Purpose

To experimentally determine forces in equilibrium and compare these results to values you determine when calculating a vector sum.

Introduction

If an object is not accelerated, that is, if it is either at rest or moving in a straight line at a fixed speed, relative to a proper coordinate system, the body is said to be in equilibrium. If the body is acted upon only by concurrent forces (i.e., forces whose lines of action intersect at a point) a single condition is necessary and sufficient for equilibrium. This condition is that the vector sum of the concurrent forces must be zero. The purpose of the experiment is to see if this condition holds for a simple set of coplanar forces acting on a body in apparent equilibrium. If the vector sum of all concurrent coplanar forces is zero, then it follows that the algebraic sum of the $x$-components of all the forces must be zero and that the algebraic sum of the $y$-components of all the forces must be zero, for any given choice of $x$-axis and $y$-axis. (Coplanar forces are forces whose lines of action lie in the same plane.)

This can be expressed as a vector equation or as the equivalent component equations.

A single two-dimensional vector equation

\[ \sum \vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \cdots = \vec{0} \]

A pair of one-dimensional component equations

\[ \Sigma F_x = F_{1x} + F_{2x} + F_{3x} + \cdots = 0 \]
\[ \Sigma F_y = F_{1y} + F_{2y} + F_{3y} + \cdots = 0 \]

Note that the condition on the components is an algebraic sum, so that some of the components must be positive and some negative such that their sum equals zero. You should be familiar with the concepts of vector addition and vector components from your work in the lecture course, PHSX 205N.

The vector $\vec{F}$ itself can be expressed in terms of its magnitude and direction or equivalently in terms of its components. In either case, a two dimensional vector must be described by two values. These can be written as an ordered pair.

Magnitude and Direction Notation

\[ \vec{F} = (F, \theta) \]

Component Notation

\[ \vec{F} = (F_x, F_y) \]

Figure 1 illustrates how the magnitude and direction are related to the components of a vector relative to a given choice for the $x$-axis and $y$-axis.

\[ F = \sqrt{F_x^2 + F_y^2} \]
\[ \theta = \arctan \left( \frac{F_y}{F_x} \right) \]

Figure 1: The functions that relate the magnitude and direction of the vector to its components depend on how the angle $\theta$ is measured. The expressions here are when $\theta$ is measured with respect to the positive $x$-axis. If $\theta$ is an obtuse angle, you may need to adjust the calculated result by 180° to get the correct value from the arctangent function.

One kind of force is the weight of an object. This is simply the gravitational attractive force on the mass of the object produced by the mass of the Earth. The magnitude is just the object’s mass $m$ multiplied by the acceleration
due to gravity \( g \); and the direction of this force (weight) is downward. If the object is supported at rest by a light string, then the tension in the string must equal the weight of the object in order to balance it. The tension in the string can thus be adjusted by hanging more or less mass from it. The string will then pull with this tension on whatever the other end of the string is tied to. In this lab, the string will run over a pulley (which is assumed to be frictionless) and tied to a ring. A diagram of the apparatus (force table) used in this experiment is shown in Figure 2. It consists of a large elevated horizontal disk whose outer rim is graduated in degrees. A ring is held approximately at the center of this table by means of a pin; strings, to which weights may be attached, pass from the ring over the pulleys mounted at the edge of the disk. The angular positions around the disk of these pulleys may be adjusted.

When the ring is centered around the pin such that it is not in contact with the pin, then the only horizontal forces acting on it are the tensions from the strings. These are concurrent forces which must add to zero if the ring is at rest. It will be your task in this lab to adjust the positions of the pulleys and the masses hanging from the strings (as described below) in order to center the ring on the pin.

The weight and mass of an object are different properties and a clear distinction must be made between them. The masses of the weights used in this experiment are stamped on one side. The relationship \( F = mg \) is used to determine the weight. And because the tension \( F_r \) in each string balances with the weight \( F_o \) when equilibrium is reached, they are of equal magnitude \( F_r = F_o \). In the SI system, force is measured in newtons (N) and mass \( m \) is measured in kilograms (kg). In this room, the accepted value for the acceleration due to gravity will be used, \( g = 9.80665 \text{ m/s}^2 \)

![Diagram of Force Table and Pulleys](image)

**Figure 2: Illustrations of the Force Table.**

---

**Procedure — Part I**

**Configuration I**

1. For String 1, set a pulley at angular position \( \theta_1 = 0^\circ \). Hang a mass \( m_1 \) of 200 grams from the string.

   ![Image](image)

   Should the support hanger be included as part of the mass hanging from the string?

2. For String 2, set a pulley at angular position \( \theta_2 = 120^\circ \). Hang a total mass \( m_2 \) of 100 grams on the string.

3. For String 3 and String 4, set pulleys at angular positions \( \theta_3 = 180^\circ \) and \( \theta_4 = 270^\circ \).

4. Experimentally determine the hanging masses \( m_3 \) at \( 180^\circ \) and \( m_4 \) at \( 270^\circ \) required to balance the system with the ring centered around the pin. Be sure that when the system is in equilibrium, each string leads directly to the center of the table.
5. Determine the uncertainties in your masses $\delta m_3$ and $\delta m_4$ by adding and subtracting small amounts of mass to see how much can be added or subtracted while still keeping the system in equilibrium. Only do this for $m_3$ and $m_4$. Assume that $m_1$ and $m_2$ are exact values.

6. Determine the fractional uncertainties $\delta m_3/m_3$ and $\delta m_4/m_4$.

7. Calculate the magnitudes of the tensions ($F_1, F_2, F_3, F_4$) in each string from the hanging mass values. The tensions should be expressed in newtons.

   ![Be careful! A newton is an SI unit, and is defined in terms of other SI units. You should make sure all your measurements are expressed with the appropriate units.]

   $$[N] = \frac{[kg][m]}{[s]^2}$$

**Configuration II**

1. The angular positions and hanging masses on String 1 and String 2 should be the same as in Configuration I.

2. Remove the all the mass (including the hanger) from String 4 so it is slack with no tension pulling on the ring.

3. Adjust the angular position of String 3 and the total mass hanging from it until the system is again balanced in equilibrium. Call the new mass value and angular position $m'_3$ and $\theta'_3$. The prime (') is used here to indicate a new value that may be different from the old value.

4. Determine the uncertainty of the mass $\delta m'_3$ by adding and subtracting small amounts of mass to see how much can be added or subtracted while still keeping the system in equilibrium. Only do this for $m'_3$. Assume that $m_1$ and $m_2$ are exact values.

5. Determine the fractional uncertainty $\delta m'_3/m'_3$.

6. Calculate the magnitudes of the tensions ($F'_1, F'_2, F'_3$) in each string from the hanging mass values. The tensions should be expressed in newtons.

**Configuration III**

1. This is not a continuation of Configuration II. Remove all the masses from the strings.

2. For String A, set a pulley at angular position $\theta_a = 150^\circ$. Hang a total mass $m_a$ of 300 grams on the string.

3. For String B, set a pulley at angular position $\theta_a = 233^\circ$. Hang a total mass $m_a$ of 200 grams on the string.

4. For String C, set a pulley at angular position $\theta_c = 300^\circ$. Hang a total mass $m_c$ of 250 grams on the string.

5. Adjust the angular position $\theta_d$ of String D and the total mass $m_d$ hanging from it until the system is again balanced in equilibrium.

6. Determine the uncertainty of the mass $\delta m_d$ by adding and subtracting small amounts of mass to see how much can be added or subtracted while still keeping the system in equilibrium. Only do this for $m_d$. Assume that $m_a$, $m_b$, and $m_c$ are exact values.

7. Determine the fractional uncertainty $\delta m_d/m_d$.

8. Calculate the magnitudes of the tensions ($F'_a, F'_b, F'_c, F'_d$) in each string from the hanging mass values. The tensions should be expressed in newtons.

**Procedure — Part II**

In this section, you will calculate the theoretical values for each configuration with which to compare your experimental results from the previous section. Do not use your experimental values from the previous Part in any of the following calculations.
Configuration I

Four forces acting on the central ring are expressed in terms of their magnitudes and directions: $\vec{F}_1 = (1.96 \text{ N}, 0^\circ)$, $\vec{F}_2 = (0.980 \text{ N}, 120^\circ)$, $\vec{F}_3 = (F_3, 180^\circ)$, and $\vec{F}_4 = (F_4, 270^\circ)$. Calculate the values of $F_3$ and $F_4$ that will bring the system into equilibrium.

Remember, if the system is in equilibrium, the sum of the $x$-components must add up to zero newtons and the sum of the $y$-components must add up to zero newtons. For example, $\vec{F}_3$ only has a component in the negative $x$-direction, thus the $x$-component of the force is expressible as $F_{3x} = -F_3$ and the $y$-component of is expressible as $F_{3y} = 0 \text{ N}$.

Be careful! Even though the entire force vector lies along the $x$-axis, the magnitude of the vector is not conceptually the same as the component of the vector. In general, $F_{ix} \neq F_i$, unless you can demonstrate otherwise; so pay attention to your subscripts.

Configuration II

Three forces acting on the central ring are expressed in terms of their magnitudes and directions: $\vec{F}_1 = (1.96 \text{ N}, 0^\circ)$, $\vec{F}_2 = (0.980 \text{ N}, 120^\circ)$, and $\vec{F}_3' = (F_3', \theta_3')$. Calculate the values of $F_3'$ and $\theta_3'$ that will bring the system into equilibrium.

Make sure you clearly define the axis and your reference angle for the direction of $\vec{F}_3'$.

Configuration III

Four forces acting on the central ring are expressed in terms of their magnitudes and directions: $\vec{F}_a = (2.94 \text{ N}, 150^\circ)$, $\vec{F}_b = (1.96 \text{ N}, 233^\circ)$, $\vec{F}_c = (2.45 \text{ N}, 300^\circ)$, and $\vec{F}_o = (F_o, \theta_o)$. Calculate the values of $F_o$ and $\theta_o$ that will bring the system into equilibrium.

Determining Uncertainties in Your Final Values

In the results section of your notebook, state the results of all three configurations of the experimental procedure in the form $F_i \pm \delta F_i$. Each $\delta F_i$ should be equal to the fractional uncertainty of the mass used to find $F_i$, multiplied by the value of $F_i$.

$$\delta F_i = F_i \times \max \left( \frac{\delta m_i}{m_i} \right)$$

Also state the results from all three configurations of your theoretical calculations. There is no uncertainty in the values from the Theoretical Procedure.

You should also address the following questions:

1. Include a free body diagram for the ring for each configuration, indicating which values were given and which values were experimentally determined. Use $0^\circ$ as the positive $x$-axis and $90^\circ$ as the positive $y$-axis.

2. Compare the experimental and theoretical values for each configuration, clearly stating the quantitative values you are comparing. Do your results for the forces you are comparing agree within their uncertainties? If not, what might be some possible reasons? Be sure to back up your claims with quantitative estimates. You should “run some numbers” and state whether or not you think your scenario(s) are reasonable, and why.