**MTL SHORT REPORT**

# The Modeling Perspective in the teaching and learning of mathematical Problem Solving at the elementary and secondary school level

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Abstract

*The purpose of this study was to examine in depth the modeling processes used by students in working with modeling activities and to examine how students' modeling abilities are changed over time. Two student populations, one experimental and one control group, were involved in the study. To examine modeling processes in students' work, experimental group students participated in an intervention program consisting of a sequence of six modeling activities. To examine students' modeling abilities, experimental and control group students completed a modeling abilities test three times. Results showed that students' models improved as students worked through the sequence of the modeling activities. Results also revealed that a number of factors, such as students' grade, students' experiences with modeling activities, and students' modeling abilities influence the modeling processes students used in their work. Results related to students' modeling abilities showed that participation in the intervention program had a significant impact on the students' modeling abilities. Finally, the study proposes a three- layer theoretical model for examining students' modeling behavior, which may have some implications on the teaching and learning of mathematical problem solving.*

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## Introduction and Theoretical Framework

A number of recent research studies documented the importance of implementing modeling activities at the elementary school level (English, 2006; English & Watters, 2005). This is important not only because elementary school students are capable of working with modeling activities, but also because modeling needs to be introduced early in the curriculum if we want to successfully implement modeling at all school levels (Blum & Niss, 1991; Doerr & English, 2003).

Modeling activities differ from traditional problem solving in two different ways. First, in solving the problems that appear in the modeling activities students need to use and interconnect mathematical concepts and operations (Lesh & Zawojewski, 2007). This can result in opportunities for students to elicit their own mathematics as they work the problems and to make sense of the realistic situations they need to mathematize. Second, in modeling activities students are encouraged to create models that are applicable to a range of similarly structured situations and as a result they can generalize and extend their solutions (English, 2006; Doerr & English, 2003).

The primary focus of many research efforts in mathematical modeling was on constructing and trying out modeling examples on teaching and applications, without contributing to the research efforts towards improving our understanding of mathematical modeling (Blum, 2002). Blum (2002) also clarified that there is a need for a shift from focusing on practice (e.g., modeling examples) to focusing on theory and he raised a number of critical questions for succeeding this shift. The present study aims to answer two of these questions, by describing and analyzing the process components of modeling (p. 11) and by examining student modeling abilities and identifying how these abilities can be developed over time (p. 12).

Modeling Processes are the processes students develop and use during their efforts to solve a real world problem. These processes include describing the problem, manipulating the problem and building a model, connecting the mathematical model with the real problem, predicting the behavior of the real problem and verifying the solution in the context of the real problem. Student Modeling Abilities include structuring, mathematizing, interpreting, and solving real world problems and the ability to work with mathematical models: to validate the model, to analyze it critically and to assess the model and its results, to communicate the model and to observe and to control self adjustingly the modeling process (Blum, 2002).

The purpose of this study was to: (a) describe the process components of modeling by examining students' models in a sequence of modeling activities and (b) examine how students' modeling abilities were changed over time through an intervention program.

## Method

One hundred and four sixth graders and 90 eighth graders from four 6$^{th}$ grade and four 8$^{th}$ grade classes participated in the experimental group. Similarly, 93 sixth graders and 116 eighth graders from four 6$^{th}$ grade and four 8$^{th}$ grade classes participated in the control group.

Experimental group students participated in an intervention program consisting of six modeling activities for a period of three months. At the same time, control group students worked with their regular mathematics textbooks. Mathematics textbooks do not include any modeling activities. The development of the modeling activities designed for the purposes of the study followed the six design principles proposed by Lesh and his colleagues (2000). According to the Model Construction Principle the solution to the activity requires the construction of an explicit explanation or procedure. The Reality Principle requires the activity to be meaningful to students from their different levels of mathematical ability and general knowledge. The Self-Assessment and Model Documentation Principles ensure the inclusion of criteria, students themselves can identify and use to test and revise their models and to create documentation that will reveal explicitly how they solved the problem. The Construct Share-Ability and Re-Usability principle, requires students to produce share-able and re-usable solutions, and the Effective Prototype principle ensures that the modeling activity will be as simple as possible yet still mathematically significant.

Three of the developed modeling activities constituted a sequence of modeling activities related to statistical concepts such as average, ranking, weighting and aggregating. The second set was related to geometrical reasoning and specifically to the concepts of area and perimeter. A short description of the six modeling activities is presented in Table 1.

Table 1

*Description of the Six Modeling Activities*

| **Title** | **Problem Context** | **Mathematical Concepts** |
|---|---|---|
| Best Drug Award Activity | Students are asked to develop a procedure for ranking five drugs based on information about the number of minutes each drug needs to act for 30 cases. | Statistical concepts such as average and frequency (i.e. average number of minutes needs to act or number of cases acts fast). |
| Where to Live Activity | Students are asked to develop a procedure for helping people choosing a city to move in. The selection was based on both qualitative and quantitative information about the number of schools, restaurants, budget available etc. | Weighting and ranking (i.e. ranking and weighting the cities according to different types of |

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|---|---|---|
| | | data). |
| University Cafeteria Activity | Students are asked to select six out of nine employees that should be rehired, based on information about the hours worked, and the money collected by the employees. | Proportional reasoning (i.e. pounds per hour). |
| Carpet Design Activity | Students are asked to develop a procedure for designing patterns of different 2D shapes for a carpet. | Geometric reasoning (i.e. area and perimeter for different shapes). |
| Car Painting Activity | Students are asked to develop a procedure for finding the amount of paint needed for a brand new car. Students are given information about the outer and interior dimensions of the car. | Geometric reasoning (area and perimeter of 2D shapes). |
| New House Activity | Students are asked to design a model of a new house. Students are given partial information about the building plot and the different rooms in the house. | Geometric reasoning (area of different figures, area and volume of solids). |

A test[2] for measuring students' modeling abilities was also developed. The test consisted of nine tasks. Three tasks were related to decision making problems, three were retrieved from system analysis and design category, and three were troubleshooting problems (OECD, 2003). The tasks reflecting the *decision making* problems presented students with a situation requiring a decision and asking to choose among alternatives under a set of conditions constraining the situation. Problems of s*ystem analysis and design* category differ from the decision-making problems in that constrains are not obvious and not all of the

[2] The Modeling Test and the Modeling Activities can be found in Mousoulides (2007)

possible options are given in the problem. Trouble-shooting problems assess students' actions when confronted with the need to specify the conditions under which a system is running properly or when a system of a mechanism is underperforming. The test was administered three times to both experimental and control group students. For the experimental group students the test was administered before the beginning of the intervention program, during (after the implementation of three activities) and after the completion of the intervention program.

## Results and Discussion

The results of the study are presented with reference to the purpose of the study; first we briefly describe the modeling processes that were evident in students' models, and second we examine the evolution of students' modeling abilities through the intervention program. The data used for describing the modeling processes were collected through experimental group students' models in the modeling activities presented in Table 1. Students' modeling abilities were measured through the administration of the test to both experimental and control group students.

### *Modeling Processes*

Data analysis revealed that a number of modeling processes appeared in students' efforts to solve the problems presented in the modeling activities. The modeling processes for each step of the modeling procedure (description, manipulation, prediction and verification) that have been identified in students' work are presented in Figure 1.

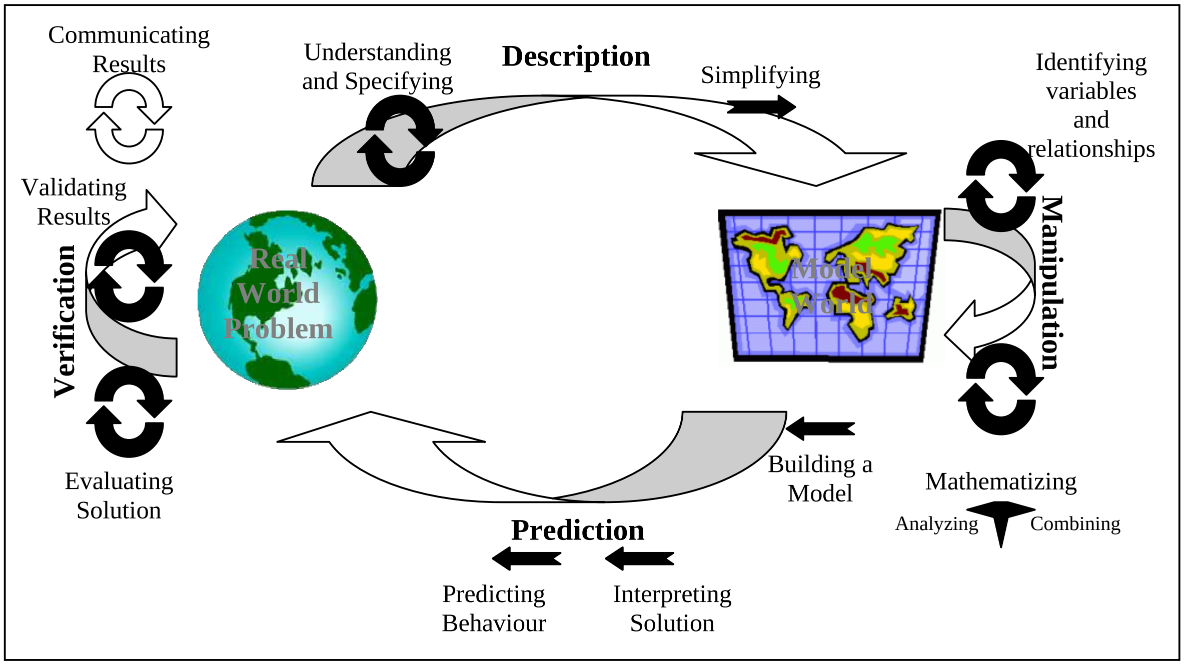



*Figure 1*. Modeling processes in mathematical problem solving.

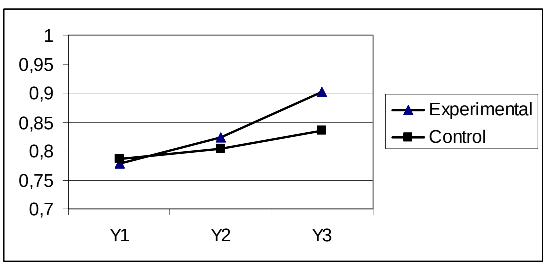
The first modeling process that was apparent in students' work in the modeling activities was understanding the main question of the problem. Understanding the core question was not an easy process, especially for the younger students. In doing so, students tried to perceive different and additional information about the problem by observing the provided data and by connecting their observations with the question of the problem. Students made these connections more easily for simpler modeling problems (e.g., Best Drug Award). In more difficult problems (e.g., University Cafeteria) these connections were not easy, nor obvious, since students could not understand how the different number of hours worked and money collected by each worker in different semesters and different time slots should be used to find the best six among nine workers. Related to problem understanding is problem specification and simplification, in which students identify a number of conditions and assumptions of the real world problem. The identification of conditions and assumptions assisted students in excluding irrelevant or less important information and directed students into simplifying the problem. In the University cafeteria activity, for example, students decided to work with the different time periods (busy, steady and slow) instead of different semesters. This helped them to exclude the unnecessary information and to simplify the problem.

The main characteristic of student work during problem manipulation was mathematizing. In mathematization, students identified variables and relationships within the mathematical entity and connected variables and relationships with prior conditions and assumptions. The identification of the necessary variables helped students to analyze and combine the necessary mathematical properties for constructing the mathematical model. In the “Where to Live” modeling activity (see Table 1), for example, students identified the necessary variables (e.g., number of buildings, facilities, budget) and combined the properties related to these variables for constructing a model for selecting the best among a number of cities.

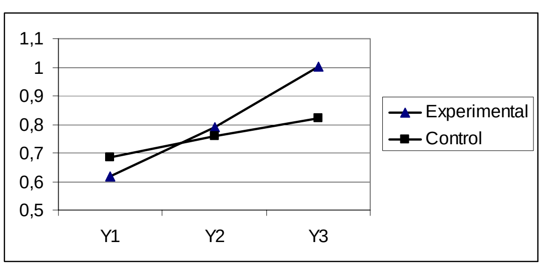
In prediction and solution verification, students’ efforts first focused on interpreting their solution (model) and on predicting the behaviour of the real problem. Finally, students validated and communicated their results. Under communication, a number of sub processes were presented; the interchange of ideas, information and instructions about the mathematical model. Communicating helped students to explain the solution of the problem, to predict the behaviour of structurally similar problems, and to finally elaborate on and to enrich their solution for solving more complex problems. In the “Where to Live” modeling activity, for example, students used their constructed model (based on the different importance of buildings, facilities, and budget) to predict how this model would result in finding the best city. In communicating their solution, students explicitly documented how they used their model to rank the different cities and how this ranking should be used for selecting the best among a number of different cities.

*Students' Modeling Abilities*

Latent Growth Modeling (LGM) analysis was conducted to analyse students' modeling abilities over time and the impact of the intervention program. The results of the LGM analysis showed that the impact of the intervention program on students' modeling abilities was significant. The analysis showed that experimental group students outperformed their counterparts in the control group. Specifically, 6th grade experimental group student modeling abilities' rate of change was two and a half times greater then the respective rate of change for the control group students. Similarly, 8th grade group student modeling abilities' rate of change was three times greater than the control group students' rate of change. Students' performance in the modeling abilities test, as it was measured during the three test administrations (Y1, Y2 and Y3) is presented in Figure 2. Sixth graders' modeling abilities results are presented in Figure 2a and the corresponding results for eighth graders are presented in Figure 2b.




(a)




(b)

*Figure 2*. Observed Trajectories for the 6th grade (a) and 8th grade (b) students.

A second finding showed that there was a significant negative relation between students' initial achievement in the modeling abilities test and their rate of change. This indicates that the intervention program was more efficient for the students with lower modeling abilities. This finding was consistent for both 6th and 8th graders. Based on the above finding, it can be claimed that the context of the modeling activities, social interactions between the students and the teacher, and the absence of direct instruction created a safe environment for students with low modeling abilities to present their ideas, to discuss possible solutions and to finally improve their abilities in solving modeling problems.

*Modeling Students' Modeling Behavior*

In putting together results from the analysis of the modeling processes appeared in students' work in the modeling activities and the results from examining students' modeling abilities, the study proposes that *Students' Modeling Behavior* can be described as the three interconnected layers that are presented in Figure 3.

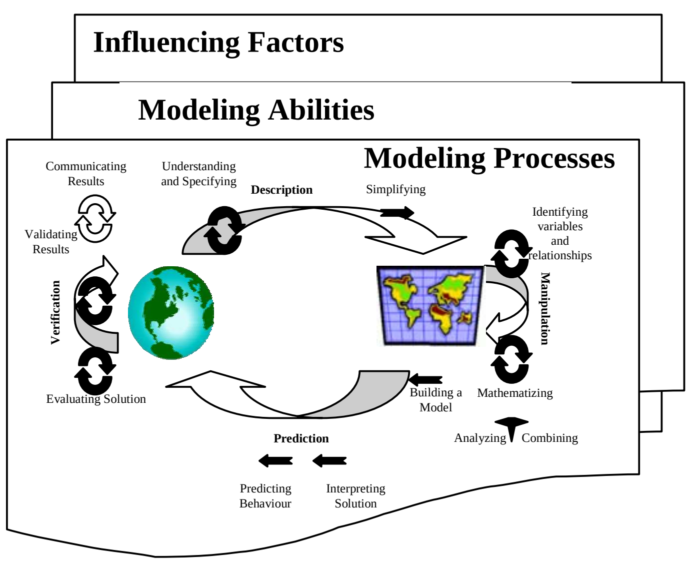



*Figure 3*. Students' Modeling Behavior.

The first layer of the proposed theoretical model presents the modeling processes that resulted in students' work in the modeling activities. The modeling cycle(s) is influenced by students' modeling abilities (see second layer) and a number of other factors that influence students' work in the modeling activities. As the third layer suggests, a number of factors influence the modeling procedure. Among these factors, the study identified the following four: (a) Student Modeling Awareness. Modeling awareness is considered as student prior modeling experiences, student mastery of modeling and problem solving skills, and student understandings of related mathematical concepts and processes. Students' models and solutions improved as students got more experienced in solving similar problems. This was evident, since students' models were not sufficient in the beginning modeling activities; in the following activities students' solutions improved and students managed to solve more complex problems. (b) Context of the problem. The created models are molded and shaped by the situation (context of the problem) in which they are created. This was evident in a number of cases in the study. In quite easy and straightforward activities, such as the Best Drug Award activity, students (even individually) easily reached satisfying solutions, did not get involved in model refinement and improvement and did not face the need to cooperate and communicate their results. On the contrary, students' results were better when activities were complex enough to challenge the students, and not so complex to discourage them. In such activities students constructed a number of different models, improved and refined them through a number of modeling cycles and finally successfully solved the problems presented. (c) Grade level. Students from different grade levels investigated and tried to solve the problems using quite different approaches, with older students being involved in a bigger number of modeling cycles. Results showed that $8^{th}$ graders constructed more complex and refined models in comparison to $6^{th}$ graders. Eighth graders also adopted and transformed more easily than $6^{th}$ graders existing models, involved in a greater number of modeling cycles, better communicated their results and reflected on their solutions. Additionally, $8^{th}$ grade students used more sophisticated formulas and approaches in constructing models for solving the problems presented in the modeling activities. Considering that students in middle school are taught mathematics in a more formal and abstract way, focusing on formulas, symbolic expressions and algorithms, it can be argued that this teaching approach might be a possible reason that $8^{th}$ graders effectively used such formulas and algorithms in solving the modeling problems. Finally, eighth graders explicitly documented their solutions by linking their suggestions with their models and by providing a pathway moving from less to more efficient models. Additionally, $8^{th}$ graders' discussions were more precise, and students really improved their ideas and models through peer interaction. In this direction, they managed to solve conflicts that arose and overcame related problems. (d) Presence of tools. When available, students effectively employed the available technological tool's capabilities and functionality, not only to make calculations quickly but also to export their data using multiple forms of representations and connecting these representations to construct a model and suggest a solution for the problem. The interconnection and the continuous interaction between the three layers shape and formulate students' "modeling behaviour" in problem solving.

## Conclusions

The findings of the study have implications for the design of modeling activities for elementary school students, for teaching mathematical modeling as a didactic means for mathematical problem solving and for teaching for improving students' modeling abilities. The range of models constructed by students suggests that teachers need to be aware that modeling activities will produce diverse strategies and thinking on the part of the students.

Finally, the design for this study has also potential to reveal details about student understanding of modeling and can serve for further research, especially for the introduction of modeling at elementary and lower secondary school level.

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