_{mid} = \frac{r_i + r_j}{2}$$

Example: $r_{mid_1} = \frac{r_2 + r_1}{2}$, $r_{mid_2} = \frac{r_4 + r_2}{2}$ etc.

16. Calculate the $\ln(E(r))$ and corresponding $\ln(r_{mid})$ for a total of 12 paired values.

17. Plot $\ln(E(r) / E_o)$ on the y-axis as a function of $\ln(r_{mid} / r_0)$ on the x-axis. Remember $E_o = 1$ V/m and $r_0 = 1$ m.

18. Find the slope of the line, $(-n)$.  

3
Part II - Determining Uncertainties in Your Final Values

In the results section of your notebook, state the result of your experiment in the form \( n \pm \delta n \). Note, \( \delta n \) should be equal to the largest fractional uncertainty from your values of voltage \( V \) and radius \( r \).

\[
\delta n = n \times \max \left( \frac{\delta V}{V}, \frac{\delta r}{r} \right)
\]

You should also address the following question:

1. Does your result for \( n \) in Part I agree within the uncertainties with the theoretical value of \( n = 1 \)? Be sure to clearly state the quantitative values you are comparing. If there are any large discrepancies, quantitatively comment on their possible origin. Make sure to “run some numbers”. For example, do you think that \( \delta V \) is being properly estimated by using the precision of the meter? If not, how would this numerically affect your results for \( \delta n \)?