A global survey of international perspectives on modelling in mathematics education

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Abstract: *In this article we survey the current debate on modelling and describe different perspectives on this debate. We relate these perspectives with earlier perspectives and show similarities and differences between these different approaches.*

ZDM-Classification: M10

The international discussion on modelling, which is amongst others documented in the last and the current issue of this journal, demonstrate that there does not exist a homogeneous understanding of modelling and its epistemological backgrounds within the international discussion on modelling.
However, this is not a new situation at all and if we go back into the earlier debate on modelling we can find astonishing similarities.

1. The earlier debate on modelling in mathematics education

Nearly twenty years ago, Kaiser-Messmer (1986, pp. 83) showed in her analyses that within the applications and modelling discussion of that time various perspectives could be distinguished, internationally and nationally in Germany or German-speaking countries as well. The following two main perspectives emerged from the discussion that time:

- A **pragmatic perspective,** focussing on utilitarian or pragmatic goals, the ability of learners to apply mathematics to solve practical problems. Henry Pollak (see for example 1969) can be regarded as a prototype of this perspective.
- A **scientific-humanistic perspective** which is oriented more towards mathematics as a science and humanistic ideals of education with focus on the ability of learners to create relations between mathematics and reality. The 'early' Hans Freudenthal (see for example 1973) might be viewed as a prototype of this approach. Freudenthal changed his position at the end of his life, as he tended to take pragmatic aims more into consideration (see for example 1981).

Although these were the main streams of the discussion on modelling further differentiations become obvious, especially interesting on a national level within the German debate, but with significant international linkages. For a better understanding of the current approaches, a brief description of these approaches will be given.

Related to the scientific-humanistic perspective the approach developed by Hans-Georg Steiner (1968) put **epistemological goals** into the foreground and emphasised the development of mathematical theory as an integrated part of the processes of mathematising. However, early attempts such as that of the French-speaking André Revuz (1971) are also important. He starts out from the triple situation-model-theory which means that models are constructed by starting from a situation which then leads to the development of a mathematical theory.

Furthermore, an **emancipatory perspective** in the discussion can be identified, which is developing into socio-critical attempts of mathematics teaching (for current approaches see for example Gellert, Jablonka & Keitel 2001).

A third stream, named **integrative perspective,** demands that applications and modelling should become subject to different levels of aims, that is to serve scientific, mathematical and pragmatic purposes but in a harmonious relation to each other. This perspective is not limited to specific aims and gets its strength from a wide range of aims and arguments (see for example Blum & Niss 1991).

The various perspectives of the discussion as reconstructed by Kaiser-Messmer vary strongly due to their aims concerning application and modelling. The appropriate references suggest various dimensions of aims. Kaiser (1995, p. 69f) distinguishes the following goals:

- **Pedagogical goals**: imparting abilities that enable students to understand central aspects of our world in a better way;
- **Psychological goals**: fostering and enhancement of the motivation and attitude

of learners towards mathematics and mathematics teaching;
- **Subject-related goals**: structuring of learning processes, introduction of new mathematical concepts and methods including their illustration;
- **Science-related goals**: imparting a realistic image of mathematics as science, giving insight into the overlapping of mathematical and extra-mathematical considerations of the historical development of mathematics.

Comparable dimensions of aims are stated by Blum (1996, p. 21f) although he identified and described the nuances differently.

Meanwhile, the current discussion on modelling has developed further and become more differentiated. New perspectives can be identified which, as it became obvious from detailed analyses, emerged from the above described traditions or partly can be regarded as their continuations.

2. The current debate on modelling in mathematics education

In the following, a classification system for present modelling approaches will be suggested by reverting to the previous differentiations summarized above but taking the current developments of the modelling discussion into consideration. This suggestion is based on recent analyses using literature mainly generated by ICMI and ICTMA activities and additional publications (see for example the reference list in the discussion document of the ICMI Study on applications and modelling in mathematics education (Blum et al. 2002, p. 279f)).

This classification distinguishes various perspectives within the discussion according to their central aims in connection with modelling and describes in short words the backgrounds these perspectives are based on as well as their connection to the initial perspectives. This ensures both a continuity for the present discussion as well as accumulates current perspectives coherently into the existing literature

Name of the perspective	**Central aims**	**Relations to earlier perspectives**	**Background**
Realistic or applied modelling	Pragmatic-utilitarian goals, i.e.: solving real world problems, understanding of the real world, promotion of modelling competencies	Pragmatic perspective of Pollak	Anglo-Saxon pragmatism and applied mathematics
Contextual modelling	Subject-related and psychological goals, i.e. solving word problems	Information processing approaches leading to systems approaches	American problem solving debate as well as everyday school practice and psychological lab experiments
Educational modelling; differentiated in a) **didactical** modelling and b) **conceptual** modelling	Pedagogical and subject-related goals: a) Structuring of learning processes and its promotion b) Concept introduction and development	Integrative perspectives (Blum, Niss) and further developments of the scientific-humanistic approach	Didactical theories and learning theories
Socio-critical modelling	Pedagogical goals such as critical understanding of the surrounding world	Emancipatory perspective	Socio-critical approaches in political sociology
Epistemological or theoretical modelling	Theory-oriented goals, i.e. promotion of theory development	Scientific-humanistic perspective of "early" Freudenthal	Roman epistemology

The following perspective can be described as a kind of **meta-perspective:**

Cognitive modelling	Research aims: a) analysis of cognitive processes taking place during modelling processes and understanding of these cognitive processes Psychological goals: b) promotion of mathematical thinking processes by using models as mental images or even physical pictures or by emphasising modelling as mental process such as abstraction or generalisation		Cognitive psychology

Table 1: Classification of current perspectives on modelling

When analysing the papers presented in this and the last issue of the Zentralblatt fuer Didaktik der Mathematik or papers presented at other opportunities (see for example the compilation of papers presented at CERME4), one finds out that the apparent uniform terminology and its usage masks a great variety of approaches.

It is remarkable that now, after a longer period of time, approaches from Roman language speaking countries were brought into the discussion on applications and modelling which start out from a more theory-related background. Partly these approaches – called epistemological perspective in the above classification - refer to the anthropological theory of didactics and to the approach of mathematical praxeologies of Chevallard emerging from anthropological theory, or they refer to approaches like that of Brousseau concerning ‚contract didactique'. In contrast to the perspective of realistic modelling, approaches such as those presented by Garcia Garcia & Ruiz Higueras (2005) or Dorier (2005) at CERME 4 or Garcia et al. (in the current issue of this journal), give less importance to the reality aspect in the examples they deal with. Both, extra-mathematical and mathematical topics may be dealt with, while the latter is then described as "intra-mathematical modelling". If the approach of praxeology becomes the main orientation, this leads to the fact that every mathematical activity is identified as modelling activity for which modelling is not limited to mathematising of non-mathematics issues.

As a consequence the approaches of the epistemological perspective show a strong connection to the science-oriented approaches of Steiner and Revuz for which mathematising and modelling is taken as part of theory development. However, these approaches are also rooted in the tradition of the scientific-humanistic perspective mainly shaped by the early Freudenthal. In his earlier work, Freudenthal (1973) understands mathematisation as local structuring of mathematical and non-mathematical fields by means of mathematical tools for which the direction from reality to mathematics is highly important. Freudenthal distinguishes local and global mathematisation, and for global mathematisation the process of mathematising is regarded as part of the development of mathematical theory.

The current approaches of the epistemological perspective continue with a distinction developed by Treffers (1987), who distinguished horizontal mathematising, meaning the way from reality to mathematics, from vertical mathematising, meaning working inside mathematics. Freudenthal (like his successors) consistently uses the term mathematising. According to Freudenthal mathematical models are only found at the lowest level of mathematising when a mathematical model is constructed for an extra-mathematical situation.

Likewise, analyses show that the approaches from the pragmatic perspective were sharpened further until they became the perspective of realistic modelling. For these kinds of approaches, authentic examples from industry and science play an important role. Modelling processes are carried out as a whole and not as partial processes, like applied mathematicians would do in practice. As central characteristic of the realistic or applied perspective formulated by Haines & Crouch (2005) at CERME 4 or Kaiser & Schwarz (last issue of this journal) can be stated that modelling is understood as activity to solve authentic problems and not as development of mathematical theory. The described empirical studies even point out that newly learned knowledge cannot be applied directly in modelling processes, only with some delay. This fact has already been pointed out in earlier reports based on anecdotal knowledge (e.g. Burghes & Huntley 1982). In general, the presented empirical studies aimed at fostering modelling competencies demonstrate underlying complexities which makes it difficult to achieve progress.

Besides these quasi polarising approaches, the realistic modelling and the epistemological modelling, there exists a continuation of integrative approaches within the perspective educational modelling which puts the structuring of learning processes and fostering the understanding of concepts into the foreground of interest. The majority of approaches developed in the area on modelling can be classified under this perspective (see for example the papers by Henning & Keune (2005), Lingefjärd (2005), Vos (2005) presented at CERME4). However, the approach of educational modelling may also be interpreted as continuation of the scientific-humanistic approaches in its version formulated by Freudenthal in his late years and the continuation done by Treffers (1987) or

respectively by De Lange (1987) for whom real-world examples and their interrelations with mathematics become a central element for the structuring of teaching and learning mathematics. Other important approaches, which emphasise a variety of goals connected with modelling and which can therefore be assigned to this position, are the approaches developed by Blum (see amongst others Blum et al. 2002) and Niss (see for example 2001), which were already classified under the integrative perspective, but which have been further developed.

Another perspective, the so-called socio-critical perspective, can be characterised as a continuation of the emancipatory approach described earlier. It refers to socio-cultural dimensions of mathematics, which are closely associated with ethno-mathematics, promoted for example by D'Ambrosio (1999). This perspective emphasises the role of mathematics in society and claims the necessity to support critical thinking about the role of mathematics in society, about the role of and nature of mathematical models and the function of mathematical modelling in society. Positions developed within this perspective such as the one by Barbosa (see the current issue of this journal) emphasise the discrimination between mathematical modelling done by professional modellers and the modelling activities carried out in school. In mathematics pedagogy the promotion of critical thinking of the students is emphasised as central goal of teaching. Therefore reflexive discussions amongst the students within the modelling process are seen as an indispensable part of the modelling process. Modelling diagrams as they are developed by different approaches, for example within the realistic modelling or the educational modelling, are seen as inadequate for the description of students' modelling activities. Based on the perspective of discursive and cultural psychology students' discourse within modelling activities come into focus, which should allow the students to develop various kinds of discussions such as mathematical, technological and reflexive discussions, latter seen as indispensable for the development of critical thinking.

The perspective of solving word problems - named contextual modelling - has a long tradition, especially in the American realm, but with the model eliciting perspective a theory based perspective has been established which is clearly going far beyond problem solving at school.
This perspective traces its lineage to the modern descendents of Piaget and Vygotsky, but also to American Pragmatists. The philosophy of this perspective (see Lesh & Doerr, 2003; Lesh & Sriraman, 2005a, 2005b) is based on the premise that:

- conceptual systems are human construct, and that they also are fundamentally social in nature (Dewey and Mead);
- the meanings of these constructs tend to be distributed across a variety of representational media ranging from spoken language, written language, to diagrams and graphs, to concrete models, to experience-based metaphors (Pierce);
- knowledge is organised around experience at least as much as around abstractions - and that the ways of thinking which are needed to make sense of realistically complex decision making situations nearly always must integrate ideas from more than a single discipline, or textbook topic area, or grand theory (Dewey);
- the "worlds of experience" that humans need to understand and explain are not static. They are, in large part, products of human creativity. So, they are continually changing - and so are the knowledge needs of the humans who created them (James).

Model eliciting activity is defined as a problem solving activity constructed using specific principles of instructional design in which students make sense of meaningful situations, and invent, extend, and refine their own mathematical constructs. In other words, while the traditional problem-solving goal is to process information with a given procedure, model eliciting is the process itself. The purpose of the process is for students to take their model elicited through solving the original problem and apply it to a new problem. The model eliciting perspective is based on the premise that modelling research should take into account findings from the realm of psychological concept development to develop activities which motivate and naturally allow students to develop the mathematics needed to make sense of such

situations. More generally, models and modelling perspectives emphasize promising aspects associated with both socio-cultural theories and theories of situated cognition, but from a different perspective. In this issue of the journal there are several papers departing from this theoretical approach.

Within the discussion on applications and modelling, the approach of cognitive modelling, which exams modelling processes under a cognitive perspective, is new. Of course, the analysis of thinking processes by means of the approach of modelling is not new and is found in many theories of learning or cognitive psychology (see for example Skemp 1987). However, the analysis of modelling processes with a cognitive focus must be regarded as a new perspective, as only recently a few studies were carried out analysing modelling processes under a cognitive perspective. This perspective is called a meta-perspective, because it is not a normative approach connected to goals of teaching modelling in school, in contrast it starts from a descriptive position. It aims to analyse various modelling processes with different types of modelling situations, varying in their degree of authenticity or mathematical complexity. One of the main goals is to reconstruct individual modelling routes or individual barriers and difficulties of students during their modelling activities. Researchers classified under this descriptive perspective of cognitive modelling such as Blum & Leiss (2005) or Borromeo Ferri (in the last issue of the journal) might be found concerning their normative approach on teaching applications and modelling within another perspective as well. For example the approach developed by Blum (see for example 2002) is classified under the perspective educational modelling.

We now classify the papers published in the last and the current issue of the Zentralblatt fuer Didaktik der Mathematik on modelling based on our classification system described in Table 1.

Approach	**Classification of the papers**
Realistic or applied modelling	Burkhardt Kaiser & Schwarz
Contextual modelling	Doerr Iversen & Larson Pierce & Stacey Sriraman & Lesh
Educational modelling; differentiated in a) **didactical** modelling and b) **conceptual** modelling	Blomhoj & Hoff Keldsen Galbraith & Stillman Lingefjärd Michelsen Maaß Michelsen
Socio-critical modelling	Barbosa
Epistemological or theoretical modelling	Garcia, Gascon, Ruiz Higueras & Bosch

As meta-perspective:

Cognitive modelling	Borromeo Ferri

Table 2: Classification of the papers published in the last two issues

3. Discussion and Implications

The classification of a paper to one category does not mean that the overall position of the researcher belongs to this category. It is possible and in a few cases even known that the overall approach of a person emphasises different aspects of modelling or that the approach of a researcher changes. Furthermore it has to be pointed out that these classifications are not based on objectifiable and operational criteria but on the analyses of texts by means of a more hermeneutic understanding of text. The analyses show that the various approach are partly incommensurable, this incommensurability might best be resolved by resorting to Lester's (2005) philosophical framework for understanding researcher's stances in educational research.

Lester (2005) suggested the idea of reconciling rival theories by viewing it within a *Hegelian* inquiry system, where antithetical and mutually inconsistent theories are developed. In other words reconciliation is only possible if we force the assumptions of each perspective or theoretical framework to be questioned.

To summarise, these analyses demonstrate on the one hand that currently significant further developments are taking place within the discussion on applications and modelling, while on the other hand it became clear that these new approaches still go along with existing traditions, and that they have developed further earlier approaches or fall back on them. However, the frequent usage of concepts from the modelling discussion should not be mistaken about the fact that the underlying assumptions and positions of the various modelling approaches differ widely. A precise clarification of concepts is necessary in order to sharpen the discussion and to contribute for a better mutual understanding. Thus, this suggestion for a analysis of the current discussion on applications and modelling is meant to be a first step into this direction.

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