

Mathematics Education and the Legacy of Zoltan Paul Dienes



edited by
Bharath Sriraman

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The Montana Mathematics Enthusiast

Monograph 2

*Mathematics Education and the Legacy
of Zoltan Paul Dienes*

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PREFACE

THE LEGACY OF ZOLTAN PAUL DIENES

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The second monograph of *The Montana Mathematics Enthusiast* contains a unique collection of articles which span five generations of mathematics educators. As the title indicates the monograph is centered on the work of Zoltan Paul Dienes (1916–), a living legend in the field of mathematics education, for his pioneering work that has spanned 50 years. Trained as a mathematician in England, Zoltan became interested in the psychology of learning in the 1950s and earned a second degree in psychology. Extant histories of the field of mathematics education are often outlined in terms of its origins in the classical tradition of Felix Klein onto the structuralist agenda influenced by the Bourbaki and Dieudonné at the Royaumont seminar in post world war II France, followed by Freudenthal's reconception of mathematics education with emphasis on the humanistic element of doing mathematics (e.g., D'Ambrosio, 2003; Skovsmose, 2005). While the approaches of Klein and Dieudonné steeped in an essentialist philosophy gave way to the pragmatic approach of Freudenthal, Zoltan's approach influenced by structuralism and cognitive psychology remains unique from the point of view of developing a theory of learning which has left a lasting

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impact on the field. The books *Building up Mathematics* (Dienes, 1960) and *Thinking in Structures* (Dienes & Jeeves, 1965) influenced mathematics educators entering the field in the late 1960's and 1970's and remain classics to date. I encountered these two books in the late 1990's in graduate school when I became interested in cognition, particularly how the process of generalization could be systematically studied and understood in the context of creating new mathematics. Dienes' name is synonymous with the Multibase blocks which he invented for the teaching of place value. Among numerous other things, he is also the inventor of Algebraic materials and logic blocks, which sowed the seeds of contemporary uses of manipulative materials in instruction. Dienes' place is unique in the field of mathematics education not only because of his theory on how mathematical structures can be effectively taught from the early grades onwards using manipulatives, games, stories and dance (e.g., Dienes, 1973), but also because of his tireless attempts for over 50 years to inform school practice through his fieldwork in the UK, Italy, Australia, Brazil, Canada, Papua New Guinea and the United States. In my conversations with Richard Lesh, it was interesting to learn from him first hand the influence Zoltan had on his generation of researchers, and the foundations of Dienes' theories on the learning of mathematics on those involved in the Rational Number Project and more recently those working in the models and modeling area of research (see Lesh et al., 1987, 1989, 1992, 2003).

I had the honor of finally meeting Zoltan Dienes in Wolfville, Nova Scotia in April 2006 after many years of correspondence. The formal interview during the visit is part of this monograph (see Sriraman & Lesh, 2007). At this time, Zoltan was still actively engaged in creative and generative activities aimed at children discovering non-trivial mathematical structures. During this visit he gave me the last existing copy of a manuscript which he had written in 1995. This unpublished manuscript entitled "*Some thoughts on the dynamics of learning mathematics*" contained the "finished" form of Zoltan's theory of learning mathematics which he began developing in the 1960's. Even though an electronic copy of this manuscript was unavailable, the manuscript was in good enough condition to be scanned and reproduced in pdf format. I was particularly pleased to be able to include this entire (un-edited) manuscript in the online version of the monograph. This manuscript revealed some of the interactions of Zoltan Dienes with Jean Piaget as well as Jerome Bruner, and the esteem in which he was held by these researchers. At the age of 91, Zoltan continues to write and publish articles in numerous journals in New Zealand and the U.K (see Sriraman & Lesh, 2007 for these citations). In this book, I am pleased to bring four articles written by Zoltan Dienes in the last six months which reveal the power of manipulatives and imaginative thinking in facilitating the discovery of beautiful mathematical structures. Dienes championed the

use of collaborative group work and concrete materials, as well as goals such as democratic access to the myriad processes of mathematical thinking, long before the words “constructivism” and “equity” and “democratization” became fashionable. As Dick Lesh states, Dienes not only studied a phenomena that later cognitive scientists have come to call embodied knowledge and situated cognition—but he also emphasized the *multiple embodiment principle* whereby students need to make predictions from one structured situation to another. And, he also emphasized the fact that, when conceptual systems are partly off-loaded from the mind using a variety of interacting representational systems that every such model is at best a useful oversimplification of, both the underlying conceptual systems being expressed and the external systems that are being described or explained (Sriraman & Lesh, 2007).

The monograph includes one chapter by Lyn English, which explores the relationship between cognitive psychology and mathematics education. “Cognitive Psychology and Mathematics Education: Reflections on the past and future” contain her reflections on issues of continued significance for the field today from the point of view of developing a coherent theory of learning. This chapter gives an overview of the history of the field of mathematics education, particularly the influence of psychology, and a summary of the valuable contributions of theorists like Jean Piaget, Jerome Bruner and Zoltan Dienes. Finally, the monograph includes a chapter on the pedagogical value of structural thinking by Jim Hirstein, who as a doctoral student at the University of Georgia intersected with Zoltan Dienes in the research meetings organized by Richard Lesh at Northwestern University in the mid 1970’s (see the commentary of Lesh in Sriraman & Lesh, 2007). I hope this book will re-initiate an interest in the seminal work of Zoltan Dienes, particularly among those unfamiliar with his research and writings as well as contribute to future generations of mathematics educators appreciating his energy and monumental undertakings that led to the birth of our field of inquiry. This monograph is dedicated to him.

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