Spectrum Analysis

Purpose:
To determine the wavelength of a sodium doublet and the wavelengths of the lines in the Balmer series experimentally and theoretically.

Introduction:
One of the first successful applications of the ideas of quantum mechanics to physical systems was the development of the Bohr theory for the atomic structure of Hydrogen. Using a mixture of both classical physics and quantum mechanical postulates, Bohr was able to derive a formula for the energy levels of the Hydrogen atom. Bohr’s theory confirmed an empirical formula which was already known which gives the wavelengths ($\lambda$) of the so-called Balmer Series:

$$\frac{1}{\lambda} = R \left( \frac{1}{2^2} - \frac{1}{n^2} \right)$$

where $R = 1.097 \times 10^7$ m$^{-1}$ is the Rydberg constant, and $n$ takes on the values 3, 4, 5, 6 for the four lines of the Balmer series (the visible lines).

In this lab we will use a diffraction grating to determine the wavelengths of the lines in the Balmer series experimentally, then compare them to the theoretical values predicted by the formula above. A diffraction grating consists of a great many narrow parallel slits. A diffraction grating is characterized by the spacing between adjacent slits, $d$, which is usually quite small. The interference maxima for the diffraction grating are given by

$$d \sin \theta = m \lambda \quad m = 0, 1, 2, 3, \ldots$$

where $m$ is the order number, and $\theta$ is the angle between the central axis and the interference maximum.

Laboratory Procedure:

Part I.I - Taking Direct Measurements to find $d$

1. First you will use the known value for the wavelength of the HeNe laser ($\lambda = 632.8$ nm) to determine the slit spacing for your grating. Note that you will use this value in later parts so it is important that you make careful measurements now. In using the laser be careful not to shine the laser in your own eye, or the eyes of any of your classmates. Although these lasers are not powerful enough to damage your eyes, it is an unpleasant experience to look directly into one. Be especially careful of stray reflections.

2. Set up the laser on the optical bench so that it shines through the grating and the $m = 1$ maxima are clearly visible on either side of the central maxima on the white screen as shown below.
3. Measure the distance from the grating to the screen, \( L \), and record this in your notebook.

4. Determine \( \delta L \) from the precision of the scale attached to the apparatus.

5. Determine the fractional uncertainty \( (\delta L/L) \) for this measurement and record this in your data table.

6. Measure the distance from the central spot \((m = 0)\) to the first order maxima on either side of the central maximum, \(y_1\) and \(y_2\). One way to do this is to make a light pencil mark on the screen at the location of the maxima, then remove the cardboard from the mount and lay it flat on the table while measuring the distances.

7. Determine \( \delta y_1 \) and \( \delta y_2 \) from the precision of the ruler.

8. Determine the fractional uncertainty \((\delta y_1/y_1 \text{ and } \delta y_2/y_2)\) for these measurements and record this in your data table.

9. Average your two \( y \) values.

10. Calculate \( \theta \) from the following equation:

\[
\tan \theta = \frac{y_{\text{avg}}}{L} \quad (3)
\]

11. Calculate \( d \) using Eq. (2) (Note: it is convenient to express \( d \) in nanometers).

12. Turn off the laser and move it out of the way.

**Part I.II - Taking Direct Measurements of Sodium Doublet**

1. Place the sodium lamp behind the screen so that the light goes through the slit in the center of the screen. The sodium lamp does not put out enough light to form a detectable real image on the screen so we will use the virtual image of the interference pattern to locate the maxima on the screen.

2. Look at the screen through the diffraction grating; first order maxima should be visible as in the figure below.

3. Measure the distance from the grating to the screen \( L \), and record this in your notebook.

4. Determine \( \delta L \) from the precision of the scale attached to the apparatus.

5. Determine the fractional uncertainty \( (\delta L/L) \) for this measurement and record this in your data table.

6. Measure the distance from the central slit on the screen \((m = 0)\) to the first order maxima on either side of the central maximum, \(y_1\) and \(y_2\). One partner can view the maxima from the sodium lamp and direct the other partner to make a small pencil mark on the screen at the location of the maximum for the bright yellow line.
7. Determine $\delta y_1$ and $\delta y_2$ from the precision of the ruler.

8. Determine the fractional uncertainty ($\delta y_1/y_1$ and $\delta y_2/y_2$) for these measurements and record this in your data table.

9. Average your two $y$ values.

10. Calculate $\theta$ using equation 3.

11. Use the value of $d$ found in part I.I to calculate the wavelength of the yellow sodium line ($\lambda_{\text{yellow}}$) using equation 2.

**Part I.III - Taking Direct Measurements of Hydrogen**

1. Replace the sodium lamp with the hydrogen lamp.

2. Measure the distance from the grating to the screen $L$, and record this in your notebook.

3. Determine $\delta L$ from the precision of the scale attached to the apparatus.

4. Determine the fractional uncertainty ($\delta L/L$) for this measurement and record this in your data table.

5. Measure the distance from the central slit on the screen ($m = 0$) to the first order maxima on either side of the central maximum, $y_1$ and $y_2$ of the red line $n = 3$ as shown below.

![Diagram of hydrogen spectral lines](image)

6. Determine $\delta y_1$ and $\delta y_2$ from the precision of the ruler.

7. Determine the fractional uncertainty ($\delta y_1/y_1$ and $\delta y_2/y_2$) for these measurements and record this in your data table.

8. Average your two $y$ values.

9. Calculate $\theta$ using equation 3.

10. Use the value of $d$ found in part I.I to calculate the wavelength of the red hydrogen line ($\lambda_{\text{red}}$) using equation 2.

11. Repeat steps 5–9 for the strong cyan (blue-green) line $n = 4$ ($\lambda_{\text{cyan}}$).

**Part II - Determining Uncertainties in Your Final Values**

In the results section of your notebook, state the results of parts I.II and I.III of your experiment in the form $\lambda \pm \delta \lambda$. Note, $\delta \lambda$ in your measurements should be equal to the largest fractional uncertainty from your values of length $L$ or distance ($y_1$ or $y_2$ ) fractional uncertainties multiplied by your value of $\lambda$. Example:

$$\delta \lambda = \lambda \times \max \left( \frac{\delta L}{L}, \frac{\delta y_1}{y_1}, \frac{\delta y_2}{y_2} \right)$$

You should also address the following questions for parts I.II and I.III:
1. Does your result for $\lambda$ in Part I.II agree within the uncertainty to the theoretical value of $\lambda = 589.3$ nm for the sodium doublet?

2. Does your result for $\lambda_{\text{red}}$ in Part I.III for the red line agree within the uncertainty to the theoretical value of $\lambda_{\text{red}}$? To determine the theoretical value of $\lambda_{\text{red}}$ use $n = 3$ in equation 1.

3. Does your result for $\lambda_{\text{cyan}}$ in Part I.III for the cyan line agree within the uncertainty to the theoretical value of $\lambda_{\text{cyan}}$? To determine the theoretical value of $\lambda_{\text{cyan}}$ use $n = 4$ in equation 1. Be sure to clearly state the quantitative values you are comparing. If there are any large discrepancies, quantitatively comment on their possible origin.