Purpose

To investigate conservation of momentum and kinetic energy in elastic and inelastic collisions in one dimension.

Introduction

When two masses collide with each other, the total momentum of both masses is conserved, regardless of the type of collision, whereas the total kinetic energy is only conserved in an elastic collision. The purpose of this experiment is to investigate the motion before and after a collision in order to test the conservation of momentum and the conservation of kinetic energy.

The linear momentum of an object is the product of its mass multiplied by its velocity \( \mathbf{p} = m \mathbf{v} \). The law of conservation of linear momentum states that the total momentum of a system remains constant if there is no net external force. For a collision between two masses, \( m_1 \) and \( m_2 \), this is expressed as the following equation:

\[
m_1 \mathbf{v}_{1i} + m_2 \mathbf{v}_{2i} = m_1 \mathbf{v}_{1f} + m_2 \mathbf{v}_{2f}
\]

where \( \mathbf{v}_{1i} \), \( \mathbf{v}_{2i} \) and \( \mathbf{v}_{1f} \), \( \mathbf{v}_{2f} \) are the velocities of the masses before and after the collision, respectively.

The kinetic energy of an object is \( \text{KE} = \frac{1}{2} m v^2 \). The law of conservation of energy states that the total energy of a system is always conserved. This does not mean that kinetic energy is always conserved, for in some instances (like inelastic collisions where two objects stick together) kinetic energy is converted into energy of another form (e.g. sound, heat, physical deformation). In perfectly elastic collisions there are no energy transformations, and the total kinetic energy of the system is conserved. For such a collision between two masses this is expressed as

\[
\frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2
\]

This equation holds true for elastic collisions only. Kinetic energy will not be conserved in an inelastic collision; it is converted into other forms of energy.

Procedure — Part I

We will be using an air track apparatus, two photogates, and the aid of a computer to calculate the velocities of two gliders before and after they collide. From the measured velocities and masses of the gliders we can test Equations 1 and 2 for different types of collisions.

\[\text{Figure 1: Air track with two gliders on it}\]
1. Be sure your 3 gliders have the appropriate velcro connections. The two large gliders should be able to connect to each other. The small glider should be able to connect to only one of the large gliders.

   It will be useful to be able to identify the gliders by name. Let Glider A be the large glider that can connect to the small glider. Let Glider B be the large glider that cannot connect to the small glider. Let Glider C be the small glider.

2. Do the following steps 3 times, once for each glider.
   
   (a) Measure the mass \( m \) of the glider.
   (b) Determine the uncertainty \( \delta m \) in the mass measurement of the glider.
   (c) Calculate the fractional uncertainty \( \delta m/m \) of the mass measurement of the glider

3. Level the air track by setting a cart in the middle of the track, turning on the air, and seeing which way the cart glides. Adjust the leveling screw at the end of the track to raise or lower that end until the cart remains at rest.

4. Position the two photogates just far enough apart so the collision can take place between the photogates. Adjust the height of the photogates so the metal prongs on top of each glider can interrupt the infrared beams of photogates. If you slide a glider through a photogate, you should see the red LED on the photogate quickly turn off and on twice (once for each prong).

5. Open the Collisions program by double clicking the icon (cyan square with the symbol \( \vec{p} \)) on the desktop. When the program is open, there will be five tabs with the headings Collision 1(a), Collisions 1(b), Collision 2, Collision 3, and Determining Friction. In each, there are display boxes that will give a reading of speed in m/s.

   The white run arrow icon by the menu bar has the functionality as with the program used for the experiment you performed when determining the magnitude of the acceleration due to gravity. Clicking the arrow will turn it black and the program will be ready to acquire data.

   If the photogates do not detect the correct number of passes (depends on which collision you are doing), the program will not complete the timing cycle and velocities will not display automatically. This likely means that you are performing the collision incorrectly. Stop the program, reread the instructions and try again.

   If you are getting “inf” for all your values, then you may need to restart the computer.

6. By far the largest source of error in the experiment is in the final speed after the collision. This is because while the carts are in contact with each other, there are small vertical forces which increase the frictional force and cause losses of both momentum and kinetic energy. We would therefore be underestimating our uncertainty in the speed if we just relied on the precision of the timer.

7. Select the Determining Friction tab.

8. Do the following steps 3 times, once for each glider.

   (a) Do the following steps 3 times with the glider you have selected.
      
      i. Click on the white run arrow icon by the menu bar. The icon will turn into a black run arrow and the values should reset to zero meters per second.
      
      ii. Starting on the far left of the air track, send the glider along the entire length of the track passing both photogates, then let it bounce off the end of the track and rebound all the way back to its starting position.

      This experiment tends to work well if you can give the glider a speed between 0.4 m/s and 0.8 m/s. This is because if your air pressure is a little too low, then friction slows the gliders more noticeably with slow speeds. If the the gliders are moving too fast, they bounce around more when they collides.

      Feel free to practice pushing the gliders at a few times!
iii. Record the initial speed $v_i$ and final speed $v_f$ of the glider.

iv. Calculate the fractional uncertainty $\delta v/v$ in the speed by taking the difference in initial and final speeds divided by the initial speed.

$$\frac{\delta v}{v} = \frac{v_i - v_f}{v_i}$$  \hspace{1cm} (3)

If the collision at the end of the track were perfect (and there were no friction), the initial and final speeds would be identical. The extent to which they are not identical gives a measure of the uncertainty in final velocities.

(b) Average the three fractional uncertainties for the glider you have selected.

$$\left( \frac{\delta v}{v} \right)_{\text{avg}} = \frac{1}{3} \left[ \left( \frac{\delta v}{v} \right)_{\text{trial 1}} + \left( \frac{\delta v}{v} \right)_{\text{trial 2}} + \left( \frac{\delta v}{v} \right)_{\text{trial 3}} \right]$$  \hspace{1cm} (4)

(c) Confirm the fractional uncertainty in the speed is larger than the fractional uncertainty in the mass for the glider you have selected.

9. You should now have 3 averaged fractional uncertainties, one for each glider.

Why did you have to find fractional uncertainties of the speeds, then average those values? Would averaging the speeds, then finding the fractional uncertainty give the same result?

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**Procedure — Part II**

Use glider A for glider 1. The identity of glider 2 will change between glider B and glider C, depending on the collision you are performing.

If glider 2 bounces off the end of the air track, you may need to catch it so that it does not interfere with any data collection no yet performed on glider 1.

**Collision Type #1: Elastic Collision with a Stationary Target**

- Case 1: $m_1 = m_2$
- Case 2: $m_1 > m_2$

1. Do the following steps 2 times. Once for Case 1, and once for Case 2.

   a. Selected the correct tab in the program: **Collision 1(a)** for Case 1, **Collision 1(b)** for Case 2.

   b. Place glider 1 at rest to the left of the photogates.

   c. Place glider 2 at rest between the photogates, $v_{2i} = 0$ m/s.

      If the metal bumpers on the gliders are facing each other.

   d. Click on the white run arrow icon. It will turn black and the display boxes will reset to zero m/s.

   e. Push glider 1 so that it will collide with glider 2.

   f. Record the initial speeds $v_{1i}$, $v_{2i}$ and final speeds $v_{1f}$, $v_{2f}$ of both gliders.

**Collision Type #2: Elastic Collision with a Moving Target**

- Case 1: $m_1 = m_2$
- Case 2: $m_1 > m_2$

1. Do the following steps 2 times. Once for Case 1, and once for Case 2.

   a. Selected the correct tab in the program: **Collision 2**.
(b) Place glider 1 at rest to the left of the photogates.
(c) Place glider 2 at rest between glider 1 and the leftmost photogate.
   Be sure the metal bumpers on the gliders are facing each other.
(d) Click on the white run arrow icon. It will turn black and the display boxes will reset to zero m/s.
(e) Push glider 2 away from glider 1; then push glider 1 after glider 2 at a larger speed. They need to both pass
   through the left photogate, collide between the photogates, then both pass through the right photogate.
   See Figure 2 for a graphical representation of the sequence of events.
(f) Record the initial speeds \( v_{1i} \), \( v_{2i} \) and final speeds \( v_{1f} \), \( v_{2f} \) of both gliders.

\[ m_1 \quad m_2 \]
\[ m_1 \quad m_2 \] \( m_2 \) passes the first photogate.
\[ m_1 \quad m_2 \] \( m_1 \) passes the first photogate.
\[ m_1 \quad m_2 \] The gliders collide.
\[ m_2 \] passes the second photogate.
\[ m_1 \quad m_2 \] Catch \( m_2 \) so it doesn’t bounce back.
\[ m_1 \quad m_2 \] \( m_1 \) passes the second photogate.

Figure 2: Sequence of events for Collision Type #2. Both gliders begin to the left of the first
photogate. Catching \( m_2 \) after it passes through the second photogate prevents it from
bouncing back through the photogate before \( m_1 \) can pass through the photogate. Although
the case shown is Case 2, the sequence of events is the same for Case 1.

Collision Type #3:  \textit{Inelastic Collision with a Stationary Target}

- Case 1: \( m_1 = m_2 \)
- Case 2: \( m_1 > m_2 \)

1. Do the following steps 2 times. Once for Case 1, and once for Case 2.
   (a) Selected the correct tab in the program: Collision 3.
   (b) Place glider 1 at rest to the left of the photogates.
   (c) Place glider 2 at rest between the photogates, \( v_{2i} = 0 \) m/s.
      Be sure the velcro connections on the gliders are facing each other.
(d) Click on the white run arrow icon. It will turn black and the display boxes will reset to zero m/s.
(e) Push glider 1 so that it will collide with glider 2.
(f) Record the initial speeds \(v_{1i}, v_{2i}\) and final speeds \(v_{1f}, v_{2f}\) of both gliders.

**Procedure — Part III**

1. Do the following steps 6 times. Once for each case of each collision you performed.
   (a) Calculate the initial momentum \(p_{1i}\) of glider 1.
   (b) Calculate the initial momentum \(p_{2i}\) of glider 2.
   (c) Calculate the final momentum \(p_{1f}\) of glider 1.
   (d) Calculate the final momentum \(p_{2f}\) of glider 2.
   (e) Calculate the total initial momentum \(p_i\) of the system.
   (f) Calculate the total final momentum \(p_f\) of the system.
   (g) Calculate the initial kinetic energy \(KE_{1i}\) of glider 1.
   (h) Calculate the initial kinetic energy \(KE_{2i}\) of glider 2.
   (i) Calculate the final kinetic energy \(KE_{1f}\) of glider 1.
   (j) Calculate the final kinetic energy \(KE_{2f}\) of glider 2.
   (k) Calculate the total initial kinetic energy \(KE_i\) of the system.
   (l) Calculate the total final kinetic energy \(KE_f\) of the system.

**Determining Uncertainties in Your Final Values**

In the results section of your notebook, state the results of all your experimental final values in the form \(p_f \pm \delta p_f\) and \(KE_f \pm \delta KE_f\). Note, \(\delta p_f\) and \(\delta KE_f\) should be equal to the largest fractional uncertainty you obtained for speeds of the gliders involved in the collision multiplied by your final value of \(KE_f\) and \(p_f\).

\[
\delta p_f = p_f \times \max \left( \frac{\delta v}{v}_{\text{glider } 1 \text{ avg}}, \frac{\delta v}{v}_{\text{glider } 2 \text{ avg}} \right)
\]

\[
\delta KE_f = KE_f \times \max \left( \frac{\delta v}{v}_{\text{glider } 1 \text{ avg}}, \frac{\delta v}{v}_{\text{glider } 2 \text{ avg}} \right)
\]

You should also address the following questions:

1. Do your results for the final values agree within their uncertainties to the initial values?
2. Are momentum and kinetic energy conserved in each collision?