Gender and Strategy Use in Proportional Situations: An Icelandic Study

Olof Bjorg Steinthorsdottir & Bharath Sriraman

Olof B. Steinthorsdottir

Assistant Professor

School of Education

University of North Carolina at Chapel Hill

CB#3500 – Peabody Hall

Chapel Hill, NC 27517

Phone: 919-962-6608

Fax: 919-961-1533

E-Mail: *steintho@email.unc.edu*

Bharath Sriraman

Associate Professor

Dept. of Mathematics

The University of Montana

Missoula, MT 59812

Phone: 406-243-6714

Fax: 406-243-2674

E-mail: *sriramanb@mso.umt.edu*

ABSTRACT

This study was conducted to investigate the influence of semantic type and number structure on individuals' use of strategies in solving missing value proportion problems, and to examine gender differences in strategy use. Fifty-three eighth graders in one school in Reykjavik, Iceland, participated in this study. Twenty-seven females and twenty-six males, were individually interviewed as they solved sixteen missing value proportion problems. The problems represented four semantic structures: Well chunked (W-C), part-part whole (P-P-W), associated sets (A-S), and symbolic (S-P). For each semantic structure there were four problems, each representing a distinct number structures: Integer-integer with an integer answer (I-I-I), integer-noninteger with an integer answer (I-N-I or N-I-I), noninteger-noninteger with an integer answer (N-N-I), or noninteger-noninteger with a noninteger answer (N-N-N). The findings in this study indicate that number structure influenced strategy use and success to a greater extent than semantic type. The easiest problems for students to solve were I-I-I tasks and the most difficult problems were the N-N-N tasks. Most students used multiplicative strategies on the I-I-I problems and I-N-I problems, but resorted to less sophisticated reasoning on the N-N-I and N-N-N problems. No gender differences were identified in the overall success rate. Girls were more successful than boys in associated sets and symbolic problems, and boys were more successful than girls in part-part-whole problems. Girls used less mature strategies for all semantic types except the case of the symbolic type. The data suggest that the semantic type influences females' choice of strategy more than that of males.

INTRODUCTION

Scholarly research related to gender and mathematics is not as frequently published as it was in the 1980’s and the 1990’s (Steinthorsdottir & Sriraman, 2007). Does it mean that we don’t have to worry about gender differences in mathematics any more? At ICME 10 (International Congress of Mathematics Education) held in Copenhagen in 2004, Topics Study Group 26 was about gender and mathematics education and 15 papers were presented. Two studies from Scandinavia showed interesting results about gender differences still in existence. In particular, a study from Sweden with 9th and 11th grade students showed that students still viewed mathematics as a male domain (Brandell, Nystrom, & Sundqvist, 2004). Another study from Finland reported that teachers held different beliefs about girls and boys in their classroom, believing that girls tended towards routine procedures whereas boys use their power of reasoning (Soro, 2004). These findings and numerous other studies from the U.S (Wiest, 2004), South Africa (Mahlomaholo & Sematle, 2004) , Australia (Forgasz, 2004) and Iran (Pourkazemi, 2004)suggest that not much has changed in terms of society’s dominant conceptions of mathematics. In addition to the aforementioned studies PISA, provided documented statistically significant gender differences in achievement in favor of boys both in the year 2000 and 2003. In the year 2000 statistically significant gender difference in achievement were found in 29 countires of the 41 participating countires. In year 2003 statistically significant gender differences in achievement were found in 27 countires of the 41 participating countires. The only country in PISA 2003 which had statistically significant gender differences in achievement <u>in</u>

favor of girls was Iceland. In PISA, many items in the content strands of "change and relationship" and uncertainty implicitly assumes facility with proportional reasoning. Sriraman & Lesh (2006) claim that proportional reasoning and estimation are two of the *most useful types of elementary mathematical thinking relevant for modeling situations.* Finally, one of the seminal influences on the field of mathematics education, Zoltan Dienes, has repeatedly emphasized the valued of proportional reasoning to cultivate pure mathematical thinking (see Dienes, 2000, 2004; Sriraman & Lesh, 2007). So, it is of great interest to the Nordic community in particular and the mathematics education community in general to carefully examine the nature of gender differences among Icelandic students in proportional situations (Steinthorsdottir & Sriraman, pre-print a).

Piaget argues that proportional reasoning emerges at the formal operational stage of learning. On the other hand he did not take into account the influence of the context in which proportional reasoning was to be applied (Saunders & Jesunathadas, 1988; Suarez, 1977; cited in Karplus et al., 1983). Therefore, Piaget, left many questions unanswered about other influences on reasoning strategies. Among these are problem context, number structure, and children's ethnomathematical knowledge. Various researchers have suggested there is a complex interaction of factors that influence performance in solving proportional problems. In fact, Resnick (1983) stated that "...a person's intelligent performance is not a matter of disembodied 'process of thinking' but depends intimately on the kind of knowledge that a person has about a particular situation in question"(p. 478).

Mastery of proportional reasoning is a key requisite for success in learning higher mathematics (Lesh, Post, & Behr, 1988). Attempts to define the domain of proportional reasoning have led to the delineation of various rational number constructs. Within these constructs, researchers have identified variables that contribute to an individual's ease or difficulty in solving problems. Problem context and number structure are among these variables and therefore may influence one's use of problem solving strategy. We have surmised that problem semantic type, number value and individual understanding of a given mathematical situation influence one's use of strategy. The first author designed a study which examined these variables. The project attempted to determine which semantic type and number structures are critical to an individual's use of strategies to solve missing value proportion problems. In addition, this study examined whether there are gender differences in strategy use. An individual's understanding is demonstrated by the strategies he or she uses. These strategies can be categorized in order of mathematical sophistication.

Number structure and semantic type exist side-by-side in proportional problems; both exert an influence on an individual's use of a solution strategy. In this study, we attempted to sort out these two factors to identify the extent of the influence of each on proportional reasoning strategies in a population of fifty three eighth graders. Specifically we asked the following three questions:

1. How does problem semantic type influence use of strategy within defined number structures?
2. How does number structure influence use of strategy within defined semantic type?
3. Are there gender differences in strategy use in solving missing value proportional problems?

In the first section of this paper we outline findings that have been reported in the field of proportional reasoning and gender differences in mathematics, important for Nordic readers unfamiliar with the extant literature in this area of research. We then describe the design of the study based on this information. Then we describe the four semantic types and four number structures which we believe will most influence one's use of strategy. We outline strategies expected to be found and briefly discuss factors contributing to development. In addition, the theoretical basis for these categories of strategies are described.

In the second section the methodology and the data analysis are described. The third section reports the results of the study, followed by discussions of the findings. In the final section, implications for instruction are discussed.

CONCEPTUAL FRAMEWORK AND BACKGROUND

In this section, the theoretical basis for the use of the semantic types, number structure, and classification of strategies in the study are discussed. The first part addresses the relation of semantic type and proportion. The second part addresses issues related to proportion and number structure. The third part examines gender differences and mathematics. The fourth part addresses the strategies used in proportional problems. Finally the development of strategies used in proportional problems are outlined.

Semantic Types and Proportion

In Vergnaud's (1994) outline of his theory of the multiplicative conceptual field, he discusses the need to critically analyze and classify problem situations to better understand cognitive tasks associated with what it means to do mathematics (Vergnaud, 1994). Based on our assumption that the semantic types influences one's ability to solve missing value proportion problems, semantic types are used as one way to analyze and classify problems. Our belief that semantic types are defined by the problem's inherent meanings of quantity is also influenced by the classification of addition and subtraction problems associated with the research based on the Cognitively Guided Instruction model (Fennema, Carpenter, Jacobs, & Levi, 1996).

The concept of ratio and proportion as applied by young people has been widely studied. Piaget and his collaborators identified proportionality within Piaget's stage of formal operational reasoning (Inhelder & Piaget, 1958). Children were found to demonstrate an intuitive understanding of proportionality before they could deal with problems quantitatively. Some of Piaget's results have been criticized for the use of complex physical tasks to assess proportional reasoning, and therefore underestimate the influence of problem semantic type (Sriraman & English, 2004). Research has shown that the student's degree of familiarity with the problem type affects problem difficulty. For example, Tourniaire (1986) reported that mixture problems are more troublesome to students than other semantic types of proportional problems. Also, given two equally familiar problem type, if one problem type is more familiar to a student as using ratio or proportion, it will more like facilitate performance then the other problem types. Furthermore, the location of the missing element in relation to the other three numbers in a proportion has an influence on children's thinking (Tourniaire & Pulos, 1985). Questions have been raised about discrete or continuous elements. Whether one element is more difficult then the other remains unresolved (Behr, Lesh, Post, & Silver, 1983; Lamon, 1989).

In an effort to examine the influence of problem type on solution strategies, Lamon (1989, 1993b) developed a framework of semantic problem types for classifying proportion problems. Lamon grouped problem situations into four categories: well-chunked measures, part-part-whole, associated sets, or stretchers and shrinkers. In individual clinical interviews with students, Lamon found the various semantic problem types elicited different levels of sophistication in solution strategies. It is not obvious from her study, however, if or how she controlled the number structure used in her problems or what influence number structure had on her results.

Number Structure and Proportion

An examination of the literature suggests that number structure plays an important role in strategies used by students to solve proportional problems. Students deal most easily with numbers between one and thirty, while numbers less than one and greater than thirty are more difficult (Hart, 1984). Also, working with whole numbers is easier for students than working with fractional numbers (Bell, Fischbein, & Greer, 1984). Unit ratios, especially 1:2 facilitate solutions more so than other fractional numbers (Noelting, 1980a, 1980b). Tourniaire and Pulos (1985) outlined three difficulty factors associated with number structure: presence or absence of integer ratios, placement of the unknown number in a problem, and numerical complexity -- that is, the size of the numbers used and the size of the ratios.

Karplus, Pulos, and Stage (1980, 1983) define relationships of the quantities used in proportion problems by focusing on whether an integral relationship exists within the ratio or between ratios. "Within" refers to the relationship of quantities in one ratio of the problem; "between" refers to the relationship between the ratios in the problem. Karplus, et.al. also defined the relationship as integer or noninteger. For example, the problem 2/4=12/x has integer multiples both within (2x2=4) ratio and between (2x6=12) ratios. A noninteger ratio, on the other hand, is when the multiplicative relationship within a ratio or between ratios is not an integer.

Abramowitz (1975) identified three kinds of number structures. The first is termed differences (equal/unequal), and refers to the presence or absence of a repeated difference between the measurement used. For example, equal differences would be 4/6=6/x where the difference between 6 and 4 (6-4 =2), is the same within the ratio and also between ratios. An example of unequal differences would be 4/6=10/x, where the difference between 6 and 4 (6-4=2), is not the same as the difference between 10 and 4 (10-4=6). The second number structure, size (larger/smaller), refers to whether the unknown number is larger or smaller than the known number. The third category is the type (simple/complex/multiple), which refers to the ratios used in the problem. This refers the number structure were there are integer or non-integer ratios and the answer is an integer or non-integer number.

These views from the literature indicate that among the crucial numerical factors in problem difficulty are the presence or absence of an integer ratio relationship and of an integer unknown. Also considered are views from the literature summarized in Abramowitz's four categories of number structure: (1) the differences (equal/unequal), (2) the size (larger/smaller), (3) the order, and (4) the type.

Gender and Mathematics

Research on gender and mathematics conducted in the last twenty years, provides evidence for the existence of gender differences (Lubienski & Bowen, 2000; Steinthorsdottir & Sriraman, 2007). Some research also implies the need to look at gender differences from a different viewpoint to provide us with a deeper understanding of the situation. Since Fennema published her first article in 1974 about gender, gender differences in mathematics education have attracted the attention of many researchers from different areas.

Causes of gender differences in learning mathematics indicated by these studies vary. Suggested causal factors include: seeing mathematics as a male domain, "math anxiety," the perceived usefulness of mathematics in one's future, teachers' differential treatment of female and male students, the perception of self as a learner of mathematics (confidence levels and attribution), classroom structures (cooperative vs. competitive), possession of autonomous learning behavior, and as well biological factors. (For more extensive review see Leder, 1992 or Oakes, 1990.)

Fennema and Sherman (1979) found that girls in early adolescence experience a drop in their self-confidence in mathematics, even prior to indications of decline in mathematics achievement. Hyde, Fennema, Ryan, Frost and Hopp (1990) reported that girls in upper grades dislike mathematics more then girls in lower grades. Hyde, et. al. also reported that girls in upper grades are less confident in doing math. Level of confidence influences students' persistence to solve high-level mathematics tasks and to study mathematics (Hyde et al., 1990). Studies have also reported differences in classroom interaction, with boys interacting more with teachers, and teachers interacting more with boys. In addition, teachers posed only a minimal proportion of higher-level questions to girls. This implies that since high-level tasks are essential for mathematical understanding, girls are being shortchanged if they are interacting less with teachers on those tasks (Fennema & Peterson, 1987; Stallings, 1985).

The concept of mathematics as a male domain also contributes to gender differences. Studies have reported that male stereotyping mathematics influences girls' choice, of whether or not to pursue mathematics (Hyde et al., 1990, Fennema & Sherman, 1979). Several studies reported that girls showed more tendencies to experience math anxiety (Tobias, 1978; Eccles, Meece, & Wigfield, 1990).

Fennema and Peterson (1985) introduced autonomous learning behavior as a possible explanation for gender differences in mathematics. Fennema and Peterson's Autonomous Learning Behavior (ALB) model combined different factors that had shown significant gender differences. The ALB model is influenced by internal factors (confidence, perceived usefulness, and attribution style) as well as external factors (teacher-pupil interaction). Fennema and Peterson's assumption was that greater participation in autonomous learning activities would lead to greater development of ALB, which in turn would lead to improved performance on high level cognitive tasks. Fennema and Peterson reported that girls who had less confidence in their mathematical ability, perceived less usefulness of mathematics in their future life, and attributed failure on mathematical tasks to lack of ability, were less likely to adapt ALB. Seegers and Boekaerts (1996) offered another model which also attempted to combine different cognitive factors and influence on gender differences in mathematics. Seegers and Boekaerts' model suggested that "trait-like self-referenced cognition" (goal orientation, attributions, and self-concept of ability) influenced "task-specific appraisals" (personal relevance, task attraction, and subjective competence) and outcomes (learning intention and task performance).

Previous work on gender differences in mathematics education has focused on the differences between male and female achievement and learning styles. The tendency in this research has been to identify the characteristics of those who do well in mathematics and then use those characteristics as an explanation for observed gender differences (Damarin, 1995; Jacobs, 1994; Willis, 1995). Those identified characteristics have been favorable to males, suggesting that the solution is for females to adapt male behavior. The current feminist critique of education have challenged this belief, denying the assumption that to become equal is to become male (Damarin, 1995; Jacobs, 1994; Willis, 1995), and has invited an alternative way to investigate gender differences. Researchers argue that substantive differences exist in male and female learning styles and ways of thinking, and that female learning styles may be equally valid (Belenky, Clinchy, Goldberger, & Tarule, 1986).

Research conducted around children's strategy use, has not attracted much attention. It has been assumed until recently, that gender differences did not exist in solution strategies. The few studies that have been conducted indicate that gender differences do exist in strategies used to solve complex problems.

Gallagher and De Lisi (1994) reported gender differences in strategy used by high ability high school students on Scholastic Aptitude Tests (SAT-M). While overall differences did not exist in the number of correct answers, differences did exist in use of strategy. Gallagher and De Lisi's study showed that high-ability girls tended to use more conventional strategies (application of commonly-taught algorithms) and high-ability boys tended to use more non-conventional strategies.

Marshall and Smith (1987) looked at the responses and performance of the same group of students in third grade and then in sixth grade. The instrument they used was the Surveys of Basic Skills, developed and administered by the California Assessment Program. Marshall and Smith's findings indicated that in both grades, girls performed better than boys in computational problems. In third grade, girls performed better in most areas evaluated. By sixth grade, strong performance by girls relative to boys declined in nontraditional items, word problems, and geometry/measurement items. In addition, the results in general showed a relation between performance on skill items and performance on corresponding application items for boys and girls in both grades. However, girls were more likely to be successful in solving the acquisition items (arithmetic computation) and less likely to solve matching application items (corresponding word problems). Boys were less likely to be successful in solving the acquisition items yet more likely to solve associated application items correctly.

Carr and Jessup (in press) documented gender differences in strategies in primary school children. In their study, girls in grades 1-3 tended to use easily observable strategies (such as counting) while boys tended to use mental strategies. Fennema, Carpenter, Jacobs, and Levi (1996) documented similar solutions. While for the most part, gender differences were not identified in the number of correct solutions, significant differences existed in use of strategies. The only problem type that showed gender differences in the number of correct solutions was extension problems. The extension problems were more complex problems and the difference was in favor of boys. Fennema, et al. documented that girls tended to use more concrete strategies, such as modeling/counting strategies, while boys tended to use more abstract strategies, such as invented algorithms.

Gender differences in proportional reasoning have been identified (Tourniaire and Pulos, 1985). In these cases, boys outperformed girls. Karplus Pulos, and Stage (1983) found no gender differences except on the noninteger problems, in favor of males. A study done by Pulos, Karplus and Stage (1981) reported that content type interacted with gender. It has also been suggested that gender differences increase with age. Linn and Pulos (1983) focused on the source of the gender differences. Their conclusion was that intelligence, spatial visualization, cognitive style, or formal reasoning could not explain the gender differences. Consequently, motivation or attitudinal factors are more likely to be the source of gender differences.

In Iceland, gender differences have not been well documented until recently. The only two sources of information is the Icelandic Standardized Test (National Institute for Educational Research, 1993, 1994, 1995, & 1996) given to all students at the end of tenth grade and the Third

International Study , TIMMS (Beaton, Mullis, Martin, Gonzalez, Kelly, & Smith, 1996). Both sources report only the achievement scores. According to TIMMS significant gender differences do not exist in achievement in seventh and eighth grades. When looking at the content areas, very small differences exist within these areas. In seventh grade, girls did better in solving algebra problems but boys did better in proportional reasoning. In eighth grade, there were no significant differences in any area, but as in seventh grade, girls did somewhat better in solving algebra problems and boys did somewhat better in proportional reasoning. Overall, students' attitudes towards mathematics were positive and no gender differences were found in their attitude. The Icelandic Standardized Test showed similar results from 1993 to 1996. There were no significant differences in achievement except on the 1996 test, where there was a significant difference in favor of girls. Looking at the content areas, overall girls did better in standard algorithms and algebra and boys did better in category labeled "mathematics related to daily life". This content area included problems about proportion. In PISA 2003 statistically significant gender differences in achievement in favor of boys was found in 27 countires of the 41 participating countries. The only country in PISA 2003 which had statistically significant gender differences in achievement <u>in favor of girls</u> was Iceland. One of the more popular explanations is so called "jokkmokk" effect. To explain it simply, jokkmokk effects refer to this "phenomena" of females outperforming males academically in rural areas. It suggest that the environment, such as the labor market, prevent males to see value in academic education, on the contrary the same environment encourage females to do well in school in the hope of achieving some status in their future or leave their hometown in search for a "better" life. Applying this idea to the Icelandic situation it probably has some effect but to believe it is the answer is naïve. Studies such as the one we are reporting in this paper, which document gender differences in learning trajectories for particular mathematical concepts, is a more rational and scientific way of explaining gender differences.

Studies conducted around children's strategy use indicate that choice of strategies when solving complex mathematics problems does differ between girls and boys. Also, in these studies there is no identified differences between girls and boys in number of correct solutions. Data from Iceland shows similar trends, that is, there in no overall differences in number of correct solutions but there are implications that girls do better then boys in different kind of problems and vice versa.

Proportional Reasoning Strategies

The test problems in this study are structured to help investigate how different semantic structures and different ratio types may influence the use of strategies. The research on proportional problem-solving suggests that there are three categories of strategies used in reasoning about proportional relationships: qualitative, additive, and multiplicative. These strategies represent different levels of sophistication in thinking about proportions. We describe each category and provide an example of how the strategy might be applied to the missing value tasks, and then briefly discuss how proportional reasoning may be developed.

Research with preadolescent students indicates that students' representation of situations involving ratio and proportion occur on an informal, qualitative basis long before students are capable of treating the topic quantitatively (Tourniaire, 1986). The qualitative reasoning strategy is based on informal or intuitive knowledge of relationships without numerical quantification (Kieren, 1993). This informal knowledge includes a visual understanding of ratio and proportion. This is seen in young children as they express comparisons among quantities, increasing/decreasing amounts and part/whole relationships. This strategy is characterized by the use of comparison words such as, bigger and smaller, more or less, to relate to the quantities in question. The following protocol, provides an example of a qualitative reasoning strategy:

> Interviewer: If a 2 foot fish needs 4 food pellets, how many food pellets does a 3 foot fish need?
> Subject: More than the 2 foot fish gets.
> Interviewer: Why? Can you explain?
> Subject: Because the fish is bigger so it needs more food.

The subject is aware that the amount of food should increase with the size of the fish but does not (or cannot) quantify the amount. This qualitative reasoning is characteristic of young children but does not disappear when more formal strategies develop. Qualitative reasoning continues to be used in proportional problem solving even by people who have the ability to reason proportionally (Behr, Harel, Post. & Lesh, 1992; Kieren, 1993; Resnick & Singer, 1993).

Additive reasoning requires quantification of the ratio relationships and is more sophisticated then qualitative reasoning (Tournaire, 1986). Additive reasoning is often called build-up strategy. It is an attempt to apply knowledge of addition or subtraction to proportion. To use the strategy, a child notes a pattern within a ratio and then iterates it to build up additively to the unknown quantity. For example, to decide how many baseball cards can be purchased for $5 if 2 can be purchased for $1, an individual might reason:

> 2 cards for $1
> 4 cards for $2
> 6 cards for $3
> 8 cards for $4
> 10 cards for $5

arriving at the solution of 10 cards. Additive strategies or build-up strategies are often observed during childhood and adolescence. This appears to be the dominant strategy for many students during these ages (Tourniaire & Pulos, 1985). The additive strategy can be used successfully to solve problems with integer ratios but can lead to error if applied to noninteger ratio problems.

Multiplicative strategies acknowledge the covariance of ratio quantities and can be applied to both integer and non-integer problems. This is a more sophisticated form of reasoning then additive reasoning. Multiplicative strategies are grounded in understanding that the two ratios in a proportion are equal. Two types of multiplicative strategies have been identified: within and between (Karplus, Pulos, & Stage, 1983; Noelting, 1980b). The within measure strategy (also called a scalar strategy) is based on applying the multiplicative relationship within one ratio to the second to produce equal ratios.

Within Measure

$$(\times\$.32)\frac{20}{\$6.40} \;:\; \frac{100}{x}(\times\$.32)$$

The student determines the within ratio multiplicative relationship is .32 and applies this same relationship to 100 to get $32.

Between Measure

$$(\times 5)$$

$$\frac{20}{\$6.40} \;:\; \frac{100}{x}$$

$$(\times 5)$$

The student determines the between ratios multiplicative relationship is 5 and applies this to $6.40 to get $32.

The between measures strategy (also called a function strategy) is based on determining the multiplicative relationship between corresponding parts of the two ratios and creating equal ratios. For example, either a within measure or a between measure strategy can be used to find out how much money is needed to buy 100 stamps when 20 stamps can be purchased with $6.40.

As noted in the previous sections, research shows that semantic type of the problem as well as the numerical relationship in the problems may influence the choice of one multiplicative strategy over an other (Karplus, Pulos, & Stage, 1983; Tourniaire & Pulos, 1985). However, students may look for the “easier” numerical comparison among the ratio relationships. They then choose a within or between strategy based on that information. This belief is supported by Thompson (1994) who states, “When we focus on the mental operation of multiplicative comparison, it is evident that it makes no difference if the quantities are of the same dimension or not” (p. 191).

Error strategies in proportional reasoning have been documented in the literature. Two types of error strategies have been frequently observed by researchers. The first error strategy is when students ignore part of the information given in the problem. For example, student might solve the problem by comparing just two of the numbers in the problem (Hart, 1981; Karplus, Pulos, & Stage, 1983). A second type of frequently used error strategy is the ratio differences, sometimes called additive strategy. In this strategy, students use the difference between the numbers within a ratio or between the ratio and then apply this difference to the second ratio to find the unknown (Hart, 1981; Inhelder & Piaget, 1958; Tournaire & Pulos, 1985). The ratio difference is often use as a fall-back strategy when dealing with a noninteger ratio (Karplus, Pulos, & Stage, 1983). A student might also use the ratio difference when treating the remainder in a problem (Tournaire & Pulos, 1985).

Development of Strategies

This study examined the influence of semantic structure and number structure on the use of qualitative, additive or multiplicative strategies. It was not designed to assess a model of development of these strategies over time. However, it is appropriate to summarize a few aspects of the literature in this respect.

Inhelder and Piaget (1958) proposed a developmental sequence of proportional reasoning. First the theory argues that students use only part of the information given in the problem to form a qualitative response. Second, students use the ratio difference to find the unknown quantity in the proportion. Next, students are capable of understanding some of the numerical relationships, such as that the differences change with the size of the number. On the other hand, students fail to recognize that the numbers form an equal ratio. Fourth is the so-called pre-proportional stage. At this stage, students understand some of the relationship between the two ratios and develop an efficient strategy. One of them is called the build-up strategy. Finally, students reach the proportional reasoning level, where the student understands both the scalar and the functional relationship between the two ratios.

Some researchers have criticized Piaget’s work. Karplus, Pulos, and Stage (1983) argue that that the Piagetian tasks assessed much more complex reasoning than proportion. Noelting’s (1980a, 1980b) answer to Piaget’s developmental sequence was to report somewhat different developmental stages in proportional reasoning. Noelting’s developmental sequence is built up on children’s understanding of integer and noninteger relationship within and between ratios.

We believe the strategies that have been outlined develop in a developmental sequence determined by the personal experiences, sense making efforts, and cognitive schemes of each child. Although children generally go through developmental levels of understanding in building strategies, each individual brings a unique set of experiences to bear on problem solving. The fact that reasoning is seen in a developmental sequence, reaching one of the more sophisticated categories does not mean a student operates consistently on that level of reasoning. The use of fallback strategies is well documented in the proportional reasoning literature (Tourniaire & Pulos, 1985; Lamon, 1993a; Kaput & West, 1994). Factors within problems, such as their semantic structure or the kind of numbers that are used, may relate to this inconsistent use of strategies. The particular strategy one uses to solve a given problem will reflect the individual's experiential background and cognitive resources at that point in time. The repertoire of resources expands and changes as the individual experiences and makes sense of increasingly complex situations.

Since many researchers believe that true proportional reasoning is defined as multiplicative, the change from the use of additive to multiplicative strategies is also a benchmark. Knowledge of the factorial structure of numbers influences this change (Resnick & Singer, 1993). Students might need multiple experiences with factorial composition and decomposition of numbers. If an individual's experience is based solely on composing and decomposing numbers additively, only additive strategies can be applied to proportional situations. Experience with factorial number structures permits use of multiplicative reasoning in proportional situations.

THE STUDY

This study was conducted to investigate the influence of semantic type and number structure on individuals' use of strategies in solving missing value proportion problems, and to examine gender differences in strategy use. Fifty-three eighth graders in one school in Reykjavik, Iceland, participated in this study. Twenty-seven females and twenty-six males, were individually interviewed as they solved sixteen missing value proportion problems. The problems represented four semantic structures: Well chunked (W-C), part-part whole (P-P-W), associated sets (A-S), and symbolic (S-P). For each semantic structure there were four problems, each representing a distinct number structures: Integer-integer with an integer answer (I-I-I), integer-noninteger with an integer answer (I-N-I or N-I-I), noninteger-noninteger with an integer answer (N-N-I), or noninteger-noninteger with a noninteger answer (N-N-N).

In this section the structure of the missing value proportional problems used in my study and the categorization of strategies is described. We first define the semantic structures of the problems. Then we define the number structures of the problems and finally the six categories of strategies used in the study are outlined.

Semantic Type and Proportion

Reading through the proportion problems used in research with children and in various math text books one finds that problems can be classified using Susan Lamon's (1993b) four semantic structures: well chunked, part-part-whole, associated sets and stretchers and shrinker. Each of these categories are defined and explained as to how they are applied to missing value proportion problems in this study. By choice, to use stretcher-shrinkers problems were not included. Instead, as the fourth category symbolic problems, i.e., problems presented in mathematical symbols were used.

The well chunked category refers to situations in which two extensive measures (for example, dollars and items) are compared to result in an intensive measure, or rate (dollars/item). This rate is well known or commonly understood, hence the name well chunked (Lamon, 1993b). For this study, in a well chunked proportional problem one intensive measure and one corresponding extensive measure was given and subjects were asked to find the missing extensive measure. For example, "If 3 candy bars cost $1, how many candy bars can you buy with $6?" (The first three problems in this category use ratios which compare two discrete quantities, distance and hours. The fourth example uses ratios which compare a continuous quantity, pounds of candy, to a discrete amount.)

The part-part-whole semantic type refers to situations in which ratios compare two subsets of one whole set. The two subsets are easily understood to be parts of one whole (Lamon, 1993b). Mixture problems (orange juice-water mixture) would fit this category. For this study, in a part-part-whole proportional problem, two parts of a whole and one corresponding part of a second whole were given and subjects were asked to find the value of the second part of the whole. An example of such a problem is, "If a certain school has 3 boys to every 4 girls in each class, how many boys are there if there are 20 girls?" In the first three problems, individuals needed to compare two discrete subsets (boys to girls, apartment to apartment) which compose a whole unit (a classroom, a building). In the fourth example they needed to compare two continuous quantities (orange juice and pineapple juice) which compose one unit (fruit juice).

The associated sets category refers to a problem type in which two elements are compared and their relationship is defined by the problem itself. The two elements have little or nothing in common without the connection being made in the problem setting

(Lamon, 1993b). This category contains problems involving ratios of cats to dogs or cars to people; they are two distinct items which are compared because of the problem setting. In the first three examples, students were given a comparison of two discrete items (number of people/animals and number of pizza/scoops of food). They were asked to determine how much food is needed to correspond to a given ratio. In the fourth example, students needed to think about a discrete number of foxes and a continuous amount of food (pounds of food).

The fourth problem category is called symbolic problems. In this category two ratios are presented in mathematical symbols and compared without context. The ratios are all in the form of an equation such as 3/7=x/28.

Several other factors are inherent in the problem's semantic structures which require different kinds of thinking in proportional situations and may influence an individual's use of strategy. We control or limit the influence of these factors as much as possible. Some research suggests that if a problem setting is not familiar, individuals may have difficulty finding a solution (Tourniaire & Pulos, 1985). Therefore settings were created in this study, which would be recognizable for all. For example, there are no physics or chemistry settings, since subjects were eighth grade students.

Also controlled, is the order in which the missing value is presented so that it is consistent with the order in which the two elements are compared in the first ratio. For example, if the situation is described as a ratio of boys to girls, the question asks, "If there are 20 boys how many girls are there?" This sequencing may guide subjects' thinking and facilitate an individual's ability to understand a proportional relationship between the two ratios.

The decision to make the fourth problem in each category represent a situation involving a continuous quantity was dictated by the number values chosen to explore. To write a proportional problem setting in which it would be reasonable to have a non-integer solution required a continuous interpretation of the unknown quantity. In other words, an individual cannot consider part of a “discrete” quantity unless he or she thinks of that quantity as being continuous. While we tried to isolate the semantic type from number structure to simplify the theoretical descriptions, it became apparent that the two aspects are intertwined, illustrating the complexity of systematically studying these constructs.

Number Structure and Proportion

In this section we consider the views from the literature summarized in Abramowitz’s (1975) four categories of number structure: (1) the differences (equal/unequal), (2) the size (larger/smaller), (3) the order, and (4) the type. In my study I applied these four number structures to equal missing value proportional problems. The first variable “difference” is constant; all problems have unequal fractions, that is, the differences between and within fractions are not equal. We believe that this does not encourage ratio difference strategies. The second variable “size” is also constant, the unknown number is always larger than the known. The third variable “order” is another constant, that is, the place of the unknown number is always the same. The fourth variable “type” is the one that we allowed to remain a variable. That is the integer and the noninteger ratio relationship.

By allowing the "type" to remain a variable, we looked specifically at how types of ratios used in the problem influence the use of strategies within each semantic type. We used four types of numerical relationships in the problem. The first was a simple whole multiple, or an <u>Integer-Integer ratio</u> with the unknown an integer. In the first type, the integer multiplier was found both within the ratio as well as between the ratios. The second was also a simple whole multiple, an <u>Integer-Noninteger ratio</u> or a <u>Noninteger-Integer ratio</u> with the unknown an integer. Test problems were created in these categories, in such a manner that the integer relationship exists either within a ratio or between ratios. By comparing equally difficult problems, one hoped to see a preference in strategy use. Karplus, Pulos, and Stage (1983) make a clear distinction between problems having a within integer ratio and those having between integer ratios. They suggest that young children prefer to use the within ratio strategy. The third type is a complex multiple, or a <u>Noninteger-Noninteger ratio</u> with the unknown an integer. The fourth type is a complex ratio, or a <u>Noninteger-Noninteger ratio</u>, with the unknown a noninteger. We consider this last type to be the most difficult one and this type is included to determine if individuals are consistent with use of strategies. Within each of the four semantic types outlined in the previous section, we have written four missing value proportional problems using the numerical relationships described above. It cause some complications to write problems that fit both structures.

Categorization of Strategies.

This section will describe in which way strategies were categorized. These categories were structured around strategies used in proportional reasoning, as presented in the literature discussed above.

We used six categories of reasoning strategies; no conceptualization, qualitative, ratio differences, additive, combined , and multiplicative strategies. This classification forms a hierarchy of reasoning sophistication. In Lamon's terminology these categories are labeled "nonconstructive" or "constructive" solution strategies (Lamon, 1993b), where the nonconstructive strategies are incorrect strategies and constructive strategies are the types of correct strategies.

The nonconstructive strategies are no conceptualization, qualitative, and ratio differences strategies. Noconceptualization was used if the subject made no attempt to solve a problem or used a number randomly and could not provide a reasonable explanation. A strategy was classified qualitative if a subject considered the numerical relationship and used estimation to quantify the answer. A ratio differences strategy was identified when subjects used the differences of the numbers in the ratios, either the difference within the ratio or between ratios, to create the second ratio.

Constructive strategies are additive, combined and multiplicative strategies. Additive strategy was identified when a subject used the given ratio to build up additively to find the missing value in the second ratio. Strategies classified as combined were when a subject used one strategy to get near the target number (the known number in the second ratio) but used a different strategy to adjust for a noninteger multiplier. A strategy was labeled mulitplicative if a subject applied only multiplicative reasoning either within or between measure spaces to achieve a solution.

METHOD

In this section the methodology employed in the study will be discussed. Before going further, it is important to provide a brief explanation about the educational system in Iceland and the population of the country. Then, various parameters such as sample used in the study, their background and the selection process, are described. Finally data collection, instrumentation and analysis methods are discussed.

The Educational System in Iceland

The educational system in Iceland requires 10 years of compulsory schooling, consisting of elementary school (first through fourth grades), middle school (fifth through seventh grades) and secondary school (eighth through tenth grade). Although there are such divisions, all ten grades are housed within one building. Nevertheless, passing from seventh to eighth grade is considered to be a difficult step for students. From first to seventh grade, the Icelandic student has the same teacher for most subjects, in addition, very frequently students have had the same teacher since fifth grade; however, beginning in eighth grade the student has a different teacher for each subject. This transition is difficult for students, in mathematics, as well as in other subjects. The amount of material covered during the school years increases enormously, along with changes in the nature of the material. Percentage, proportion, and algebra are the main focus, which is mostly new to them. In addition, the presentation in the textbooks changes, from the child-like, informal books of the earlier grades to more substantial scholarly books beginning in eighth grade. Finally, when students enter eighth grade they start to feel pressure, from both parents and school, due to the standardized test which will come at the end of tenth grade.

Another issue which is important is that Iceland is very homogeneous. For that reason one need not look into multicultural issues concerning students. Around 98% of Icelanders are white Nordic people and around 98% belong to the same church. For a more detailed history see Bjarnadottir (2007). It should also be mentioned that the definition of economic classes is not as distinct as in the United States. Icelanders usually do not define themselves as belonging to on class or an other. Nevertheless, differences do exist in incomes and living standards which can often influence children.

The students in the study were aged 13-14 and had just started their second semester in eighth grade. The Icelandic National Curriculum dictates that in the second semester of sixth grade and seventh grade, students must have been introduced to proportion and ratio (Icelandic National Curriculum, 1989, pp. 142-156). Here are some typical problems the students had been working with:

- kg of butter cost 1200 kr
  1 kg of butter cost _____kr
  kg of butter cost _____kr

- 4 chickens weigh 5.2 kg
  3 chickens weigh _____kg

- For 6 parcels you need 4.2 m. of ribbon to tie around it.
  So for 2 parcels we would need ______m of ribbon

- A group of kids are going on a cycling trip. The journey is 30 km. They travel at an average 12 km. per hour. How long will it take them to reach the end assuming that they will not stop?

- In a class there are 24 students, 1/6 of them have brown eyes, how many students are there with brown eyes?

- Draw a graph and read from it:

price per kg
Icelandic krona per foreign currency
km per hours

- Use road maps and its scale to find the real distances.

At the time the interviews were conducted, the students had not been formally introduced to the algebraic form of solving missing-value proportional problems.

Sample Selection

The population for this study, which included 27 females and 26 males, consisted of eighth-grade students in one school in Reykjavik, the capital of Iceland. This school is one of the largest compulsory schools in Reykjavik. It is in a neighborhood that is one of the newest resident areas in Reykjavik. Families with a wide range of income live there, ranging from government supported housing project for low income families and single parent families to upper middle class families.

The school where the study was conducted had four eighth grade classes, each with a different mathematics teacher. They were all mixed classes, that is, the range of students was from students with learning disabilities to outstanding students, with approximately same number of females and males. Two of the four classes were randomly selected to participate in the study. All students of the two classes except one gave permission to be interviewed. The one exception asked his parents for permission not to participate in the study because of a learning disability. Other students with learning disabilities or difficulties did participate and their results were included in the data analysis.

Interview Procedure

Each of the 53 students was interviewed individually by the first author (native of Iceland) and audiotaped. The students were asked to solve 16 missing value proportional problems. In the process of solving the problem field notes where taken to capture students work. Individual students required between forty to eighty minutes to complete the 16 problems. The 16 problems were categorized by semantic structure and number structure. There were four semantic types: well chunked (W-C), part-part whole(P-P-W), associated sets (A-S), and symbolic (S-P). There were also four number structures, integer-integer with an integer answer (I-I-I), integer-noninteger with an integer answer (I-N-I or N-I-I), noninteger-noninteger with an integer answer (N-N-I), and noninteger-noninteger with a noninteger answer (N-N-N). Each semantic type had four problems, each with different number structure (table 1). The problems' content was selected to equally favor both boys and girls.

The simplest problem in each semantic type had an integer-integer ratio relationship with an integer unknown (I-I-I). Each I-I-I task had a given ratio from the equivalency class of 1/3, the largest denominator used being 15. Integer relationships existed both within and between ratios and the unknown is an integer with a value of 15 or less.

The second problem in each semantic type has an integer-noninteger or noninteger-integer ratio relationship with an integer unknown (I-N-I or N-I-I). Each I-N-I or N-I-I problem had a given ratio in which the numerator and denominator differed by two. No denominator in a given ratio was larger than 7. Integer relationships were found in one direction only (within <u>or</u> between ratios). The unknown was an integer with a value of 15 or less.

The third problem in each semantic type had a noninteger-noninteger ratio relationship with an integer unknown (N-N-I). Each N-N-I problem had a given ratio in which the numerator and denominator differed by two, four, six, or eight. No denominator in a given ratio was larger than 16. There were no integer ratio relationships either within or between ratios. The between ratio relationship was always a integer plus a half, X.5. The unknown was an integer with a value of 28 or less.

The most complex problem in each semantic type had a noninteger-noninteger ratio relationship with a noninteger unknown (N-N-N). Each N-N-N problem had a given ratio in which the numerator and denominator differed by 1. No denominator in a given ratio exceeded 6. There were no integer ratio relationships either within or between ratios. The unknown was a noninteger less than 18.

Throughout the interview, in the problems used were both given and unknown numbers less than 45. This placed a limitation on the number values in some of the problems. Other aspects of number structure in the problems were also controlled. The unknown was in the same position in every problem and was always larger than the known. (See Table 1).

The sixteen problems were presented in a random but predetermined order during individual interviews with each student. Each student was asked the questions as they are numbered in Table 1. If the student did not understand the question, the question was rephrased. Follow-up questions were sometime used, depending on the student's previous response(s), to determine if the student truly understood the proportional relationship in the problem. Frequently, the follow-up questions were to discuss whether the students could find another method to solve the problem. In some cases, when it was clear that a student was struggling and getting frustrated with a particular problem, they were provided assistance in solving the problem but marked it as unsolved for further analysis.

Students were provided with paper and pencil to write what they felt necessary. They were repeatedly encouraged to describe their thinking as they solved each problem, whether in writing, drawing, orally, or a combination of these. If their solutions and strategies were not clear, they were asked to explain their thinking further. The students were also told that if they could not understand the question they could ask for clarification, or if they could not solve a problem they could ask to proceed to the next question. Field notes were taken during the interview. In addition, the interviewed the tape recorded.

Data Analysis

An individual's response to each problem was examined using field notes, tape recordings, and any written work produced by the students. The interviews were not transcribed, but used as reference if field notes were not clear. The reasoning strategy used in each problem were categorized even if it did not lead to a correct numerical answer. All student responses fit into one of six classifications (see Table 2) which formed a hierarchy of reasoning sophistication. Lamon's terminology was borrowed in labeling these categories 'nonconstructive' or 'constructive' solution strategies (Lamon, 1993b). Problem number fifteen, a part-part-whole, N-N-I problem was excluded from the analysis. This was because of the consensus that it did not represent children's proportional reasoning in this category, due to their difficulties of understanding the context of the problem. This question had to be reframed for all the students interviewed. In classifying strategies stringent criteria was consistently followed and whenever necessary, students' responses were consolidated.

The strategy that the student used to give his or her final answer was coded. When students explained their solution using two different strategies, their response was recorded as belonging to the more sophisticated strategy.

Table 2

Students' Strategies for Solving Missing Value Proportional Problems

Strategies	
	Nonconstructive Strategies
No Conceptualization	This classification was used only if a subject made no attempt to solve a problem or if an individual used numbers randomly and could not provide, in spite of prompting on the part of the interviewer, a reasonable explanation of thinking.
Qualitative	This classification was used when subjects considered the numerical relationships and used estimation to quantify the problems.
Ratio Difference	Subjects using this strategy calculated the difference between the numbers in the known ratio and used this information to create a second ratio with the same difference. This strategy indicates a search for patterns in numbers but is not productive in finding a solution to a missing value proportion problem.
	Constructive Strategies
Additive	A subject's strategy was classified as additive when an individual calculated a unit ratio or used a given ratio to build up additively in an attempt to reach a target number (the known number of the second ratio). When this number could not be reached with the additive scheme the subject selected, various levels of sophistication for dealing with this problem became obvious. On the most basic level, individuals chose to ignore the discrepancy, accepting as accurate an answer close to the target number. Others acknowledged the discrepancy and applied the difference between the two numbers in the given

	ratio to their answer. Still others applied qualitative reasoning to the discrepant amount and adjusted the solution using estimation. The most sophisticated reasoning involved calculating the unit ratio to adjust the final answer.
Combined	Some subjects used multiplication to get near a target number (the known number of the second ratio), but resorted to addition, ratio difference, or qualitative thinking to adjust for non-integer multipliers. These strategies were classified as mixed.
Multiplicative	Strategies were labeled as multiplicative if a subject applied only multiplicative reasoning either within or between measure spaces to achieve a solution. The multiplicative strategy is most sophisticated in that it takes into account the covariation of the two ratios in the most efficient way.

After classifying the strategy used in each student response according to Table 2, the results were organized, first by semantic type and then by number structure. Also classified were the number of correct answers, first by semantic type and then by number structure. A similar table was also created by gender. All the tables, except Table 1 and 2, show the percentage of problems in that particular category.

RESULTS

The first part of the section will look at overall findings, from the interview data. First we describe the trends in strategy use within each semantic type, followed by discussion of trends in strategy use within number structures. Next we report the success rate, both within semantic types and number structures. The second part[1] of this section will look at the findings related to gender differences. First we report the success rate within semantic types and number structures, followed by a description of strategy use by gender.

Overall differences in strategy use

Semantic Type and Strategy

In this study, it is found that there are three semantic structures from which the students constructed different understandings of missing value proportional problems (see Table 3). First, the well chunked (W-C) and the associated sets (A-S) problems call for similar interpretation from students and the pattern of strategy use is very similar. On the other hand, the part-part-whole (P-P-W) problems and the symbolic problems (S-P) indicate a somewhat different pattern of strategy use.

In the W-C and the A-S problems, students used less multiplicative and ratio differences strategies and more additive and combined strategies, than in the other two semantic types. Forty six percent of the W-C and fifty one percent of the A-S problems were solved with multiplicative strategy and thirty one percent of the W-C and thirty percent of the A-S were solved with combined strategy. The overall frequency of additive

Table 3

Total Percentage of Strategies Used by Semantic Type

Strategies	Semantic Types			
	Well Chunked	Part-Part-Whole	Associated Sets	Symbolic-Problem
No Conceptualization	10%	4%	7%	7%
Qualitative	0%	1%	0%	2%
Ratio Difference	6%	20%	4%	10%

[1] The results can be reported in a second part paper if space considerations are an issue

Additive	8%	4%	8%	0%
Combined	31%	16%	30%	13%
Multiplicative	46%	55%	51%	67%

Note. Well-Chunked, Associated Sets, and Symbolic Problem; total n=212
Part-Part-Whole; total n=159 (n=number of problem solved in each semantic type)
column does not add up to 100 due to rounding error

strategy was small. However, the A-S and W-C problems elicited the most frequency, where eight percent of the problems in both types were solved with additive strategies. Further, thirteen students used additive strategies in at least one W-C or A-S problem. whereas forty-eight students used combined strategies in at least one of the W-C and A-S problems.

For the P-P-W problems, students tended to rely on a ratio differences strategy if they could not successfully use a multiplicative strategy. Fifty five percent of the problems were solved with multiplicative strategies whereas twenty percent were solved with ratio differences strategies. The students used very few additive strategies solving the P-P-W problems, or four percent. Further, only five students used additive strategies in one of the P-P-W problems, but twenty students used ratio differences in one or more of the P-P-W problems. P-P-W problems accounted for the fewest cases of no conceptualization (misconception).

The symbolic problems had the highest use of multiplicative strategies, sixty seven percent were solved with mulitplicative strategies. These problems differed from the others since they were not word problems. Similar, to the P-P-W problems, the students used ratio differences strategies when multiplicative strategies failed or ten percent of the solutions. No student used additive strategies for these problems. The symbolic problems also elicited that lowest rate of combined strategies, with only thirteen percent of the solutions used combined strategies.

The combined strategy was over ninety percent a combination of a multiplicative strategy and a less sophisticated strategy. In these tasks, many students used multiplication to attempt to reach a target number (the known part of the second ratio). When this number could not be reached with an integer multiplier, students used the nearest multiplier and applied a less sophisticated strategy on the remainder.

The findings of this study indicate that there was a difference in dealing with this remainder between semantic types, therefore different combination of strategies. In thirty five percent of the cases in combined strategies in both A-S and W-C, students found the remainder using the additive strategy. In all these cases, the remainder was half of the known number in the first ratio. It appeared to be easy for students to see the half relationship and add that number to their target number. In twenty percent of the cases, the students unitized the remainder and added that number to reach the target number. In about twenty three percent of the cases students used the relationship of the numbers in the ratios to estimate the remainder. But only in about eleven percent of cases did students use ratio differences to deal with the remainder.

Twenty-four students turned to combined strategies to solve the P-P-W problems, or sixteen percent of the problem. The pattern of the combined strategies was also different. Only four percent of the students used additive strategies to deal with the remainder, which had the relationship of half compared with thirty five percent in the W-C and A-S problems. In twenty eight percent of the cases, students unitized the remainder and added that number to their target number. On the other hand, in about forty percent of the cases, students used estimation by considering the relationship between the numbers in the ratios. Twenty-four percent used ratio differences to deal with the remainder.

The combination of strategies was also different from the other semantic types. In twenty one percent of the items, additive strategies were used to deal with the half relationship. No one unitized the remainder. Twenty one percent of the cases used estimation, but fifty percent of the cases accounted for ratio differences dealing with the remainder.

Number Structure and Strategy

A clear pattern occurred of decreased usage of multiplicative strategies and increased usage of ratio differences as the number structure became more difficult (Table 4).

For the I-I-I and I-N-I tasks, nearly all responses were multiplicative, eighty two percent of the I-I-I solution and seventy two percent of the I-N-I / N-I-I solutions. The more complex number structures (N-N-I and N-N-N) generated less sophisticated strategies. Students partly relied on combined strategies to reach an answer for the more complex structures even though they had used multiplicative strategies on nearly all other problems. The use of fall-back strategies has been identified in previous research on proportional reasoning (Tourniaire & Pulos, 1985; Karplus et al., 1983; Lamon, 1993a;

Table 4

Total Percentage of Strategies Used by Number Structure

	Number Structure			
Strategies	I-I-I-	I-N-I or	N-N-I	N-N-N

		N-I-I		
No - Conceptualization	4%	7%	11%	8%
Qualitative	0%	0%	3%	1%
Ratio Difference	5%	7%	10%	16%
Additive	9%	4%	6%	1%
Combined	0%	10%	34%	50%
Multiplicative	82%	72%	36%	25%

Note. I-I-I and I-N-I and N-N-N total n=212
N-N-I: total n=159 (n=number of problems solved in each number structure category)
I-I-I = Integer-Integer ratio relationship with an Integer unknown
I-N-I or N-I-I = Integer-Noninteger ratio relationship with an Integer unknown
N-N-I = Noninteger-Noninteger ratio relationship with an Integer unknown
N-N-N = Noninteger-Noninteger ratio relationship with a Noninteger unknown
column does not add up to 100 due to rounding error

Kaput & West, 1994). In these tasks, most students used multiplication to attempt to reach a target number (the known part of the second ratio).

For the I-I-I problems, students used multiplicative strategies in eighty two percent of all cases. The percentage of problems solved with an multiplicative strategy decreased as the number structure became more complex, falling from eighty two percent for the I-I-I problems to twenty five percent for the N-N-N problems. The use of combined strategies increased as the numbers got more complex. In solving the I-I-I problems, no one used combined strategies, but in the N-N-N strategy, fifty percent of all the N-N-N problems were solved with combined strategies. There was also a decreased use of additive strategies for more complex number structure, with the most use in the I-I-I problems or nine percent of solutions, wile only one student used an additive strategy on the N-N-N problems. There was also an increased use of ratio differences. Combined strategies also differed among number structures. As mentioned earlier, students used combined strategies when there was not a whole number relationship within the ratio or between ratios. In most cases they used multiplicative strategies to get to their target number, but then tended to fall back to less sophisticated strategies to deal with the remainder.

No combined strategies were used in the I-I-I problems, simply because there was no remainder to handle. Seventy-three percent of the I-N-I problems had to do with half relationships. The left over was half of the whole number in the known ratio. The students' continuous reliance on multiplicative strategies suggests that the fraction ½ is so common to them that it might present less difficulties. Perhaps students operate with the fraction of ½ nearly as easily as they can with natural numbers. Ten percent of the students used ratio differences for the remainder and fourteen percent dealt with the remainder by unitizing it and then adding it to the target number.

In the N-N-I problems sixty five percent of the problems solved with a combined strategy had the half relationship, which students added to their target number. Eleven percent of the students used estimation, using the relationship of the numbers in the ratios. Thirteen percent used ratio differences and nine percent choose to ignore the fact that there was a remainder. Only one student tried to unitize the remainder and find the exact answer using that method. In the N-N-N problems, forty three percent of the problems that were solved with combined strategies used multiplicative strategy and estimation. Twenty-seven percent of the students, choose to fall back to ratio differences to find the remainder. Twenty-seven percent also choose to unitize the left over and come up with a number to add to their target number.

Correct and Incorrect Answers

Number structure most clearly determines the difficulty level of missing value proportion problems (Abramowitz, 1975; Tourniaire & Pulos, 1985). The results in this study support this finding. The number structure of the problems in this study clearly affected students' abilities to respond with correct answers more so than semantic type. (See Table 5.) Students did better on the A-S problems than any of the other or seventy three percent of correct solutions.

Table 5

Percentage of Correct Responses by Semantic Type

	Gender		
Semantic Types	Female	Male	Total
Well- Chunked	66%	70%	68%
Part-Part Whole	57%	71%	64%
Associated Sets	74%	71%	73%
Symbolic Problem	68%	63%	65%

Note. Well-Chunked, Associated Sets, and Symbolic Problem:
female n=108; male n=104; total n=212
Part-Part-Whole: female n=81; male n=78; total n=159
(n=number of problem solved in each number structure category
columns do not add up to 100 due to rounding error

Table 6

Percentage of Correct Responses by Number Structure

	Gender		
Number Structure	Female	Male	Total
I - I - I	92%	91%	92%
I - N - I or N - I - I	82%	88%	85%
N - N - I	60%	59%	60%
N - N - N	31%	34%	32%

Note. I-I-I and I-N-I and N-N-N female n=108; male n=104; total n=212
N-N-I: female n=81; male n=78; total n=159
(n=number of problem solved in each number structure category)
I-I-I = Integer-Integer ratio relationship with an Integer unknown
I-N-I or N-I-I = Integer-Noninteger ratio relationship with an Integer unknown
N-N-I = Non-integer-Noninteger ratio relationship with an Integer unknown
N-N-N = Noninteger-Noninteger ratio relationship with a Noninteger unknown
columns do not add up to 100 due to rounding error

The other semantic types were all quite similar. On the other hand, if we look at the correct answers by number structure (Table 6), I-I-I problems were clearly the easiest with ninety two percent correct solutions. N-N-N problems were the most difficult ones, with only thirty two percent of all the N-N-N problems solved correctly.

Gender Differences in Strategy Use

Correct and Incorrect Answers

No overall differences were identified in the rate of correct answers between boys and girls (χ^2=0.19, df = 1, p>0.05). Females solved sixty-six percent of the problem correctly and males solved sixty-eight percent of the problems correctly. Semantic type had a bigger impact on girls' success rate (Table 5) than on boys' success; that is, the success rate for females was not as similar across semantic types as it was for males. It is also interesting to note that girls did better in the symbolic problems than boys, while the boys did better on P-P-W problems. The number structure did not indicate any differences by gender. Both boys and girls showed a similar pattern of decreasing number of correct answers, as the complexity of the number structure increased. (See Table 6.)

Strategy Used

Boys tended to use more multiplicative strategies than girls. Girls tended to use more additive strategies than boys. Boys had higher rates of no conception and qualitative strategies. Gender differences were not identified in pattern of strategy use by number structure (see Table 7). The more complex the number structure becomes the less sophisticated the strategy becomes by both genders.

Table 7

Percentile of Strategies Used by Gender -- Number Structure

Strategies	Number Structure							
	I-I-I-		I-N-I or N-I-I-		N-N-I		N-N-N	
	Female	Male	Female	Male	Female	Male	Female	Male
No Conceptualization	2%	6%	5%	9%	9%	14%	7%	9%
Qualitative	0%	0%	0%	0%	0%	5%	1%	2%
Ratio Difference	6%	3%	11%	3%	15%	5%	20%	11%
Additive	13%	6%	6%	2%	7%	4%	1%	1%
Combined	0%	0%	10%	11%	35%	33%	51%	48%
Multiplicative	79%	85%	68%	76%	35%	38%	20%	30%

Note. I-I-I and I-N-I and N-N-N total n=212
N-N-I: total n=159 (n=number of problems solved in each number structure category)
I-I-I = Integer-Integer ratio relationship with an Integer unknown
I-N-I or N-I-I = Integer-Noninteger ratio relationship with an Integer unknown
N-N-I = Noninteger-Noninteger ratio relationship with an Integer unknown
N-N-N = Noninteger-Noninteger ratio relationship with a Noninteger unknown
column does not add up to 100 due to rounding error

Boys tended to use more multiplicative strategies than girls in all semantic types except for the symbolic problems (Table 8). The higher use of multiplicative strategies on symbolic problems by girls reflect that the girls succeeded to solve these problems more often than the boys. W-C and A-S problems show similar patterns in strategy use by boy and girls and P-P-W and S-P have different patterns. In W-C and A-S problems, girls tended to use more additive strategies than boys, and in P-P-W and symbolic problems, they tended to use ratio differences more often than boys. There are indications that strategy use varies among semantic types more for girls than for boys. Differences in solution pattern between semantic types is more extreme with girls than boys.

Looking at the combined strategy there seem to be no differences between girls and boys, both genders have similar patterns.

Table 8

Percentile of Strategies Used by Gender -- Semantic Type

	Semantic Types							
Strategies	Well Chunked		Part-Part-Whole		Associated Sets		Symbolic-Problems	
	Female	Male	Female	Male	Female	Male	Female	Male
No Conceptualization	9%	11%	1%	6%	5%	10%	6%	9%

Qualitative	0%	0%	1%	0%	0%	1%	0%	5%
Ratio Difference	7%	5%	30%	10%	6%	1%	13%	7%
Additive	12%	3%	5%	4%	10%	5%	0%	1%
Combined	33%	28%	12%	19%	33%	26%	11%	15%
Multiplicative	38%	54%	51%	60%	45%	58%	71%	63%

Note. Well-Chunked, Associated Sets, and Symbolic Problem; total n=212
Part-Part-Whole; total n=159 (n=number of problem solved in each semantic type)
column does not add up to 100 due to rounding error

Summary

The students relied heavily on multiplicative strategies in all semantic types, although W-C and A-S problems were solved with multiplicative strategies less frequently then both P-P-W and S-P. The highest frequency of multiplicative solutions were found within symbolic problems. The pattern of strategy use was similar for W-C and A-S problems, whereas P-P-W and S-P had unique patterns of strategy use.

In this study the number structure influenced students choice of strategy to greater extent than the semantic type. Students used multiplicative strategies most frequently on the I-I-I problems, but resorted to less sophisticated reasoning on N-N-N problems. Furthermore, the number structure influenced students' success rate, with a clear progression of problem difficulty from I-I-I tasks (easiest), I-N-I or N-I-I, N-N-I, to N-N-N tasks (most difficult).

No overall gender differences were identified in the number of correct solutions. There are indications that the semantic type had more influence on girls' choice of strategy than on boys' strategies. The number structure did not promote substantial gender differences in strategy use. The same pattern was identified, showing decreased usage of multiplicative strategies as the number structure became more complex.

DISCUSSION AND CONCLUSIONS

The purpose of this study was to identify whether the semantic structure or the number structure of missing-value proportional problems has a greater influence on student's choice of solution strategy. A second purpose was to investigate gender differences in strategy use, within semantic type as well within number structure. In this section we discuss the major findings and questions raised by this study.

The major results of this study are the following:

1) The number structure influenced students' use of strategies for solving missing value proportional problems more than did semantic type.
2) The number structure most clearly determines the level of difficulty of the problem.
3) There were no overall gender differences in number of correct solutions.
4) There are indications that the semantic type influences girls' use of strategy for solving missing-value proportional problems more so than it did boys' use of strategy.

Three questions were raised in the beginning, asking 1) How does the semantic type influence use of strategy in solving proportional problems? 2) How does the number structure influence use of strategy in solving proportional problems? 3) Are there any gender differences in strategy use in solving proportional problems? This final section addresses these questions.

The influence of semantic type in students choice of strategy.

The results indicate that number structure, more than semantic type, influenced students' use of strategies for solving missing value proportional problems. In addition, number structure most clearly determined the level of difficulty of the problem. Consequently, the number structure of the problem appeared to affect students' ability to respond with correct answer. The influence of the semantic type should not be overlooked.

In examining the semantic types, the well chunked and associated sets problems show similar patterns in strategy use. Some aspects of these problems call for a very similar interpretation from students. Well chunked problems compare two extensive measures, elements that often go together, which result in an intensive measure, whereas the associate sets problems compare two extensive measures, elements that have little in common outside the problem setting, which result in another extensive measure. This did not appear to make any difference for students. They viewed both problem types as problems involving two discrete quantities that need to be compared and for them, whether the outcome is extensive or intensive was not important. The context of these problem types are familiar to the students; students deal with speed, price, and amount of food in their daily lives. The familiarity of the context is what well chunked and associated sets have in common. These types also show the fewest number of "pure" multiplicative strategies (Table 3). I call it "pure" because the combined strategies were usually a combination of multiplicative and other strategies. If combined strategies are taken into account then these two semantic types do in fact elicit the most frequent use of mulitplicative strategies. This is consistent with the number of correct solutions (Table 5), since well chunked and associated sets have the highest number of correct solutions. It can thereby be argued that students use their most sophisticated strategies in problems set in context that they are familiar with and do understand.

For students, the part-part-whole problems called for a different interpretation. In the part-part-whole problems, the two elements in a given ratio are subsets of one whole. By looking at the number of multiplicative strategies and combined strategies it can be said that part-part-whole item elicited the fewest solutions with multiplicative strategies. It is interesting, that for students additive strategies were not a practicable way to solve these problems, but ratio differences were.

For symbolic problem students relied on multiplicative strategy, and if that failed then they used ratio differences. It is interesting to note that no attempts were made to use additive strategies. Most students treated these problems as fractions, not ratios. The students were familiar with how to make two fractions equal and most knew that they were supposed to multiply. Use of additive strategies was not considered, since they had nothing to build up, no elements that made sense to them. Also, students probably remembered from their textbook problems that to add on was not something you did when making two fractions equal.

It is difficult to talk about symbolic problems without taking into account the number structure. Since the students treated this as two fractions, it was difficult for them to see the answer as a mixed number, since they were not familiar with complex fraction. A very common answer when dealing with the N-N-N problem was, “this is not possible, the answer is not a whole number”. When faced with this difficulty, many students tried something else that would give them a whole number answer. Also interesting is that the students who used combined strategies, used multiplicative strategies to reach the target number and then used ratio differences for the remainder. We believe that this is connected with students’ understanding of fractions, which is limited when dealing with complex fraction.

The influence of number structure in students choice of strategy.

The number structure was carefully manipulated within planned parameters of complexity. The number complexity formed a parallel hierarchy among the semantic type. Consequently, the pattern of strategy use was in a hierarchical order. The more complex the number structure, the less sophisticated were strategies used by students. The frequent use of multiplicative strategy in the I-N-I problems might support the idea that students chose the “easier” relationship in the problem. Two of the problems had an integer multiplier between ratios and two within ratios. But when looking at the problems, it can be seen that the problem that had an integer relation within, had a 1/2 relationship between. That is, the multiplier between ratios was an integer plus a half. Students’ reliance on multiplicative strategies might also suggest that the fraction 1/2 is so common to them that it might have presented fewer difficulties. It is well documented that students operate with the fraction of 1/2 nearly as easily as they can with natural numbers (Noelting, 1980). On the other hand, with the N-N-I problems, which all had 1/2 relationship between ratios, there is a considerable drop in use of “pure” multiplicative strategies. The difference between these two number structures is that in three of the N-N-I problems, the denominator in the known ratio was larger than ten and the known quantity in the second ratio was larger than 30. For example, in N-N-I problem 7 (8/12=x/42) to see that 3*12=36 and the left over is 6, which is the half of 12, is not as obvious as in I-N-I problem 3 (2/4=x/22) where 5*4=20 and 2 left which is the half of 4. According to the literature, problems with numbers larger than 30 influence students' proportional reasoning (Lamon, 1989). The results corroborate the literature.

The last number structure N-N-N, showed the fewest cases of correct solutions as well as the least usage of multiplicative strategy. Students had to rely partly on other strategies to reach an answer even though they used multiplicative strategies on nearly all the other problems. The use of fall-back strategies has been found in previous research on proportional reasoning (Tourniaire & Pulos, 1985; Karplus et al., 1983; Lamon, 1993a; Kaput & West, 1994).

In these tasks, many students used multiplication to attempt to reach a target number (the known part of the second ratio). When this number could not be reached with an integer multiplier, they used the nearest multiple and applied a less sophisticated strategy to the remainder. It was this remainder issue which caused difficulty for many of the students on the N-N-N tasks. The remainder issue could have been more difficult for students because they had to give fractional answers of thirds, fourths, fifths and sixths. This would suggest that these fraction families were less understood by the students interviewed. While students could deal more easily with halves of a unit or item, they had more difficulty with other fractional parts.

The fall-back strategy of closing in on a target number using multiplication and then using an additive or other strategy for the remainder, also demonstrates that most of the students interpreted the items mentioned in each problem as discrete units. This interpretation was also demonstrated by some students who used multiplication to approach a target number and then continued to use a multiplicative strategy to deal with the remainder. Only a few of these students had a continuous interpretation of the original item mentioned in the problem. They did not use one multiplicative strategy to deal with the complete answer. If they had a continuous interpretation of the original quantity (and perhaps more familiarity with various fraction families), they could have used one mixed number as a multiplier and demonstrated more efficiency and sophistication in their strategies. Studies have reported different proportional reasoning in discrete versus. continuous problems (Pulos, Karplus, & Stage, 1981).

An alternative way to look at the N-N-N problems is to combine the number of multiplicative and combined strategies. By doing this one can see that approximately seventy five percent of problem were solved by a strategy that was multiplicative in nature, since the combined strategies were a combination of multiplicative and one other strategy. In the method the students used to reach the target number, as described above, it was not their lack of proportional reasoning that hindered them in determining the correct answer but their lack of computational skills. On the basis of my results, it is not possible to determine whether students had difficulties with proportional reasoning in N-N-N problems. One could argue that a student who had a preference for using a multiplicative strategy understood some aspects of proportion, and also understood that the same relationship applies in more complex number structures. He or she might not have used computational knowledge to solve a complex calculation, or might not have seen the need to come up with an exact number in some cases. It might not make any sense for the student to go through a complex manual calculation when a close estimation can be obtained

Gender differences in strategy use in solving proportional problems.

As expected, the data indicated no over all gender differences in number of correct solutions. Girls did better then boys solving the symbolic problems and associated sets problems and girls tended to use less mature strategies, except in the symbolic problems. These findings are supported by the literature about gender differences, whereas girls do better than boys in computation and algebra (Marshall and Smith, 1987).

An important factor to look at, is that the symbolic problems were very familiar to the students. These problem were common in the textbooks and the students most likely had solved this kind of problem, even though their recognition was related to fractions and not proportion. For girls it appears important to consider what they had learned before. It might be that girls paid attention to what was taught and are good learners. There are studies that support the importance of instruction for girls, that girls do better then boys on content that has been covered in the classroom (Kimball, 1989). Another study implies that there is a greater effect of semantic type for female than males (Pulos, Karplus, & Stage, 1981). A noticeable difference exist in this data in the variation of strategy used by male and female (see Table 7). There was also a differences in the success rate (see Table 5). If it is true that women are more connected knowers than men

,as Belenky, Clinchy, Goldberger, and Tarule (1986) argue, this supports the findings in this study. The associated sets were problems about how much food is needed for a certain number of animals or people. That situation is most likely very common for children in their daily life. Well chunked problems involve problems related to speed and price. These are also common circumstances in daily life. The symbolic problems might not have been connected to students' daily life but most likely the students had learned a procedure that helped them to solve this type of problems. On the other hand, the part-part-whole problems are not as much of students daily life activities. For students to wonder about a number of people in a group and how much linear expansion would be if some particular part of the group grows was less likely to be part of student's daily practice. Consequently, girls did better and used more mature strategies on problems set in a context that were familiar to them

SUMMARY

In this study, students solved problems of three semantic types: 1)Well-chunked and associated sets, 2) part-part-whole, and 3) symbolic. The W-C and A-S problems. involved combining sets of discrete quantities, the P-P-W problems gave an extensive measure of a single subset of a whole in terms of the extensive measure of two groups of which it is composed, and the symbolic problems were presented as two fractional numbers.

Well-chunked and associated sets problem elicited the fewest number of multiplicative strategies and highest number of combined strategies. However, since combined strategies were always a combination of multiplicative strategy and one other strategy it can be argued that W-C and A-S elicited the most frequent use of multiplicative strategies. The number of correct solutions supports that interpretation, since W-C and A-S problem had the highest success rate.

P-P-W problems showed the lowest number of multiplicative strategies and most frequent use of ratio differences strategies. For symbolic problems students used mostly multiplicative strategies, and did not use additive strategies at all. Students perceived these problems as two fractions for which they needed to find a common denominator, consequently multiplying the numerator and denominator of one of the fractions by a factor was a common way to solve the problem. Additive strategies were not used because, there were no concrete elements to build-up. The N-N-N number structure within symbolic problems had a very low success rate which suggest limited fractional understanding by students.

The data suggest that students used their most mature strategies on these problems which they clearly understand and could easily explain. It also seems plausible that students used more mature strategies on problems involved a "set" of discrete items, rather than a continuous quantity.

The findings in this study indicate that number structure influenced reasoning ability and success to a greater extent than semantic type. The easiest problems for students to solve were I-I-I tasks and the most difficult problems were the N-N-N tasks. Students' strategies varied according to problem difficulty. All students used multiplicative strategies on the I-I-I and most students on the I-N-I / N-I-I problems, but many resorted to less sophisticated reasoning on the N-N-I and N-N-N problems.

Frequent use of multiplicative strategies when solving the I-N-I / N-I-I problems can be interpreted in two ways. First, students did indeed find the “easiest” way and looked for the integer relationship in the problem. Secondly, the noninteger relationship was a half relationship and for students half is a familiar easily applied fraction. One explanation for why the half relationship in the N-N-I problems did not show the same success as in the I-N-I / N-I-I problems is that the size of the numbers used in the N-N-I problems were larger then in the other number structures.

These findings suggest that the students interpreted the N-N-N quantities as discrete amounts, using a multiplicative strategies to deal with a part of the quantity and a fall-back strategy to handle the remainder. It could be argued that for some of the students, finding an exact answer was not a feasible way of solving the problem. A close estimate made more sense to some students than a noninteger answer. Consequently, this finding can not necessarily be interpreted as a lack of proportional reasoning on the part of these students.

No gender differences were identified in overall success rates. However, there were differences in success rates for different semantic types. The success rate of the girls was greater than that of boys in associated sets and symbolic problems. On the other hand, the success rate of boys was greater than girls in part-part-whole problems. A plausible explanation could be that girls learn what they have been taught or experienced. Associated sets problems maybe familiar to girls in their daily lives, and if they perceived symbolic problems as two fractions, it would be closely related to material covered in their mathematics classes. Girls used less mature strategies than boys for all semantic types except in the case of the symbolic type. The data suggest that the semantic type influences females' choice of strategy more than males.

IMPLICATIONS FOR INSTRUCTION

Though further research is needed to examine the interaction of the number structure and semantic type, this study has implications for teaching and learning, including suggestions related to numbers and semantic type. In this section we highlight a few factors that need to emphasized or added to the school curricula for students to engage in proportional reasoning. We discuss these issues in the following section.

Fractions, Equivalence Classes, and Fraction Families

Since proportion is the equivalence of two ratios, the understanding of fractions and equivalence classes is essential to understanding proportion (Behr, Lesh, Post, & Silver, 1983; Post, Cramer, Behr, Lesh, & Harel, 1993). The results of this study are supported by other researcher (Post et al., 1993) suggesting that a child's understanding of fractions is very limited. This study and the literature suggests that a child's experience with fractions and equivalence classes needs to be expanded beyond halves and fourths. A child's understanding of whole numbers is based on the fact that they have first hand experience with whole numbers, both inside and outside of the classroom. This understanding is important when dealing with fractions; students need more physical, visual and conceptual experience with fractions. Children often have difficulties understanding and visualizing fractions as a quantity (Post et al., 1993). The results of this study also indicate that children's failure to view fractions as a quantity hinders their understanding of proportion. The avoidance and difficulty students demonstrated with relations to noninteger number solutions is also related to students' lack of understanding of discrete and continuous interpretation of the problems.

Discrete vs. Continuous Interpretation of the Problems

The data in this study indicate that students interpreted almost all quantities as discrete, even though continuous interpretation might have led to a more efficient solution strategy. For children to be able to treat discrete and continuous quantities in an effective and reasonable manner, they require a flexible understanding of quantity. Students need to understand that a whole number x can be interpreted both as one unit of x and as x units of ones. A similar approach is needed for a noninteger number x/y, which could be interpreted as one quantity x/y and as x pieces out a total of y. Not only do students need to understand these differences of interpretation, they also need to have the ability to go back and forth between interpretations to fully understand the continuous and discrete concept of a quantity (Lamon, 1993). Behr, Lesh, Post, and Silver (1983) hypothesized that two cognitive structures were involved in understanding the discrete and continuous nature of problems. If two knowledge structures are involved both have to be developed and connections made between the two. A broader range of tasks involving various kinds of problems is needed. Students must be exposed to situations that challenge them to look beyond discrete interpretations and make connections between the discrete and continuous nature of problems.

Within versus Between -- Functional versus Scalar Relationship

A fourth concept essential to the development of proportional reasoning is the comparison of quantities. In proportional problems, quantities have multiplicative relationships both within and between measure spaces. An individual's ability to make comparisons either within or between measure spaces allows more flexibility when solving proportion problems. This flexibility in thinking allows an individual to choose which comparison would be most efficient for a particular problem context and/or number structure. When a student can demonstrate proficiency in making both within and between measure comparisons (or use both scalar and functional operations), he or she has demonstrated proficiency in proportional reasoning (Abramowitz, 1975; Inhelder & Piaget, 1958; Karplus, Pulos, & Stage, 1983). Since most students in my sample frequently relied on within measure comparisons, it appears that they need to develop their understanding of functional relationships between quantities. This between measure comparison will not only help develop students' proportional reasoning, it also has broader implications for the development of the concept of rate and algebra applications. The number structure inherent in a problem can be manipulated to encourage a functional relationship interpretation.

Estimation versus Exact Answer

The results of this study of students' proportional reasoning also provide evidence on general area which should be considered during math instruction, the use of estimation. The evidence indicated the type of problems that encourage estimation and thus improve students' sense of number. The results indicated that while semantic types may not have encouraged a particular strategy for solving proportion problems, it did help students make sense of the number values and number relationships. Thus, it is essential that proportions and other math concepts be taught within a problem content. Students need to be encouraged to use estimation to come close to a correct solution but they also need exposure to situations which demand an exact answer. Problem contents can be created to encourage an exact solution to a particular problem.

Symbols

A final concept which the results of this study indicate regarding the need for instructional attention, is the use of mathematical symbols. Mathematics is frequently defined as a science, an art, and a language. Mathematics as a language is communicated in written form largely through the use of symbols (Dienes, 2000; Sriraman & Lesh, 2007). The manner in which an individual learns mathematics is in some respects, similar to the manner in which one first learns a language. The concept of an object is introduced through some kind of visual or tactile representation. The concept and the object is then associated with the oral name for that object. It is only after an individual has this understanding that the written word can generate meaning. Like a language, mathematics must first be understood at a conceptual level and associated with visual and/or tactile representations. After that, a particular concept can be understood and represented by an oral interpretation and communicated through words and pictures (Dienes, 2004). Once an individual understands a math concept and can explain it orally and in an informal written form, proper mathematical symbols can be understood and are more likely to be meaningful (Mack, 1990; Post, et al., 1993).

Mathematical symbols associated with proportional problems (the fraction bar as well as other operation and equality symbols) should be introduced to students once they have demonstrated understanding of proportion concepts and procedures through oral and written forms (such as drawings and operations involving natural numbers). The symbols are abstractions. Students need to understand the meaning of the concepts and procedures before they can generalize. If symbols are introduced too early in the learning process, they can hinder understanding of math concepts and procedures and may result in the development of "buggy algorithms."

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