An Empirical Investigation of Sixth grade Students' Modelling Processes in Cyprus

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Abstract

Results from the recent PISA 2003 study documented three types of problem solving activities, namely decision making, system analysis and design and trouble shooting (OECD, 2004). The present study conducted in Cyprus provides a sound theoretical foundation for modelling processes involved in mathematical problem solving of young learners by elaborating on PISA's results and considering previous innovative contributions. Within this framework, we develop a test for measuring the modelling processes involved in different types of problems in mathematical problem solving. We investigate sixth grade students' modelling processes in problem solving in Cyprus and propose an empirical model that operationalizes and encompasses most of the previous research in the area. We aim to generate a categorization of modelling processes students need to master in succeeding in different problem solving situations which require engaging in modelling processes.

Keywords: Problem solving, modelling processes, mathematics teaching, structural equation modelling.

1. Introduction

Today's global society and economy demand for school graduates to be successful problem solvers, to effectively use new technological tools and to possess flexible and creative, mathematical skills. Unfortunately, the results from the PISA 2003 study showed that only one in five 15 year olds could be considered a reflective, communicative problem solver (Organization for Economic Co-Operation and Development [OECD], 2004), being able to analyse a situation and make decisions, manage multiple conditions simultaneously, think about the underlying relationships in a problem, developed and test models, modify and refine their work and communicate the results (OECD, 2004). While PISA was focussed on problem solving in general, many of the items could be classified as mathematical problem solving. Similarly, student results in mathematical problem solving as reported in TIMSS 2003 international survey were also disappointing, indicating that the majority of 8^{th} grade students performed poorly in mathematical problem solving (Mullis et al., 2004). Among the numerous well reported reasons for the above findings is the failure of mathematics teaching to provide students with opportunities to study mathematics in real world contexts and to construct models in exploring and understanding problem situations. Student work with models and modelling involves mathematical processes that although are important, are under-represented in traditional mathematical activities (English, 2003; Greer, 1997).

The importance of mathematical modelling in developing problem solving ability in students has been documented with emphasis by Blum & Niss (1991): "fostering overall explorative, creative and problem solving capacities (such as attitudes, strategies, heuristics, techniques, etc.)" (p. 43). Modelling interesting and non trivial situations can become an effective medium for students to be actively engaged in acquiring mathematical knowledge (Blum & Niss, 1991) in experientially real contexts (Gravemeijer, Cobb, Bowers, & Whitenack, 2000; Mousoulides, Pittalis, & Christou, 2006). A significant number of studies have provided valuable insights into: (a) foundations of the modelling perspective in mathematics teaching and learning (Blum & Niss, 1991; Lesh & Doerr, 2003; Lesh & Sriraman, 2005a, 2005b; Sriraman, 2005; Sriraman& Lesh, 2006), (b) principles for developing modelling activities (Lesh et al., 2003), (c) the modelling perspective on teacher development (Doerr & Lesh, 2003; Schorr & Lesh, 2003) and (d) the modelling perspective in problem solving and learning (English, 2003; Doerr & English, 2003). The latter category, that is, the modelling perspective in problem solving provides the foundation for this study. Very recently, Kaiser, Blomhøj & Sriraman (2006) claimed that "the theory of teaching and learning mathematical modelling is far from being complete…[and] more research is needed, especially in order to enhance our understanding on micro levels, meaning teaching and learning problems which occur in particular educational settings where students are engaged in particular modelling activities" (p.82). However, in order to further any teaching goals related to modelling, it is first necessary to empirically identify the areas in which students' encounter the most difficulty when confronted with problem solving requiring modelling. The present study provides a theoretical foundation for the modelling processes involved in problem solving in mathematics by young learners by elaborating on PISA's results and considering previous innovative contribution in the area (e.g., Blum and Kaiser, 1997; Lesh et al., 2003). Following the three types of problem solving activities documented by PISA 2003 study, we developed a test for measuring modelling processes in decision making, system analysis and design and trouble shooting problems in mathematical problem solving. Specifically, the present study investigated Cypriot students' modelling processes in problem solving by generating a structural equation model that operationalizes most of the previous research in the area. More specifically, the study aimed to generate and verify a categorization of modelling processes which appear in the decision making, system analysis and design and trouble-shooting problems. Additionally, the study attempted to trace developmental trends in modelling processes used by 6th grade students in problem solving. The paper is organized as follows: First, we briefly outline our theoretical framework, namely the modelling perspective in mathematical problem solving. We, then, present the methodology and the results of the study conducted in Cyprus. Finally, the conclusions of the study and their implications for educational planning and teaching are discussed

2. Theoretical Framework

2.1 Problem Solving in Mathematics

Recent research findings in mathematics education raise important questions about students' mathematical problem solving and problem posing ability (Christou, Mousoulides, Pittalis, Pitta, & Sriraman, 2005; Doerr & English, 2003; Schoenfeld, 1992). How well prepared are young learners to solve the problems that they will encounter beyond school? How can students work with unfamiliar problems by thinking flexibly and creatively (Lesh & Doerr, 2003)? An interesting definition of what problem solving is comes from Lesh and Doerr (1998) pointing that in mathematical modelling as problem solving "students must refine/transform/extend initially inadequate (but dynamically evolving) conceptual models in order to create 'successful' problem interpretations" (p.9). Modelling as problem solving activity move beyond traditional problem solving experiences, by addressing the processes, developments and models students require in working with increasingly sophisticated systems and applying their models and solutions in a range of similar structure problems (Lesh, & Lehrer, 2003; English, 2003). Modelling activities require students to confront world based problems by understanding problem information, identifying important features and relationships between problem variables, constructing and applying appropriate representations and, finally, evaluating, justifying and communicating results as a means to further understanding the problem (Zawojewski & Lesh, 2003). To succeed in improving students' problem solving abilities teachers should avoid working in a strict linear fashion, because real world problems are not linear (see figure 1). Therefore, developing simplistic linear models and solutions fail to account for everyday problem solving because these solutions are not suitable for the dynamic and multifaceted world based problems (Roth, & McGinn, 1997). A successful model for solving a real world problem involves a number of trial procedures between givens and goals. Neither givens or goals are clearly defined and the student has to decide upon the data used, procedures employed and when the solution is appropriate (Zawojewski & Lesh, 2003).

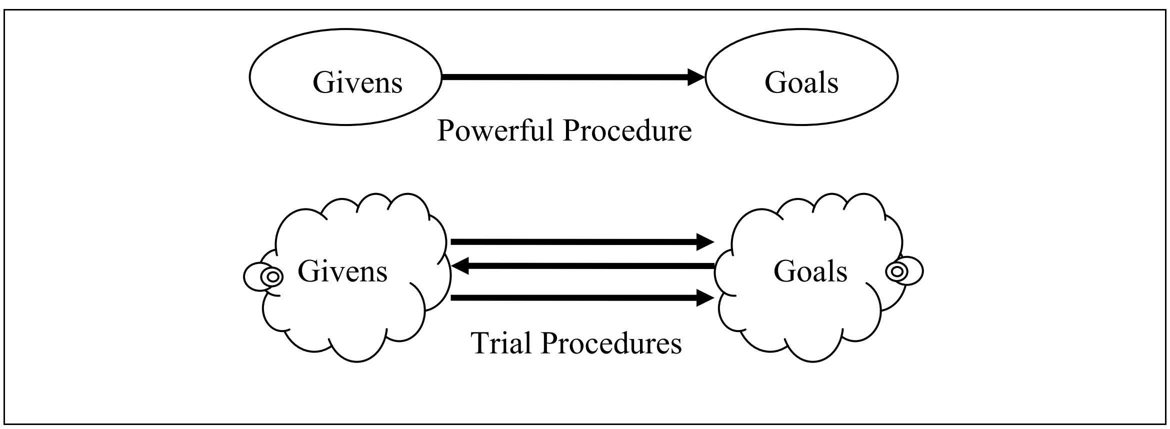


Figure 1: Linear versus a modelling approach to problem solving.

2.2. Modelling Processes in Problem Solving

A number of relevant works (Lesh & Doerr, 2003; Blum & Niss, 1991) have documented the different processes involved in mathematical modelling in problem solving. In an attempt to summarize previous work in modelling processes in problem solving, and to provide a theoretical framework for the present study, the authors adopted Lesh and Doerr's (2003) interpretation of the modelling procedure, incorporating the related modelling processes (see Figure 2). To this end, a test for measuring modelling processes in problem solving was designed such that the results would describe the degree to which students are able to confront, structure, represent and solve problems by applying effectively the necessary modelling processes.

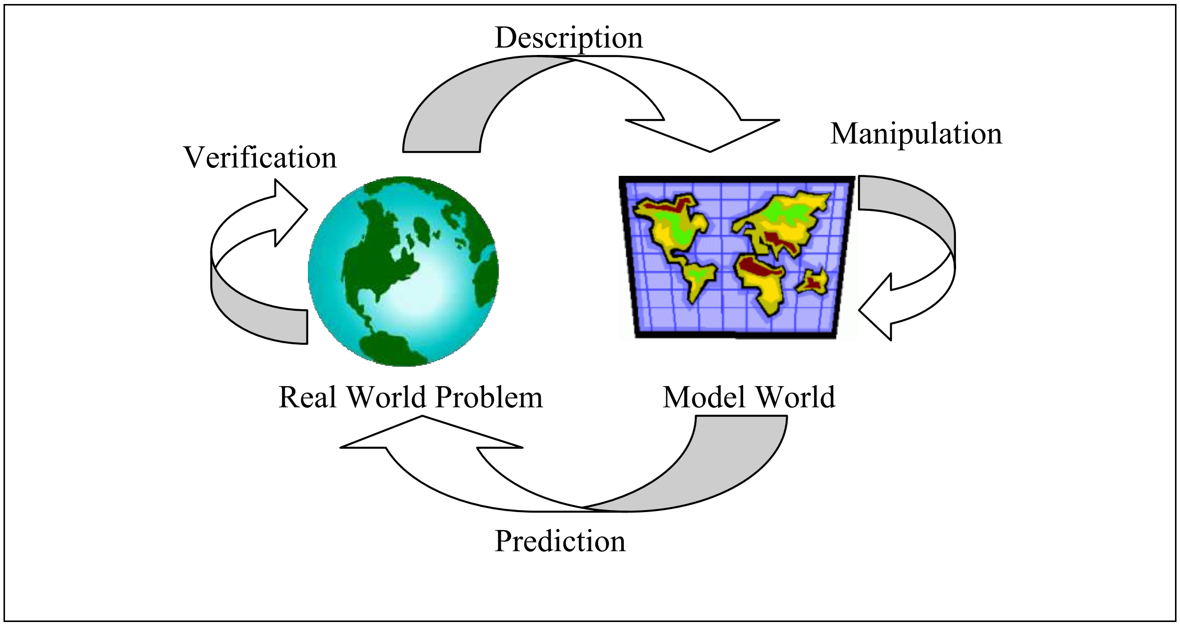


Figure 2: Modelling Procedures/Processes in Problem Solving (Lesh & Doerr, 2003)

In figure 2, the terms used have the following meanings:

Description : by understanding and simplifying the problem.

Manipulation: by mathematizing (identifying variables and relationships, building a model, making the selection among alternatives)

Prediction: by interpreting the solution, predicting the behaviour of the problem.

Verification: by checking and evaluating the solution, validating and communicating results.

In particular, in problem solving situations which involves mathematical modelling, students have to engage in the following modelling processes: (a) Understand and simplify the problem. This includes understanding text, diagrams, formulas or tabular information and drawing inferences from them; demonstrating understanding of relevant concepts and using information from students' background knowledge to understand the information given. (b) Manipulate the problem and develop a mathematical model. These processes include identifying the variables and their relationships in the problem; making decisions about variable relevancy; constructing hypotheses; and retrieving, organising, considering and critically evaluating contextual information; use strategies and heuristics to mathematically elaborate on the developed model. (c) Interpreting the problem solution. This includes making decisions (in the case of decision making); analysing a system or designing a system to meet certain goals (in the case of system analysis and design); and diagnosing and proposing a solution (in the case of trouble shooting tasks). (d) Verify, validate and reflect the problem solution: This includes constructing and applying different modes of representations to the solution of the problem; generalize and communicate solutions; evaluating solutions from different perspectives in an attempt to restructure the solutions and making them more socially or technically acceptable; critically check model (Sriraman & Lesh, 2006; Blum & Kaiser, 1997; Lesh & Doerr, 2003).

An important distinction should be made here. Although the above modelling processes reported in the literature appear to be the same in all problems, there is evidence that students' successful use of modelling processes is influenced by problem type (OECD, 2004). In an attempt to categorize these modelling processes, we followed PISA's results to present the modelling processes in three types of problems as they appear in table 1. As can be noted from table 1, the modelling processes described in Figure 2 appear, as it is expected, in all three problem types. However, different problem types require a different mastery depth for each modelling process.

Decision making	**System analysis & design**	**Trouble shooting**
Understanding a problem situation with several alternatives and constraints and a specified task	Understanding the information that characterises a given systemand the requirements associated with a specified task	Understanding the main features of a system or mechanism and its malfunctioning, and the demands of a specific task
Identifying relevant constraints	Identifying relevant parts of the system	Identifying causally related variables
Representing the possible alternatives for solving a task	Representing the relationships among parts of the system	Representing the functioning of the system, focusing on causally related variables
Making a decision among alternatives	Analysing or designing a system that captures the relationships between parts of the system	Diagnosing the malfunctioning of the system and/or proposing a solution
Checking and evaluating the decision	Checking and evaluating the analysis or the design of the system	Checking and evaluating the diagnosis/solution
Communicating or justifying the decision	Communicating the analysis or justifying the proposed design	Communicating or justifying the diagnosis and the solution

Table 1: Modelling Processes in the three categories of problems

In this study, we hypothesize that students' modelling processes in problem solving have distinct aspects that are represented in the three types of problems (decision making, system analysis and design and troubleshooting). We further propose to investigate whether these modelling processes, as required in the solution of different problems formulate a general form of modelling ability (ability to master different modelling processes) in mathematics as it unfolds through the responses of 6th grade students to problem solving modelling tasks. Specifically, the aims of the study were the following: (a) to investigate whether modelling processes in different types of mathematical problems formulate distinct factors of students' modelling ability and to examine whether these factors formulate a general modelling ability measure,

(b) to trace different groups of students that reflect different competencies in mastering modelling processes and to identify the main characteristics of each group, and

(c) to examine the structure and relationships amongst these modelling processes as they are presented through 6th grade students' responses.

3. Methodology

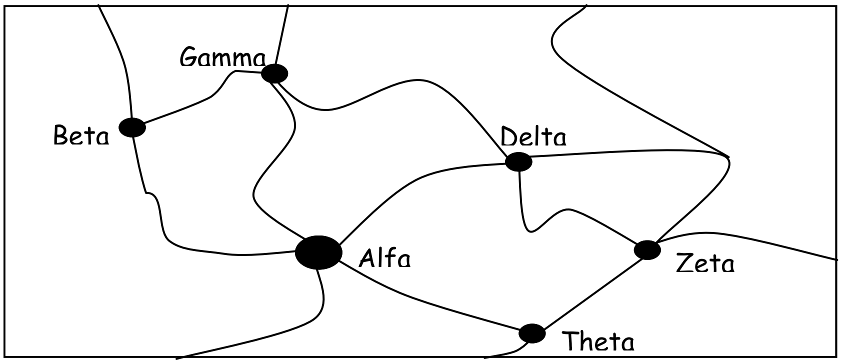
3.1. Participants and Tasks

Data reported in this study was collected by a test measuring modelling processes in mathematical problem solving. The test was administered to 201 6th grade (12 year olds) students in Cyprus in February 2006. The test included six units with a total of nine tasks. Four out of the six units were modified versions of problem solving tasks from the PISA 2003 study (OECD, 2004). Two units aimed to investigate students' modelling abilities in decision making, two in system analysis and design and two in trouble shooting problems. The units reflecting the decision making problems present students with a situation requiring a decision and asking to choose among alternatives under a set of conditions constraining the situation (see Table 1). For example, in “Holidays” unit, students were asked to calculate the shortest possible distance between two cities that were far away, given the map of the area and a table presenting the distances between cities. Students have to understand the situation provided, identify the constraints, possibly translate the way in which the information is presented, make a decision based on the alternatives under the constraints given, check and evaluate the decision, and then communicate the required answer. The factors creating difficulty in decision-making problems are the number of constraints the student has to deal with in working through the information provided and the amount of restructuring a student has to do in sorting through the information along the way to develop a solution. The two units representing decision-making were the “Holidays” and the “Energy Needs”. The test included two units assessing students’ modelling processes to solve problems involving system analysis and design. Problems of this category differ from the decision-making units in that constrains are not obvious and not all of the possible options are given in the problem. Students’ priority in the system analysis and design problems is to develop an understanding of the problem, beginning with the identification of the relationships existing between the parts of the system, or to design a system with certain relationships among its main features. The next step is to test the system with the developed model and finally, students are involved in justifying their analysis. The two units representing system analysis and design were the “Course Design” and the “Children’s Camp”. The third category of units was drawn from the topic of trouble shooting. The units representing trouble shooting were the “Hospital Management” and the “Veterinary Clinic”, including in total four items. Trouble-shooting units assess students’ actions when confronted with the need to specify the conditions under which a system is running properly or when they face a system of a mechanism that is underperforming in some way. To solve such problems, the student must be able to understand the main features of the system and the actions or responses that are expected of each of these features. Based on this understanding, the student must then be able to identify the causal-response relationships between interrelated parts and the role that such links play in the overall function of the mechanism or system of interest. Finally, students may need to communicate their solution in writing or through a diagram to explain their thinking and their recommended course of action. Such problems are complicated by the number of interrelated variables involved and the varied number of representations and translations that one might have to make in understanding the system or mechanism from directions or instruction booklets.

Tasks

Decision Making

This is the map of an area. The table below indicates the actual distances between the various towns.



Alfa						
Beta	55					
Gamma	50	30				
Delta	30	85	55			
Zeta	55		100	45		
Theta	30	85	80	60	25	
	Alfa	**Beta**	**Gamma**	**Delta**	**Zeta**	**Theta**

Calculate the shortest possible distance between towns **Zeta** and **Beta**.

System Analysis

A college is offering the following 11 subjects for a 3-year-study. A student needs 9 subjects, representing 20 credits to complete studies. Each student can take 3 subjects every year in order to complete the 3-year study.

No	Code for subject	Subject and level of subject	Credits
1	M1	Mechanical Studies Level 1	2
2	M2	Mechanical Studies Level 2	3
3	E1	Electrical Studies Level 1	2
4	E2	Electrical Studies Level 2	3
5	BM1	Business Management Level 1	1
6	BM2	Business Management Level 2	2
7	BM3	Business Management Level 3	3
8	IT1	Information Technology Level 1	2
9	IT2	Information Technology Level 2	3
10	MIS1	Management Information Systems Level 1	2
11	MIS2	Management Information Systems Level 2	3

Regulations : (a) A student can take a subject of level 2 or 3 only if he/she has already completed the lower level(s) of the subject on the year before. **(b)** A student can take **Electrical Studies Level 1** if he has/she completed **Mechanical Studies Level 1**. A student can take **Electrical Studies Level 2** if he/she has completed **Mechanical Studies Level 2.**
What subjects should a student attend for each of the 3 years of study?

Trouble-shooting
The cardiology department at a local hospital employs 5 doctors. Every doctor can work from Monday to Friday and examine 10 patients per day. In a whole year (365 days, 52 weeks) a cardiologist can have 25 days for holiday, 26 days off for attending seminars and the weekends. **Can the 5 cardiologists deal with the 10000 patients that are expected to arrive at the hospital during the following year? If not, can you suggest two alternative solutions to the hospital's manager?**

Table 2: Sample problems from the test.

3.2. Coding, Scoring and Analysis

Students' fully correct responses were marked with 20, 21 etc with each coding representing a different fully correct response. Partly correct responses were similarly coded as 10, 11, 12, while incorrect responses with 70, 71 etc. Fully correct responses were scored as 1, partly correct responses as 0,5 and incorrect as 0. The confirmatory factor analysis (CFA), which is part of a more general class of approaches called structural equation modelling, was applied in order to assess the results of the study. CFA is appropriate in situations where the factors of a set of variables for a given population is already known because of previous research. In the case of the present study, CFA was used to test hypotheses corresponding to the different types of modelling processes presented in the three types of problems. Specifically, the aim was not to determine the factors of a set of variables or to find the pattern of the factor loadings. Instead, our purpose of using CFA was to investigate whether the established structure of modelling processes fits our data, coming from a different age group, comparing to PISA's sample. Mplus (Muthen & Muthen, 2004), a structural equation modelling software, which is appropriate for discrete variables, was used to test for model fitting in this study. In order to evaluate model fit, three fit indices were computed: The chi-square to its degree of freedom ratio (x^2/df), the comparative fit index (CFI), and the root mean-square error of approximation (RMSEA) (Marcoulides & Schumacker, 1996). The observed values of x^2/df should be less than 2, the values for CFI should be higher than .9, and the RMSEA values should be less than .08. Mplus was also used to trace classes (groups) of students who could reflect the different modelling processes, such as understanding the real problem and setting up a mathematical model. To this end the study employed latent class analysis (LCA), a statistical method for finding subtypes of related cases (latent classes) from multivariate data. The results of LCA can also be used to classify cases to their most likely latent class. That is, given a sample of subjects measured on several variables, one wishes to know if there is a small number of basic groups into which cases fall. Once the latent class model is estimated, subjects can be classified to their most likely latent class by means of recruitment probabilities. A recruitment probability is the probability that, for a randomly selected member of a given latent class, a given response pattern will be observed.

4. Results & Discussion

The results are presented in relation to the aims of the study. First, we examined the hypothesis, implied by the first aim of the study, i.e., whether the modelling processes required in solving decision making, system analysis and design and trouble shooting problems constitute distinct modelling processes in mathematical problem solving. In the second part of the results we are presenting the findings of tracing multiple groups of students according to the way in which they solved the problems. It was hypothesized, that each group of students could reflect a different achievement level. In other words, we wanted to examine the different characteristics of groups of students, not only by their total score in the test, but in terms of their solution characteristics, focusing on the use of different modelling processes. Finally, the third aim of the study, i.e., to examine the structure of the types of modelling processes presented in the three types of problems, which may indicate a developmental trend of these processes.

4.1. The Proposed Model of Modelling Processes in Problem Solving

In this paper, we propose a model, which may enable students' modelling processes to be described across three factors, namely the modelling processes involved in decision making factor, in system analysis and design factor and in trouble-shooting factor. As it is highlighted in Figure 3, the proposed model consists of three first-order factors and one second-order factor. The first order factors are the decision making factor involving three items, the system analysis and design factor with two items and the trouble-shooting factor with four items. These factors were hypothesized to construct a second order factor "modelling processes in problem solving", which was hypothesized to account for any correlation or covariance between the first order factors. Figure 3 represents the model which best describes data fit to our theory driven model. Confirmatory factor analysis (CFA) was used to evaluate the construct validity of the model. CFA showed that each of the tasks employed in the present study loaded adequately (i.e., they were statistically significant since z values were greater than 1.96) on each factor, as shown in Figure 3. It also showed that the observed and theoretical factor structures matched for the data set of the present study and determined the "goodness of fit" of the factor model (CFI=0.967, x^2= 29.39, df= 23, x^2/df=1.27, p>0.16, RMSEA=0.044), indicating that modelling processes in decision making, system analysis and design and trouble-shooting problems can represent three distinct categories of modelling processes in mathematical problem solving. A focus in this study was to address the hypothesis that the modelling processes required to successful problem solving representing the aforementioned factors constitute students' modelling processes in problem solving. The structure of the proposed model addresses the appropriateness of such a hypothesis, indicating that modelling processes in decision making, system analysis and design and trouble-shooting factors formulate a general construct measuring modelling processes for young learners.

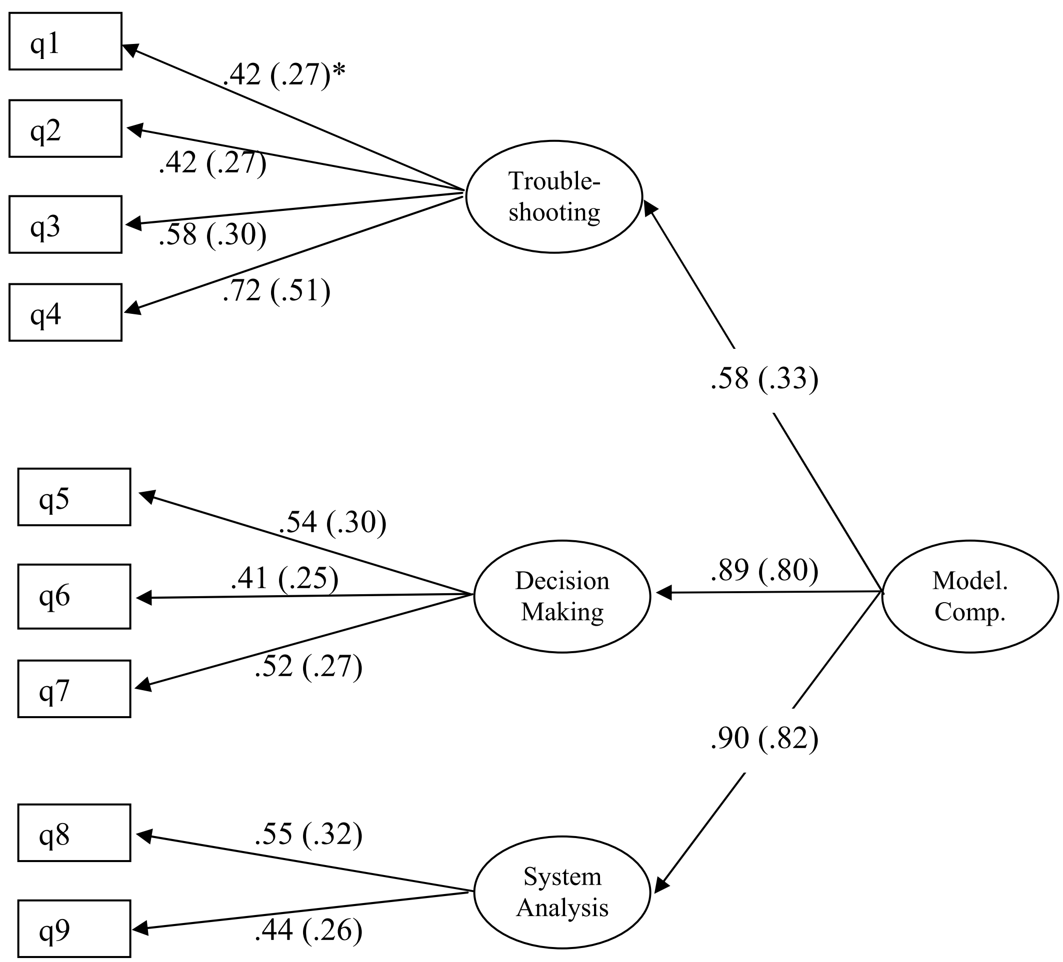


Figure 3: Confirmatory factor analysis

4.2. Analysis of Students' Answers: Groups of Students

The second aim of the study focuses on the types of students' answers. Specifically, we examined whether the different types of answers provided by students could be grouped in a basis of similar characteristics. Mixture growth modelling (Muthen & Muthen, 2004) and qualitative analysis of students' answers were used to answer this question. The statistical modelling here used a stepwise method-that is, the model was tested under the assumption that there are two, three, and four groups of subjects. The best fitting model with the smallest AIC (647.867) and BIC (731.216) indices (see Muthen & Muthen, 2004) was the one involving three groups. As it can be seen in Table 3, the average group probabilities indicate that the three groups are quite distinct, that is each group of students has its own characteristics. The means and standard deviations of each of the three types of problems (decision making, system analysis and design and troubleshooting) across the three groups are shown in Table 4.

	Group 1	Group 2	Group 3
Group 1	0.981	0.019	0.000
Group 2	0.000	0.999	0.001
Group 3	0.000	0.021	0.978

Table 3: Average Group Probabilities by Group

Table 4 shows students' results in each of the three categories of problems. It can be noted that students in Group 3 outperformed students in Group 1 and 2 in all categories, while students in Group 2 outperformed students in Group 1 in all categories. Table 3 also shows that Group 1 students had low means in all three categories, indicating that these students had difficulties in all tasks of the test. Group 2 students' highest mean performance was in decision making tasks. Group 3 students' highest mean performance was in decision making units, their performance in system analysis tasks was two times as their Group 2 counterparts, while they were not so successful in the trouble shooting tasks.

Group	**Decision Making**	**System Analysis**	**Trouble-shooting**
Group 1 (*N*=27)	0,07 (0,15)	0,12 (0,18)	0,02 (0,18)
Group 2 (*N*=62)	0,42 (0,14)	0,21 (0,22)	0,06 (0,13)
Group 3 (*N*=112)	0,76 (0,21)	0,38 (0,32)	0,14 (0,23)

Table 4: Means and standard deviations of students' performance in each Group

In this section we present some qualitative findings from the three categories of problem solving units, in light of the three different groups of students formulated by the Latent Class Analysis. Due to space constrains, student responses from only one unit from each category will be presented here. In "Holidays" unit (see Table 2) the majority of students in Group 1 and 2 did not select the correct way. On the contrary, 60% of Group 1 students and 48% of Group 2 students performed wrong calculations, without noticing that their purpose was to calculate the shortest way, showing that they did not identify the relevant constraints. In line with that, 15% of Group 1 students and 20% of Group 2 students made correct calculations for finding a possible way between the two cities, but they did not notice that their solution (Zeta, Delta, Gamma and Beta) was not the shortest one, indicating that they did not identify the possible alternatives. On the contrary, 60% of Group 3 students solved correctly the problem by identifying the optimal solution. Of interest is the finding that 15% of these students transferred the table data on the map, before calculating the different ways, in an attempt to check and evaluate their decision. Quite interesting are the findings in the "Course Design" unit, in system analysis and design category. None of Group 1 students reached a correct solution, while the percentages of Group 2 and Group 3 students that reached a correct solution to this problem were almost the same. The common mistake for most of group 1 students was their efforts to complete tables randomly, indicating that they did not understand the provided information and restrictions, that characterises the given system and the requirements associated with the courses offered in the college. Group 2 students wrongly completed the tables by selecting that two (or even three) levels of the same course should be offered in the same year. This finding indicates students' inability to identify relevant parts of the system, that is, the courses take place at the same time period during an academic year. The percentage of Group 3 students that completed the table randomly or selected that two levels of the same course in the same year was much smaller than their counterparts; however they failed to grasp specific relationships among parts of the system by ignoring the first or the second regulation (see table 2). It can be claimed that Group 1 students failed even to understand the characteristics of the problem, Group 2 students did not identify relevant parts of the system while Group 3 students did not design a course schedule that captures the regulations of the problem and failed to evaluate their solution according to the problem's requirements (e.g., the necessary credits). The majority of students in all three groups did not solve the problems in trouble-shooting category. More specifically, in the "Hospital Management", none of Group 1 students reached a solution because they could not even properly calculate how many days a cardiologist works, failing to understand the main features of the system presented. Half of them just exhibited random calculations while the rest of them ignored that doctors do not work in weekends. Group 2 students also ignored that doctors do not work in weekends and in case they consider that, they could not properly manipulate data, showing that they could not identify the causal variables between days of work and weekends. It should also be pointed that only half of Group 2 students that calculated how many days a cardiologist works reached a complete solution because they did not relate the days that a cardiologist works with the total amount of patients that can be treated in the hospital. The majority of Group 3 students understood the main features of the problem and identified causal relations within the problem framework. However, they could not represent these relations and as a result they manipulated the variables and their relationships in an unsuccessful way.

4.3. The Structure of the Developmental Trend

The presence of a consistent trend in the difficulty level across the decision making, system analysis and design and trouble-shooting problems provides support for the hypothesis of the existence of a specific developmental trend in the modelling processes students need to master in solving such problems. The data of the study imply that students firstly grasp and successfully employ the modelling processes in the decision making problem solving units and secondly by further elaborating and enhancing these modelling processes in analyzing and designing systems. It appears that students finally incorporate modelling processes in trouble shooting problems. To further examine this sequence, we tested a model for specifying the nature of the developmental trend of students' modelling processes in problem solving. Structural confirmatory analysis was used to examine model's fit to the empirical data. As can be seen in Figure 4, model's fit indices indicated support for the hypothesis (CFI=.952; x^2 = 33.76; df=22; x^2/df=1.53; RMSEA=0.04). These results reaffirm the developmental trend as described above and indicate that students are more fluent in employing modelling processes firstly on decision making, secondly on system analysis and design and thirdly on trouble shooting problems.

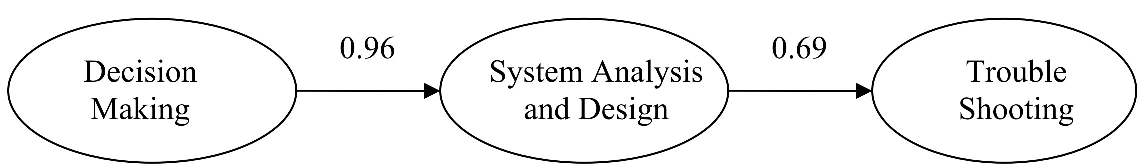


Figure 4: A model representing the developmental trend in students' modelling processes in problem solving

5. Concluding Points

Our present analysis of 6th grade students in our sample indicates that there are not any students that can be characterized as efficient problem solvers in mathematics. On the contrary, many students appear to be weak or emergent problem solvers. These are the students included in the 1st group of the analysis conducted (with very low means in every task). This finding has several implications for problem design aimed at furthering student competencies on the different types of problems in mathematics problem solving. There are also a number of implications for teachers incorporating such activities in the classroom. Since students were more fluent in employing modelling processes firstly on decision making problems, secondly on system analysis and design and thirdly on trouble shooting problems, it makes sense that problem solving activities which involve modelling be designed in such a way that "trouble shooting" receives more emphasis in the classroom. This entails problems in which students are required to spend considerable time with understanding the main features of a system or mechanism, the demands of a specific task, identifying causally related variables, diagnosing malfunctions in the system, checking solutions and communication. Many of these processes are higher order thinking processes which are normally not emphasized in routine problems. Students' ability to solve world based problems is currently discussed as an importing factor not only for school mathematics but also in the workplace (Gainsburg, 2006). Given the importance of problem solving in mathematics (Schoenfeld, 1992), investigating how students perform in different types of problems in problem solving and how students incorporate modelling processes to solve the aforementioned types of problems appears to be of great interest, not only for researchers but also for teachers. One of the aims of the study was to propose a model for explaining the structure and development of modelling processes in problem solving for young learners. Results from the present study validated a theoretical model to help researchers build new understandings about the modelling processes required by students in solving world based problems. The model, elaborating on the modelling procedure as proposed by Lesh and Doerr (2003), integrates most of the modelling processes identified from existing problem solving research (Blum & Kaiser, 1997). The model extends the literature in a way that these specific modelling processes are recognized as important components of problem solving competencies. Additionally, the model shows how modelling processes differ in different types of problems (decision making, system analysis and design and trouble shooting). This finding can be proved to be important for classroom teachers. Despite the fact that problem solving is probably the most important component of mathematical activity teachers are rarely informed on successful practices towards improving students' ability in solving demanding problems. Conclusions arising from the results of the study lead to a practical suggestion for unfolding problem solving activities in schools. A number of teaching implications arise from these findings. The study offers classroom teachers a means to examine the complexity and sophistication of modelling processes in problem solving. From teachers' perspective, the model may be used in order to include in instruction the development and use of appropriate modelling tasks incorporating the different modelling processes. Within the structure of the problems presented here, it can be argued that problem solving activities can begin with decision making tasks. Modelling processes required to successfully solve system analysis and design and trouble shooting problems seem to be more demanding and thus more focus is needed, during instruction, in order for students to effectively apply them. From the researchers' perspective, the model of modelling ability in problem solving situations as well as the developmental trend in different types of problem solving can be useful as a prototype for further analyses of the modelling processes of problem solving, as well as for the purposes of constructing problems which align with developmental trends of students. Given, the fact that we have used a quantitative methodology, our hope is that use of structurally similar problems with similar age group students in other geographic locations will allow for ease in comparing developmental trends. We consider this generalizability aspect vital for Cyprus in relation to North America and other European Union countries who participated in PISA and other international assessments, and are trying to improve student competencies in mathematical problem solving. Finally, the study can be considered as a step towards integrating and empirically verifying existing theories on mathematical modelling.

References

Blum, W., & Kaiser, G. (1997). Vergleichende empirische Untersuchungen zu mathematischen Anwendungsfähigkeiten von englischen und deutschen Lernenden. Unpublished application to Deutsche Forschungsgesellschaft.

Blum, W., & Niss, M. (1991). Applied mathematical problem solving, modelling, applications, and links to other subjects – state, trends, and issues in mathematics instruction. *Educational Studies in Mathematics*, 22(1), 37-68.

Christou, C., Mousoulides, N., Pittalis, M., Pitta-Pantazi, D, Sriraman, B. (2005). An empirical taxonomy of problem posing processes. *Zentralblatt für Didaktik der Mathematik* (*International Reviews on Mathematical Education*), 37(3), 149-158.

Doerr, H. M., & Lesh, R. (2003). A modelling perspective on teacher development. In H.

M. Doerr & R. Lesh (Eds.), *Beyond constructivism: A models & modelling perspective on mathematics problem solving, learning & teaching* (pp. 125– 140). Hillsdale, NJ: Lawrence Erlbaum Associates, Inc.

Doerr, H., & English, L. (2003). A Modelling perspective on students' mathematical reasoning about data. *Journal for Research in Mathematics Education*, 34(2), 110-136.

English, L.D. (2003). Reconciling theory, research, and practice: A models and modelling perspective. *Educational Studies in Mathematics*, 54, 225–248.

Freudenthal, H. (1991). *Revisiting Mathematics Education. China Lectures*. Dordrecht: Kluwer Academic Publishers.

Gainsburg, J. (2006). The mathematical modelling of structural engineers. *Mathematical Thinking & Learning*, 8(1), 3-36.

Gravemeijer, K., Cobb, P., Bowers, J., & Whitenack, J. (2000). Symbolizing, modelling and instructional design. In P. Cobb, E. Yackel, & K. McClain (Eds.), *Symbolizing and communicating in mathematics classrooms* (pp. 225-274). Mahwah, NJ: Lawrence Erlbaum Associates.

Greer, B. (1997). Modelling reality in mathematics classrooms: The case of word problems. *Learning & Instruction*, 7, 293–307.

Kaiser, G., Blomhøj, M & Sriraman, B. (2006). Towards a didactical theory for mathematical modelling. *Zentralblatt für Didaktik der Mathematik*, 38(2), 82-85.

Lesh, R., & Doerr, H. (1998). Symbolizing, communicating, and mathematizing: Key components of models and modelling. In P. Cobb & E. Yackel (Eds.), *Symbolizing,* *communicating, and mathematizing*. Hillsdale, NJ: Lawrence Erlbaum.

Lesh, R., & Doerr, H.M. (2003). *Beyond Constructivism: A Models and Modelling Perspective on Mathematics Problem Solving, Learning and Teaching*, Lawrence Erlbaum, Hillsdale, NJ.

Lesh, R., & Sriraman, B. (2005a).John Dewey Revisited- Pragmatism and the models-modelling perspective on mathematical learning. In A. Beckmann, C. Michelsen & B. Sriraman [Eds.], *Proceedings of the 1st International Symposium on Mathematics and its Connections to the Arts and Sciences*. May 18-21, 2005, University of Schwaebisch Gmuend: Germany.Franzbecker Verlag, pp. 32-51.

Lesh, R., & Sriraman, B. (2005b). Mathematics Education as a design science, *Zentralblatt für Didaktik der Mathematik*, 37(6), 490-505.

Lesh, R., Cramer, K., Doerr, H. M., Post, T., & Zawojewski, J. (2003). Model development sequences. In H. M. Doerr & R. Lesh (Eds.*), Beyond constructivism: A models&modelling perspective on mathematics problem solving, learning & teaching* (pp. 35–58). Hillsdale, NJ: Lawrence Erlbaum Associates, Inc.

Lesh, R.A., & Lehrer, R. (2003). Models and modelling perspectives on the development of students and teachers, *Mathematical Thinking and Learning* 5(2&3), 109–130.

Maaβ, K., (2006). What are modelling competencies, *Zentralblatt für Didaktik der Mathematik* , 38(2), 113-142.

Marcoulides, G. A., & Schumacker, R. E. (1996). *Advanced structural equation modelling: Issues and techniques*. NJ: Lawrence Erlbaum Associates.

Mousoulides, N., Pittalis, M., & Christou, C. (2006). Improving Mathematical

Knowledge through Modelling in Elementary School. *Proceedings of the 30th Conf. of the Int. Group for the Psychology of Mathematics Education*. Prague.

Mullis, I.V.S., Martin, M.O., González, E.J., and Chrostowski, S.J. (2004). *TIMSS 2003* International Mathematics Report: Findings from IEA's Trends in International Mathematics and Science Study at the Eighth and Fourth Grades. Chestnut Hill, MA: Boston College.

Muthen, L. K. & Muthen, B. O. (2004). *Mplus User's Guide*. Third Edition. Los Angeles, CA: Muthen & Muthen.

Organization for Economic Co-Operation and Development [OECD] (2004). *Problem Solving for Tomorrow's World – First Measures of Cross Curricular Competencies from PISA 2003*, http://www.pisa.oecd.org/dataoecd/25/12/3400900 0.pdf. Retrieved 29.09.2005.

Roth, W., & McGinn, M. (1997). Toward a new perspective on problem solving. *Canadian Journal of Education*, 22(1), 18-32.

Schoenfeld, A. H. (1992). Learning to think mathematically: Problem solving, metacognition, and sense making in mathematics. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 340-370). New York: Macmillan.

Schorr, R., & Lesh, R. (2003). A modelling approach for providing teacher development. In R. Lesh & H. M. Doerr (Eds.), *Beyond constructivism: Models and modelling perspectives on mathematics problem solving, learning, and teaching*. Mahwah, NJ: Lawrence Erlbaum Associates.

Sriraman, B. (2005). Conceptualizing the notion of model-eliciting. *Proceedings of the 4th European Congress of Mathematics Education*, St. Feliu de Guixols, Spain, Feb 16-22, 2005.

Sriraman,B., & Lesh, R. (2006). Beyond traditional conceptions of modelling. *Zentralblatt für Didaktik der Mathematik*, 38(3), 247-254.

Wyndhamn, J., & Säljö, R. (1997). Word problems and mathematical reasoning – A study of children's mastery of reference and meaning in textual realities. *Learning and Instruction*, 7(4), 361-382.

Zawojewski, J. S., & Lesh, R. (2003). A models and modelling perspective on problem solving. In R. Lesh & H. M. Doerr (Eds.), *Beyond constructivism: Models and modelling perspectives on mathematics problem solving, learning, and teaching*. Mahwah, NJ: Lawrence Erlbaum Associates.