Measurement of Earth’s Magnetic Field

Purpose:
To experimentally determine the horizontal component of the Earth’s magnetic field.

Introduction:
In the first section of chapter 20 of your textbook there is a paragraph and a drawing that illustrate the general behavior of the magnetic field generated by the Earth. We are going to measure the strength of the magnetic field here in Missoula. There are three aspects of the field that should be understood. The first is that the magnetic field does not align with the line of longitude passing through Missoula. That is to say, a compass will not point at the north geographic pole. Here in Missoula it pointed about 21° east of north in 1902. This year it is about 14.25° east of north. This angle is called magnetic declination. If you like, you can get current magnetic field information at this site: http://www.ngdc.noaa.gov/geomagmodels/struts/calcIGRFWMM. Therefore when we set up our experiment you should expect your compass to not point directly at the north wall of the lab.

The second aspect of the Earth’s magnetic field that we need to be aware of is the inclination or the angle of dip. The field does not run parallel to the surface of the Earth except in a very few places. We will be measuring only the horizontal component of the field and in order to compute the total field we must know the angle between the horizontal \((B_{eh})\) and the total field \((B_{e \text{ total}})\). The angle of inclination is currently about 70.5° here in Missoula.

The third aspect is that the strength of the Earth’s magnetic field changes over time. The field is currently changing fairly rapidly on a geological time scale. Therefore we must routinely look up the theoretical value of the total field.

Our apparatus consists of a coil of wire. We will produce a current in the wire and from the right hand rule we can determine that the direction of the magnetic field of the coil is perpendicular to the plane of the coil. If we align the plane of the coil parallel to the horizontal component of the earth’s magnetic field we can determine a relationship between the magnetic field of the coil \((B_c)\) and the horizontal component of the magnetic field of the earth \((B_{ex})\).

\[
\tan \theta = \frac{B_c}{B_{ex}}
\]

Figure 1: Horizontal Components of Magnetic Fields.

In class you have seen the equations for the magnetic field outside a long straight wire, and the magnetic field for a solenoid. For a coil of wire, as used in this laboratory, the magnitude of the magnetic field is:

\[
B_c = \frac{\mu_0 IN}{2r}
\]

\(I\) is the current in amperes, \(N\) is the number of turns in the coil, and \(r\) is the radius of the coil in meters. Using the two expressions from above we will be able to calculate \(B_c\) by changing angle \(\theta\) a known amount and determining
Laboratory Procedure:

Part I - Taking Direct Measurements

1. Measure the diameter of the coil $d$ and record this value.

2. Determine $\delta d$ from the precision of the ruler.

3. Determine the fractional uncertainty ($\delta d / d$) for this measurement and record this in your data table.

4. Calculate the radius of the coil $r$ by dividing the diameter in half.

5. Place the coil as far away from the power supply as possible. The power supplies have strong and varying magnetic fields that can interfere with the magnetic field we are establishing within the coil thus possibly corrupting our data.

6. Place the compass at the center of the coil with zero degrees of the compass aligned along the plane of the coil.

7. Rotate the coil so that the plane of the coil is parallel to the horizontal component of the earth's field (i.e. red arrow and white arrow on the compass are aligned).

8. Connect the ammeter and coil in series as the circuit diagram shows in Figure 4 using only 100 turns ($N=100$) of the coil. Make sure the voltage knob on the power supply is fully counter clockwise (0 V).
9. Turn on the power supply and slowly increase the voltage until the compass needle is deflected 20°.
10. Record the angle ($\theta_1$) and determine its uncertainty ($\delta\theta_1$) from the precision of the compass.
11. Determine the fractional uncertainty ($\delta\theta_1/\theta_1$) for this measurement and record this in your data table.
12. Record the current $I_1$ and determine $\delta I_1$ from the precision of the ammeter.
13. Determine the fractional uncertainty ($\delta I_1/I_1$) for this measurement and record this in your data table.
14. Switch the leads at the power supply and again adjust the voltage until the compass needle is deflected 20° in the opposite direction. We take two measurements in opposing directions because the compass has friction. If you were to simply turn the current on and off several times, you would find that the needle does not return to exactly the same angle each time. By swinging the compass needle in both directions we keep it swinging more freely.
15. Record the angle ($\theta_2$) and determine its uncertainty ($\delta\theta_2$) from the precision of the compass.
16. Record $I_2$ and determine $\delta I_2$ from the precision of the ammeter.
17. Repeat steps 9–16 for $\pm 30^\circ$, $\pm 40^\circ$, $\pm 50^\circ$, $\pm 60^\circ$ and $\pm 70^\circ$.
18. You should now have recorded twelve different values of $I$ for 100 turns of the coil.
19. Repeat steps 9–16 again but this time set up the coil for 200 turns in the wire.
20. You should now have recorded twelve different values of $I$ for 200 turns of the coil ($N=200$).

21. Repeat steps 9-16 for 300 turns in the coil.

22. You should now have recorded twelve different values of $I$ for 300 turns of the coil ($N=300$).

23. Make a graph of $I$ (y-axis) as a function of tan $\theta$ (x-axis) for all turns in the coil. There will be three lines on the graph, one each for 100, 200, and 300 turns of the coil.

24. Draw a best fit line for each plot and calculate the slope ($m_N$).

25. We can now combine equation 1 and 2 to find $B_{ez}; \mu_o = 4\pi \times 10^{-7} \frac{T m}{A}$,

\[ B_{ez} = \frac{\mu_0 N m N}{2r} \]  \hspace{1cm} (3)

26. Record your answers in Tesla ($1 T = 1 \frac{N s}{C m}$).

27. Calculate the average value of $B_{ez}$ from your three measurements.

**Part II - Determining Uncertainties in Your Final Values**

In the results section of your notebook, state the result of your experiment in the form $B_{ez} \pm \delta B_{ez}$. Note, $\delta B_{ez}$ in your measurements should be equal to the largest fractional uncertainty from your values of radius $r$, angle $\theta$, or current $I$ fractional uncertainties multiplied by your value of $B_{ez}$. Example:

\[ \delta B_{ez} = B_{ez} \max \left( \frac{\delta r}{r}, \frac{\delta \theta_1}{\theta_1}, \frac{\delta \theta_2}{\theta_2}, \frac{\delta I_1}{I_1}, \frac{\delta I_2}{I_2} \right) \]

Calculate and record $B_{ez_{total}} \pm \delta B_{ez_{total}}$ ($\delta B_{ez} = \delta B_{ez_{total}}$) and compare to the theoretical value of 55 $\mu T$ in Missoula.

\[ B_{ez_{total}} = \frac{B_{ez}}{\cos 70.5^\circ} \]

If there are any large discrepancies, **quantitatively** comment on their possible origin.