An Empirical Taxonomy of Problem Posing Processes

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Abstract: This article focuses on the construction, description and testing of a theoretical model of problem posing. We operationalize processess that are frequently described in problem solving and problem posing literature inorder to generate a model. We name these processes **editing** quantitative information, their meanings or relationships, **selecting** quantitative information, **comprehending** and organizing quantitative information by giving it meaning or creating relations between provided information, and **translating** quantitative information from one form to another. The validity and the applicability of the model is empirically tested using five problem-posing tests with 143 6th grade students in Cyprus. The analysis shows that three different categories of students can be identified. Category 1 students are able to respond only to the comprehension tasks. Category 2 students are able to respond to both the comprehension and translation tasks, while Category 3 students are able to respond to all types of tasks. The results of the study also show that students are more successful in first posing problems that involve comprehending processes, then translation processes and finally editing and selecting processes.

Kurzreferat:.

ZDM-Classifikation:

1. Introduction

Problem posing is an important aspect of both pure and applied mathematics and an integral part of modelling cycles which require the mathematical idealization of real world phenomenon. Scientists are continuously posing problems that, if solved, advance the current state of knowledge in their fields (Mestre, 2002). For example, the advancement of mathematics requires creative imagination, which is the result of raising new questions, new possibilities, and viewing old questions from a new angle (Ellerton & Clarkson, 1996). For this reason, problem posing and problem solving have been identified to be central themes in mathematics education. Recent recommendations for the reform in mathematics education suggest the inclusion in instruction of problem posing i.e., of activities in which students generate their own problems in addition to solving pre-formulated problems (English, 1997a; NCTM, 2000; Silver & Cai, 1996). Given the importance of problem-posing activities in school mathematics, some researchers started to investigate various aspects of problem-posing processes (e.g., Silver, 1994; English, 1998; English, 2003). One important direction for such investigation is to examine thinking processes related to problem posing (Brown & Walter, 1990). Recently Root-Bernstein (2003) proposed the concept of nepistemology, i.e., the negation of epistemology and its relevance for understanding how innovative individuals engage in problem posing (generation) and evaluation in order to get a grasp into the unknown. In mathematics, the classical example is that of mathematicians classifying problems as P or NP. Simplistically stated, class P problems are those which are solvable on a deterministic sequential machine in polynomial time. The class NP consists of problems whose solutions can be verified in polynomial time on a non-deterministic machine. Today, the biggest open question in theoretical computer science is whether P = NP. Although the solution of this problem has numerous implications for computer science, the crucial point that Root-Bernstein (2003) makes for education is the lack of our focus on trying the understand processes that constitute problem posing, an important thinking trait of highly innovative individuals.

The present study adds to the research literature on problem posing in a number of ways. First, it investigates students' processes in problem posing by proposing a model that encompasses most of the previous research in the area. Second, the study provides researchers and teachers with a starting point in better understanding the nature of processes which underlie students' problem posing behavior. We begin by reviewing the research that has a bearing on this study, and then discusses four processes that seem to describe students' patterns of thinking on different types of problem posing tasks. These four processes are operationalized via the construction of a theoretical model whose robustness is empirically tested. Finally, the results and the conclusions of the study are presented and discussed. The rationale for choosing a quantitative design is discussed further under the methodology section.

Theoretical Considerations

In this section we describe three strands of research studies on problem posing in mathematics instruction. The first strand of research discusses the effectiveness of

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problem posing on students' understanding of mathematical concepts; the second strand describes the development of students' problem posing processes and abilities, and the third one refers to the classification of problem posing tasks.

Problem Posing and Problem Solving

Several studies have reported approaches to incorporate problem posing in instruction. These studies provided evidence that problem posing has a positive influence on students' ability to solve word problems (Leung, 1996; Silver, 1994), andprovided a chance to gain insight into students' understanding of mathematical concepts and processes (English, 1997a; English, 2003). It was found that students' experience with problem posing enhances their perception of the subject, and produces excitement and motivation (English, 1998; Mestre, 2002; Silver, 1994; Winograd, 1991). Specifically, English (1997a, 1998) asserted that problem posing improves students' thinking, problem solving skills, attitudes and confidence in mathematics and mathematical problem solving, and contributes to a broader understanding of mathematical concepts. Kilpatrick (1987) provided the theoretical argument that the quality of the problems that students pose serve as a prediction variable of how well they can solve problems. This theoretical argument provided the direction for further studies that probed the links between problem solving and problem posing. Cai (1998), for example, found a strong relation between problem posing and problem solving, while Silver and Cai (1996) showed that students' problem solving performance was highly correlated with their problem posing performance. More recently, one important direction for investigation is probing the links not only between problem solving and problem posing but also between problem posing and mathematical competence in general. For example, Mestre (2002), using problem posing as a tool for studying cognitive processes, asserted that problem posing can be used to investigate the transfer of concepts across contexts, and to identify students' knowledge, reasoning, and conceptual development. Nevertheless, despite the importance of problem posing and its contribution to conceptual understanding of mathematical ideas, little is known about the nature of the underlying thinking processes that constitute problem posing, and the schemes through which students' mathematical problem posing can be analysed and assessed.

Problem Posing Abilities and Processes

The second strand of the research refers to the development of students' problem posing abilities and processes. According to English (1997a), there is a lack of research studies on students' abilities to create their own problems in numerical or non-numerical contexts. To this end, English (1997a, 1997b, 1998) investigated students' abilities in generating problems in three studies involving third, fifth and seventh graders. English (1998) found that third graders were only able to create several change/part-part-whole problems by altering the contexts of the original problems and by focusing on the operational and not the semantic structure of the problems. The understanding of the semantic structure of problems seems to be the result of instruction. English (1997a, 1997b) found that fifth and seventh graders, who participated in specific programs on problem posing, exhibited greater facility in creating solvable problems than their counterparts that did not participate. Most of the students in the programs created quite sophisticated problems using semantic relations in their problems. Specifically, she found that, through these programs, fifth graders improved their abilities to model a new problem on an existing structure and to diversify the story context of the problem. She also found that fifth and seventh graders developed their abilities to perceive the problem structure as independent of a particular context, providing them with greater flexibility in their problem creations.

Moreover, Silver and Cai (1996) conducted a study in which a large number of sixth and seventh grade students were asked to pose questions to given story problems. Silver and Cai classified the problems in terms of mathematical solvability, linguistic complexity and mathematical complexity and the responses were also examined for the presence of previously defined semantic structural relations (Marshall, 1995). It was found that most students in Silver and Cai's study were able to pose appropriate mathematical questions when presented with a story situation as a stimulus for question generation and to generate syntactically and semantically complex mathematical problems. Most important was the finding that students who generated questions to story problems used the process of association, i.e., many students appeared to pose their second and third responses by using their first response as a cue. The process of association was also obvious in students' posed problems, since they tended to generate related problems that moved from simpler to more complicated related problems. However, the study did not allow for definitive analyses of the thinking of students and the processes they used as they generated their problem sequences.

Classification of Problem Posing Tasks

The third strand of research discusses the classification of problem posing tasks. Although there is a wide variety of problem posing tasks (Silver & Cai, 1996), research so far indicates only a few ways to classify them. Stoyanova (1998) identified three categories of problem posing experiences that increase students' awareness of different situations to generate and solve mathematical problems: (a) free situations, (b) semi-structured situations, and (c) structured problem-posing situations. In free situations students pose problems without any restriction. Lowrie's (1999) tasks, in which students are encouraged to write problems for friends to solve or write problems for mathematical Olympiads, exemplify the free problem posing situation. Semi-structured problem posing situations refer to ones in which students are provided with open-ended problems or are asked to write problems that are similar to given problems or to write problems based on specific pictures and diagrams. Structured problem posing situations refer to situations where students pose problems by reformulating already solved problems or by varying the conditions or questions of given problems.

Silver (1995) classified problem posing according to whether it takes place before (presolution), during (within-solution) or after problem solving (post-solution). He argued that problem posing could occur (a) prior to problem solving when problems are being generated from a particular stimulus such as a story, a picture, a diagram, a representation, etc., (b) during problem solving when an individual intentionally changes the problem's goals and conditions, such as in the cases of using the strategy of "making it simpler", (c) after solving a problem when experiences from the problem solving context are applied to new situations. Thus, Silver considers problem posing as closely related to problem solving and his classification assumes that problem posing is an important companion to problem solving.

Stoyanova (1998) and Silver (1995) classified problem posing tasks in terms of the situations and experiences which provide opportunities for students to engage in mathematical activity. Both classifications involve five categories of problem posing tasks, used throughout the studies reviewed so far: Tasks that merely require students to pose (a) a problem in general (free situations), (b) a problem with a given answer, (c) a problem that contains certain information, (d) questions for a problem situation, and (e) a problem that fits a given calculation.

It is acknowledged that there are a variety of ways to analyze problem posing tasks and each may give a different understanding of the process. However, there is a need for a framework that can be used on responses from a wide range of tasks and from different age groups so that inter-task study and development of problem posing behaviour can be investigated. The model proposed in the present study synthesizes most of the ideas articulated in previous studies, including a classification scheme of the underlying processes. The focus of the proposed model is on the processes that students use in order to pose their own two-step addition and subtraction problems, but the model can be applied to many other areas of mathematics such as problem solving in algebra, geometry, and measurement. The decision to base the model on numerical situations is justified by prior research in this specific area (English, 1997a, 1998; Silver, 1994). We build on and expand on this previous work on numerical problem posing.

The Proposed Model

Notwithstanding the extent of research into students' thinking in problem posing (English, 1998; Silver & Cai, 1996), recent research has not investigated systematically the quantitative information of the problem posing tasks in combination with the thinking processes used in each task. Accordingly, the literature does not provide the kind of coherent picture of students' problem posing thinking that is desirable for current approaches to instruction. Based on the reviewed literature, we propose a model, which enables young students' problem posing thinking to be described by four processes. As highlighted in Figure 1 (in the next section of the paper), the processes that are postulated to occur when a person engages in problem posing are **editing** quantitative information, **selecting** quantitative information, **comprehending** and organizing quantitative information, and **translating** quantitative information from one form to another. We speculate that these basic thinking processes correspond to specific problem solving tasks presented in iconic, tabular or symbolic form. It is possible for a process to correspond to more than one task, but for clarity and simplicity, we incorporate in the model the most prominent process for each task. Editing quantitative information is mostly associated with tasks that require students to pose a problem without any restriction from provided information, stories or prompts (Mamona-Downs, 1993). Selecting quantitative information is associated with tasks that require students to pose problems or questions that are appropriate to specific, given answers. The given answer functions as a restriction, making selecting more difficult than editing, because students need to mainly focus on the structural context and the relations between the provided information (English, 1998). Comprehending quantitative information refer to tasks in which students pose problems from given mathematical equations or calculations. Comprehending requires understanding the meaning of the operations and students usually follow an algorithmic process focusing on the operational and not the semantic structure of the problems (English, 1998; Silver & Cai, 1996). Translating quantitative information requires students to pose appropriate problems or questions from graphs, diagrams or tables. Theoretically, the application of translating is more demanding than comprehending since it requires the understanding of the different representations of the mathematical relations.

In order to capture the nature of problem posing, our model incorporates forms of semi-structured and structured situations (Stoyanova, 1998) in which students are asked to generate problems from a presented stimulus (pre-solution phase). The stimulus situations involve quantitative information of tasks, which contain representations either in iconic or symbolic form. For example, students posing problems based on a picture are handling information in iconic form. Similarly, students are handling quantitative information in iconic form if they are given graphs and diagrams. Students posing problems based on words or phrases or calculations are handling quantitative information in symbolic form. Examples of the tasks that correspond to each cognitive process are shown in Table 1.

Table 1
Task Corresponding to Each Process

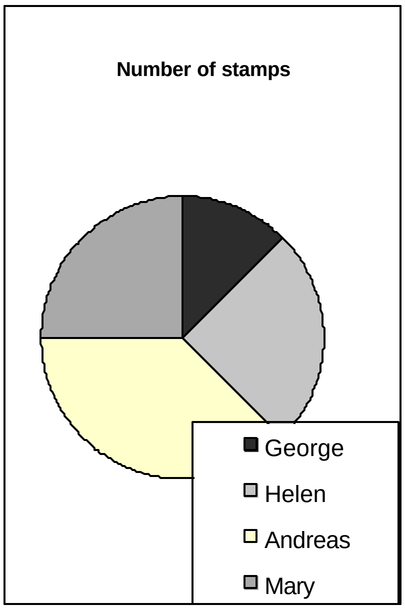
Tasks (*PROCESS*)

SELECTING

Write a question to the following story so that the answer to the problem is "385 pencils".
"Alex has 180 pencils while Chris has 25 pencils more than Alex".
Write a question to the following story so that the answer to the problem is "75 pounds".
"Jason had 150 pounds. His mother gave him some more. After buying a book for 25 pounds he had 200 pounds."

TRANSLATING

Write a problem based on the following diagram whose solution would require on addition and one subtraction:



Write a problem based on the following table whose solution would require one addition and one subtraction:

Children	Bank savings
John	340
Helen	120
Joanne	220
Andrews	110
George	280

COMPREHENDING

Write an appropriate problem for the following:

(2300+1100)-790=n
5100–(2400+780)=n

EDITING

Write a problem based on the following story:

In 1492 A.D. Columbus started his long journey to India. In his first ship, Santa Maria, he had 250 kg of meat, 600 kg of flour and 1200 kg of potatoes. Unfortunately, due to an accident, 245 kg of potatoes were damaged. In his second boat, Pinta, he had 300 kg more meat than in Santa Maria. Columbus did the greatest discovery in the history; He discovered America!

Write a problem based on the following picture:

The purpose of the present study is twofold: First, to validate the proposed model, i.e., to confirm that problem posing consists of the proposed cognitive processes, and second to search for a possible developmental trend in students' abilities to pose problems based on the editing, selecting, comprehending, and translating cognitive processes and to find out meaningful differences in students' processes in generating problems.

Methodology

The sample for this study consisted of 143 Grade 6 students from six classes at elementary schools in an urban district in Cyprus. Seventy-nine students were males and 64 females. The school sample is representative of a broad spectrum of socioeconomic backgrounds. Prior to the start of this study, none of the students had been exposed to problem posing instruction.

Instruments

Each student completed five problem posing tests, which contained situations that help students to perceive mathematical context in diverse ways. Test 1 consisted of 4 tasks in which students were required to complete problems with the missing question in such a way as to fit the provided answer. Test 2 involved 3 tasks, which required from students to write problems that fit to given equations. Test 3 consisted of 3 tasks, which presented three pictures with mathematical information. Students were asked to use information from the pictures to write problems whose solutions would require specific operations, i.e., two additions or one addition and one subtraction. Test 4 had the same structure as Test 3 but the mathematical information was presented in tabular form. In Test 5 students had to pose problems based on interesting stories. The first story referred to a daily TV serial, while the second one described how America was discovered by Columbus. For these 15 tasks, students were required not only to pose questions or problems but also to justify their answers by writing the mathematical solutions of the constructed problems or the mathematical equation, which corresponds to their own problems

The tests were administered to students by researchers in five 20-minute sessions during mathematics class. Prior to the administration of the test, which lasted 10 working days, one researcher visited the classes involved in the project and worked with the students on problem posing for approximately 40 minutes.

The purpose of choosing a quantitative design for this study was four-fold. First, unlike qualitative studies, quantitative studies are more easily generalizable provided researchers in other geographic locations maintain instrumentation consistencies. Second, we find that numerous qualitative studies on problem-solving and problem posing often use descriptors for underlying thinking processes that are neologisms of one another, yet readers are uncertain about the applicability of these descriptors beyond the particular qualitative study from which they were generated. Third, our purpose was to complement the work of existing qualitative studies by introducing a model derived via a quantitative methodology with clearly operationalized variables that could be applied to code data in qualitative studies that attempted to describe thinking processes in similar problem situations. Fourth, our present study was an attempt to empirically verify the findings reported in numerous qualitative studies such as those summarized in the literature review.

Data Analysis

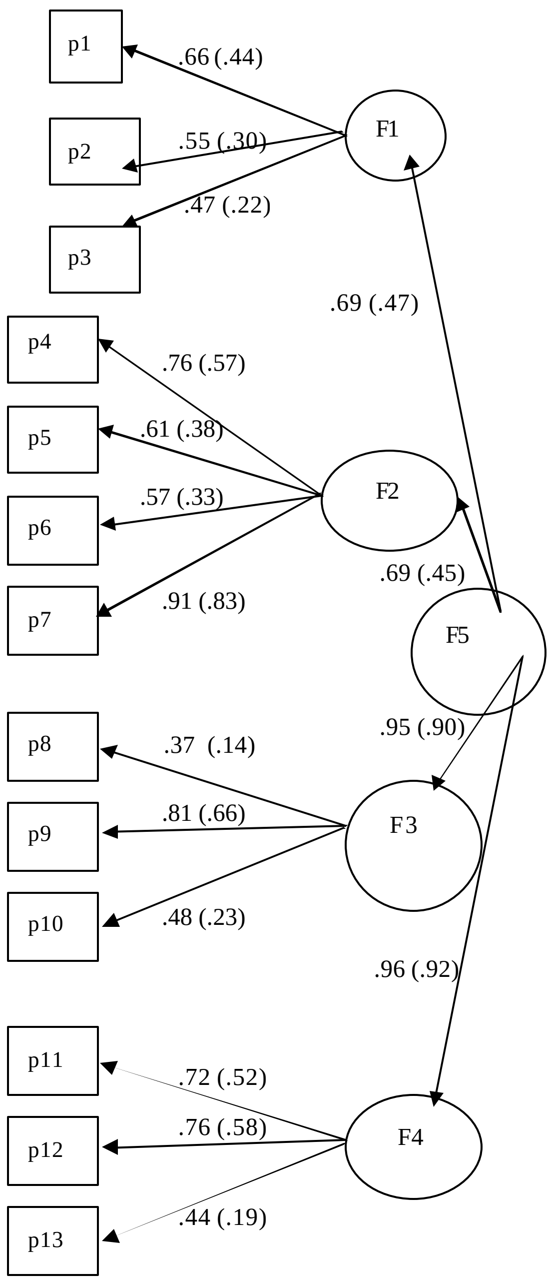
The goals of the analysis were first to estimate the relative strength of the proposed model and second to trace the developmental trend of students' abilities in problem posing. Because we proposed a theoretically driven model about the components of problem posing cognitive processes, our first interest was in the assessment of fit of the hypothesized a priori model to the data. The assessment of the proposed model was based on confirmatory factor analysis, which is part of a more general class of approaches called structural equation modeling. Confirmatory factor analysis is used to test measurement models in which observed variables define latent constructs or latent variables. After establishing that the measurement model was valid, we tested the structural models to examine relationships among constructs. Essentially, the measurement model provided an assessment of convergent and discriminant validity, and the structural model provided an assessment of the predictive validity (Anderson & Gerbing, 1988).

One of the most widely used structural equation modeling computer programs, MPLUS (Muthen & Muthen, 2004), was used to test for model fitting in this study. In order to evaluate model fit, three fit indices were computed: The chi-square to its degree of freedom ratio (x^2/df), the comparative fit index (CFI), and the root mean-square error of approximation (RMSEA). These three indices recognized that the following needed to hold true in order to support model fit (Marcoulides & Schumacker, 1996): The observed values for ?^{2}/df should be less than 2, the values for CFI should be higher than .9, and the RMSEA values should be close to or lower than .08.

Testing of an a-priori general structure model

In this study, we posited an a-priori (initial) structure and tested the ability of a solution based on this structure to fit the data. In the present study, the a priori model consists of four first-order factors and one second-order factor. The four first-order factors represent the cognitive processes: the comprehending (F1), the translating (F2), the editing (F3), and the selecting (F4). The editing, the selecting and the comprehending factors were measured by three tasks each, while translating was measured by four tasks. F1, F2, F3, and F4 were hypothesized to construct the second order factor (F5) "problem posing processes", which was hypothesized to account for any correlation or covariance between the first order factors (see Figure 1). Figure 1 makes easy the conceptualisation of how the various components of problem posing cognitive processes relate to each other. The first-order factors lead to the second order factor.

The second purpose of the analysis was to identify a developmental trend between the processes of problem solving. To this end, latent profile analysis, a person-centered analytic strategy, was used to explore students' cognitive processes to different types of problem posing processes, allowing for the subsequent description of those patterns in the context of dealing with different forms of problem posing tasks.



Fit Indices: CFI=0.950, x^2/df=1.25, RMSEA=0.05
Note: F1=Comprehending, F2=Translating, F3=Editing, F4=Selecting and F5=Problem posing abilities, p1-p13 refer to the problems assigned to students.
* The first number indicates factor loading and the number in parenthesis indicates the corresponding r^2.

Figure 1: Problem Posing Processes Model

Results

In this section, we refer to the main issues of the study. First, we present the results of the analysis, establishing the validity of the latent factors and the viability of the structure of the hypothesized latent factors. Second, we present the exploration of the data for meaningful categories with respect to problem posing processes, and then working up from those categories, we present the organization of the information about students' thinking processes in problem posing. Thus, the second issue is descriptive rather than quantitative in its intent and it shows a developmental trend in students' thinking processes.

The Validation of the Model

Figure 1 presents the structural equation model with the latent variables and their indicators. The descriptive-fit measures indicated support for the hypothesized first and second order latent factors (CFI=0.950, x^2/df=1.25, RMSEA=0.05). The parameter estimates were reasonable in that all factor loadings were large and statistically significant (see Figure 1). Specifically, the analysis showed that each of the tasks employed in the present study loaded adequately on each of the four thinking processes (see the first order factors in Figure 1), indicating that selecting, editing, comprehending and translating can represent four distinct functions of students' thinking in problem posing. Furthermore, the r-squares (shown in the parentheses in Figure 1) also illustrate that modest to large amounts of variance are accounted for all tasks corresponding to each cognitive process. This means that the four thinking processes (editing, selecting, comprehending, and translating) can model the performance of students on problem posing.

The structure of the proposed model also addresses the differential predictions of the four processes for the problem posing abilities. Considering the effects among the processes reveals that the selecting and the editing cognitive processes were the primary source explaining students' abilities to generate problems (r^2=.92 and r^2=.90, respectively). The translating and comprehending processes had a moderate significant effects on students' abilities to pose problems (r^2=.45, and r^2=.47, respectively).

Categories of Students and Developmental Trend

The second aim of the study concerns the extent to which students in the sample vary according to the answers they provided in the four processes. Specifically, we examined whether there are different types of students in our sample who could reflect the selecting, the editing, the comprehending and the translating processes. Mixture growth modeling was used to answer this question (Muthen & Muthen, 2004), because it enables specification of models in which one model applies to one subset of the data, and another model applies to another set. The modeling here used a stepwise method-that is, the model was tested under the assumption that there are two, three, and four categories of subjects. The best fitting model with the smallest AIC and BIC indices (see Muthen & Muthen, 2004) was the one involving three categories. Taking into consideration the average class probabilities as shown in Table 2, we may conclude that categories are quite distinct, indicating that each category has its own characteristics. The means and standard deviations of each of the cognitive processes across the three categories of students are shown in Table 3. It is shown that students in Category 3 outperformed students in Category 2 and Category 1 in all processes tasks, while students in Category 2 outperformed their counterparts in Category 1. The percentage of success of students in Category 1 in the editing, selecting and translating tasks was below 50% showing that these students have difficulties in problem posing. However, most of the students in Category 1 ($\bar{X}$ =.75) were successful in comprehending tasks, reflecting, to an extent, the instructional emphasis of Cyprus mathematics textbooks which include activities of problem posing based on given equations (Cyprus Ministry of Education, 2000). Actually, these results reaffirm previous studies (English, 1998; Lowrie, 2002), which indicated that students tend to pose problems that mirror school experiences or problems that simply are variations of those found in textbooks.

Table 2

Average Latent Class Probabilities

Latent Class Membership	Latent Class 1	Latent Class 2	Latent Class 3
Category 1	0.94	0.06	0.00
Category 2	0.03	0.95	0.01
Category 3	0.00	0.03	0.97

Category 2 students had difficulties in the editing and selecting abilities since their success percentage was lower than 67%. Editing and selecting were more demanding processes than comprehending since their applications pre-assume the ability to generate problems that are syntactically and semantically correct (Silver & Cai, 1996). These students were successful in most of the comprehending tasks (88%) and solved correctly 76% of the translating tasks. Finally, Category 3 (N=61) students seem to be successful in all tasks.

Table 3

Means and Standard Deviations of the Three Classes of Students in the Cognitive Processes

	Comprehending	Translating	Editing	Selecting
Category 1				
Mean	0.75	0.18	0.46	0.43
SD	0.32	0.22	0.36	0.37
Category 2				
Mean	0.87	0.79	0.66	0.63
SD	0.23	0.20	0.25	0.29
Category 3				
Mean	0.97	0.97	0.96	0.99
SD	0.11	0.07	0.10	0.04

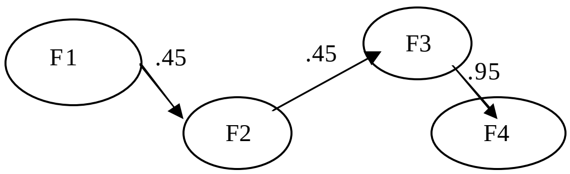
From Table 4, which shows the problems solved by more than 67% of the students in each category, it can be deduced that there is a developmental trend in students' abilities to complete the assigned tasks because success on any problem by more than 67% of the students in a category was associated with such success by more than 67% of the students in all subsequent categories. Thus, Category 2, which was the second largest category (N=36), can be considered as the translating and comprehending category. Category 1 students (N=20) solved more than two thirds of the comprehending problems, and thus we can consider Category 1 as the comprehending category. Students in Category 1 seem to pose more comprehending tasks, but their ability in operating in all other processes is much less than the ability of students in higher categories.

Table 4

Problems Solved by More than 67% of the Students in Each Category

	Comprehending	Translating	Editing	Selecting
Category 1	■			
Category 2	■	■		
Category 3	■	■	■	■

The presence of a consistent trend in the difficulty level across the editing, the selecting, the comprehending and the translating processes supports the hypothesis for the existence of a specific developmental trend. The data imply that students firstly grasp the comprehending processes and secondly they are able to apply the translating processes. The selecting and editing processes are grasped after the conceptualization of the translating process. To further examine this sequence, we tested a path analysis model for specifying the nature of the developmental trend of students in posing problems (Figure 2). From the analysis, we can deduce that the model fits the data in an excellent way (CFI=0,952, x^2/df=1,36, RMSEA=0,05.



Fit Indices: CFI=0.952, x^2/df=1.36, RMSEA=0.05

Note: F1=Comprehending, F2=Translating, F3=Editing, and F4=Selecting.

Figure 2: The Sequence of Path Analysis for the Developmental Trend of Problem Posing Processes

Discussion, Conclusion and Implications

Problem posing is currently discussed as a function of complex and concomitant growth in a knowledge base, strategies, motivation, and metacognition (English, 1998). Given the importance of problem-posing activities in school mathematics, some researchers investigated various aspects of problem-posing processes (e.g., Silver, 1994). It was argued in this study that few models exist to help educators explain how problem posing actually develops. Hence, the first goal of this study was to articulate and empirically test a theoretical model to help educators build new understandings about the thinking processes required by students in generating problems. The model integrated most of the abilities and tasks from existing problem posing research (Silver & Cai, 1996; English, 1997a). Specifically, although the tasks within which subjects posed problems were somewhat structured in order to focus the study, the tasks were open-ended in the sense that subjects could pose any problems they wished so long as the problems met the constraints delineated in each task. The tasks could fall in almost all situations identified by Silver (1995) and Stoyanova (1998), i.e., students were asked to pose problems with given answers, problems that contain specific information in iconic or symbolic form, and problems that fit given calculations. The model extended the literature in a way that these specific processes were recognized as important components of problem posing abilities. The model proved to be consistent with the data, leading to the conclusion that the four processes (selecting, editing, comprehending, and translating) mediate the ability to pose problems. The second aim concerned the extent to which students in the sample vary according to the tasks provided in the test. The analysis illustrated that three different categories of students can be identified. Category 1 students were able to respond only to the comprehension tasks. Students in this category seemed to reason algorithmically. That is, when posing problems they focused on contexts that are typically used in traditional instruction for constructing problems involving number equations. Specifically, students in this category tended to pose problems that mirrored school experiences, i.e., they tended to pose traditional word problems that were simply variations of those found in textbooks (English, 1997a; Lowrie, 2002).Category 2 students were able to respond to both the comprehension and translation tasks, whereas category 3 students were able to respond to all types of tasks, comprehension, translation, selection and editing tasks. Category 2 students were able to construct problems not only following the numbers in a provided situation but also in situations where the information was provided in tabular and graphical forms. Students in Category 2 were able to consider the relationships among the provided data, which was presented in different representational forms, and offer meaningful problems. In this category, students were expected to translate the information provided in tables or graphs into a solvable problem. The posing of this kind of problem required the ability to synthesize two important actions. First, students had to reorganize the provided information from one form to another and secon d to articulate specific relations between the provided information in order to write a solvable problem. Editing and selecting were the processes that distinguished students in category 3 from students in the other categories. In editing tasks, students were provided with a written story situation or a picture, which involved a large amount of information and were asked to construct meaningful problems. In selecting tasks, students were asked to add a question or a statement that would turn the problem situation into a problem that could have a specific answer. The two processes differed in where the understanding appeared: (1) in the editing tasks, understanding meant extracting information from the story context and (2) in the selecting tasks, understanding meant perceiving mathematical relationships among the provided bits of information in the story context. The two tasks were similar, however, in that the desirable outcome was a problem that was linked to specific operations/answers (i.e., a problem that could be solved by some specified operations/answers). **Both the editing and selecting processes characterized the most able students**. Thhis finding is related to and confirms numerous qualitative studies on the mathematical thinking of gifted students (e.g., Sriraman, 2002, 2003)

A number of teaching implications arise from these findings. The model used in this study offers teachers and researchers a means to examine the complexity and sophistication of problem posing. From the perspective of teachers, the model may be used in order to include in their instruction the development of the four cognitive processes. Within the theoretical framework of this study, it can be argued that posing mathematical problems begins

with the comprehending processes. This might be due to the teaching approaches in the classroom, which emphasize the algorithmic ways of thinking in the expense of the translating, the editing and selecting processes. The editing and the selecting processes seem to be more demanding and thus more focus is needed, during instruction, in order for students to apply them. From the perspective of researchers, it is likely that the model could be useful as a prototype for further analyses of the cognitive processes of problem posing. For example, future research may reveal whether the learner moves sequentially through the cognitive processes of comprehending, translating, editing and selecting and whether there are sequential phases of learning to help students move from one cognitive process to another.
Another implication for future research is to create a metric with the operationalized variables editing-selecting-comprehending-translating whereby one could assess the structure of problems posed by students which utilizes quantitative information. While student generated problems very often do connect to mathematics which students are interested in (English, 2005), one danger of unstructured problem posing is that it makes it harder for teachers to talk about elements of problem design and discuss the differences between poorly posed and well posed problems. This recommendation promulgates the call to reduce student anxiety (Healy, 1993) by using a structured quantitative setting through which students can engage in problem posing.

Finally we would like to state that our ideas on designing research by operationalizing existing theory and testing it empirically is by no means new and one of the standard research methodologies in mainstream psychology. We hope that future researchers will consider a similar approach towards integrating and empirically verifying existing theories. Such work will help make reported mathematics education findings more generalizable and testable in other geographic locations with similar populations.

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