On the internal structure of goals and beliefs

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Abstract: *The theory ‚Teaching-In-Context' (abbreviated here as KGB), first introduced by Schoenfeld in 1998, has the objective of making the actions of a teacher in mathematics lessons rationally understandable. According to this theory, it suffices to locate the behavior as a function of the following three parameters: goals, beliefs and available teacher's knowledge. Schoenfeld´s hypothesis implies that spontaneous alterations in the teaching trajectories can be explained through shifts in the structure of goals and beliefs. In the following we discuss a particular videoed classroom lesson with a remarkable turning point on the background of this approach.*

1. Introduction

The topic of our analysis is an unexpected turning point in a videotaped lesson going beyond the originally intended context, namely an introduction to linear functions. The question arising for us is how the related processes can be rationally explained or understood (Cobb, 1986). There are a number of approaches for the analysis of such a situation. First of all, one can refer to theoretical approaches on everyday practice in mathematics lessons (Andelfinger & Voigt, 1986; Krummheuer & Fetzer, 2004); we however have opted for Schoenfeld's KGB framework.

The KGB framework of Schoenfeld (1998) characterizes teachers' spontaneous decision-making in terms of available knowledge, high priority goals and beliefs. Schoenfeld´s fundamental assumption is that decision-making is typically accomplished from the inner perspective (Schoenfeld, 2000; 2003). The category *knowledge* is employed as teacher's knowledge in the sense of Leinhardt and Greeno (1986), which is seen as primarily based on scripts (Sherin et al., 2000). We consider it worth mentioning that available knowledge – that is, action scripts through which lessons are designed or carried out – must be viewed as representing limitations and restrictions inherent in the beliefs and goals. For the analysis of the lesson presented in this paper, we therefore primarily focus on beliefs and goals and their interaction.

As already mentioned, we are dealing here with a videotaped lesson on introduction to linear functions, which was compared to an equivalent lesson in a Dutch school and discussed within the framework of a teacher training course. Besides the topic 'Introduction to Linear Functions', the responsible teacher was allowed to design the lesson free of any directives or restrictions. The lesson started rather open and problem-oriented using examples such as petrol consumption of vehicles or temperature changes in dependency of the height. Thus, units like miles, gallons and Dollar were converted into kilometers, liters and Euro, and feet and degrees Fahrenheit were converted into meters and degrees Centigrade. Students worked in small groups of three or four using Excel. However, as the lesson developed and time seemed to run out, the teacher suddenly changed her teaching style in favor of a more traditional approach. That is, she switched to a monologue on definitions in a formalized structure. These observations have challenged the question whether this turn in the teaching trajectory and the discontinuities involved could be understood rationally. Additional sources of information include an interview with the teacher after the lesson and a questionnaire that was handed out to the students.

2. Theoretical Framework

In the following we focus on the terms beliefs and goals, central to Schoenfeld's theory, and discuss these constructs against the background of the existing literature.

2.1. Beliefs

In the literature beliefs have been described as a "messy construct" with different meanings and accentuations (Pajares, 1992), and indeed the term belief has not yet been clearly defined. However, there is some consensus that mathematical beliefs are considered as personal philosophies or conceptions about the nature of mathematics as well as about teaching and learning mathematics (Thompson, 1992). Following Schoenfeld (1998), beliefs can be interpreted as "mental constructs that represent the codification of people's experiences and understandings", and he continues to state that "people's beliefs shape what they perceive in any set of circumstances, what they consider to be possible or appropriate in those circumstances, the goals they might establish in those circumstances" (p. 19). The teacher's beliefs about the mathematical content and the nature of mathematics as well as about its teaching and learning have an influence on what he or she does in the classroom, and what decisionshe or she takes.

Beliefs can not be regarded in isolation; they must always be seen in coherence with other beliefs. In the literature this phenomenon is described by using the term *belief system*. Green (1971) points out that "beliefs always occur in sets or groups. They take their place always in belief systems, never in isolation" (p. 41). Aguirre and Speer (2000) introduce the construct *belief bundle* which "connects particular beliefs from various aspects of the teacher's entire belief system (beliefs about learning, beliefs about teaching, etc.)" (p. 333). Furthermore, they consider the activation level of certain beliefs by stating that "a bundle is a particular manifestation of certain beliefs at a particular time" (p. 333).

2.2 Goals

Typically, a teacher begins a lesson with a specific agenda, in particular with certain goals he or she wants to accomplish. With regard to these goals the underlying structure of a lesson can be identified, especially the teacher's choices can be modeled. Schoenfeld (2003) distinguishes three categories of goals: overarching goals, major instructional goals, and local goals which occur at different grain sizes.

> Overarching goals […] are consistent long-term goals the teacher has for a class, which tend to manifest themselves frequently in instruction. […] Major instructional goals may be oriented toward content or toward building a classroom community. Such goals tend to be more short-term, reflecting major aspects of the teacher's agenda for the day or unit. Local goals are tied to specific circumstances. […]; a goal becomes active when a student says something that the teacher believes needs to be refined in some way. (p. 20)

All these goals have different and altering priorities in a given situation during a lesson. This re-prioritization of goals is the topic of our reflections. Furthermore, our objective is not only to identify the beliefs and goals of the teacher but in particular to document the duality of both constructs. Cobb (1986) has already pointed out that beliefs are allocated the link between goals and the actions arising as a consequence of them:

> The general goals established and the activity carried out in an attempt to achieve those goals can therefore be viewed as expressions of beliefs. In other words, beliefs can be thought of as assumptions about the nature of reality that underlie goal-oriented activity. (p. 4)

Essentially, we base our remarks on the goals expressed by the teacher in the interview. We choose a categorization deviating from Schoenfeld´s by differentiating between formal goals, methodological-pedagogical goals, and content goals (see 4.1 to 4.3). We also occasionally refer to the questionnaire with the students of this class.

2.3 Dependencies between beliefs and goals

Most research on beliefs and goals has focused on these constructs quite isolated from one another (Aguirre & Speer, 2000). Nevertheless, there are several interdependencies between the set of beliefs and the one of goals. A teacher's goals are part of his or her action plan for a lesson. He or she enters the classroom with a specific agenda, in particular, with a certain constellation of goals, which might change in relation to the development of the lesson. Looking at these goals elucidates the teacher's actions. A shift in a teacher's goals provides an indication of the beliefs the teacher holds. Beliefs influence both the prioritization of goals when planning the lesson *and* the pursuance of goals during the lesson. Furthermore, beliefs serve to re-prioritize goals when some of them are fulfilled and/or new goals emerge (Schoenfeld, 2003).

In the literature beliefs are often given priority over goals: "A teacher's beliefs and values shape the prioritization both of goals and knowledge employed to work toward those goals" (Schoenfeld, 2003, p. 8), or "they [beliefs] shape the goals teachers have for classroom interactions" (Schoenfeld, 2000, p. 248). We are, however, more reserved concerning this assertion and prefer to speak of the two-sidedness of the constructs to capture the interdependencies between goals and beliefs.

Whereas the interdependencies between goals and beliefs are sometimes mentioned in the literature, these ideas are not explicitly worked out (Cobb, 1986; Schoenfeld, 1998; 2000; 2003). Also, one finds only a few clues on suitable internal modelling within the set of beliefs and the one of goals (Cooney et al., 1998; Törner, 2002) – what we label here as intradependencies. Prioritizations, hierarchies and other dependencies seem to be relevant in this context; Green (1971) for example, refers to a quasi-logical relation between beliefs.

> We may, therefore, identify three dimensions of belief systems. First there is the quasi-logical relation between beliefs. They are primary or derivative. Secondly, there are relations between beliefs having to do with their spatial order or their psychological strength. They are central or peripheral. But there is a third dimension. Beliefs are held in clusters, as it were, more or less in isolation from other clusters and protected from any relationship with other sets of beliefs. (p. 47/48)

It is apparent that the set of goals can be modeled in the same sense.

3. Methodology

We examine here the relationship between goals and beliefs in the practice of one high-school teacher. We therefore employed qualitative methods and triangulated the data wherever possible. The lesson was professionally videoed using three cameras and transcribed. Additional sources of information include an interview with the teacher, the discussion of the lesson within the scope of a teacher in-service training, and a questionnaire handed out to the students.

The primary source for the analysis of the turning point in the lesson is the open interview with the teacher in which she clearly expressed some of her beliefs and goals. Furthermore, we refer to students' statements documented in the questionnaire.

The lesson in question was taught by a teacher with thirty years of professional experience. She has mainly been teaching in grade 11 and 12 but also in grade 5 to 10 in a German high-school. Remarkably, she has attended numerous in-service teacher training courses, in particular for using Computer Algebra Systems (CAS) in mathematics teaching.

4. Analysis of the lesson goals and the involved beliefs

The comments of the teacher in the interview clearly show that goals and beliefs can hardly be separated. Goals are founded in beliefs. Beliefs do not always imply direct actions but influence or induce defined goals.

The interview conducted by one of the authors reflects the lesson process in retrospect. When reading through the interview one can identify statements which can be regarded either as goals or as beliefs. In particular, the open interview style incites the teacher to justify her goals – partly without explicitly being asked to do so. Subjective convictions hereby become evident, which we understand as beliefs.

As mentioned above, goals and beliefs present a complex network of intra- and interdependencies. The videotaped lesson reveals an abrupt change in the teaching style after approximately 20 minutes which leads us assuming that, at this point, new prioritizations and shifts occur in the networks of goals and beliefs.

Our aim is to present the complexity of these networks on the one hand, and to clearly point out the characteristics of these goals and beliefs on the other hand. It would make sense to deal with the goals and beliefs separately and just simply list them in chronological order as they occurred in the interview or the lesson. However, this separation would strengthen the impression that goals can be understood in isolation. On the contrary, there exist reciprocal correspondences and argumentative relations going beyond the individual sections.

Whereas Schoenfeld (1998, 2005) distinguishes overarching goals, major instructional goals, and local goals occurring at different grain sizes, we choose a different characterization which appears to us to be more suitable in our context.

4.1 Formal und functional goals

The primary goal (G_{formal} 1) does not change during the lesson: the production of a complete video sequence to the theme 'Introduction to Linear Functions'. For this purpose a professional video team was present.

G_{formal} 1: More or less comprehensive video material has to be produced at the end of a 45 min. lesson.

G_{formal} 2: The content of the recorded lesson is an 'Introduction to Linear Functions' in grade 8.

These goals entail implicit beliefs on the realizability of the formal demands into concrete actions, which additionally requires of the teacher a certain amount of self-confidence.

4.2 Pedagogical and methodological beliefs und goals as networks

Apparently, distinguishing between pedagogical goals/beliefs on the one hand and between methodological goals/beliefs on the other hand would be rather artificial; thus we subsume the two aspects under a common title and speak of pedagogical and methodological goals/beliefs which may also include curricular aspects. In accordance with our initial hypothesis, beliefs und goals correspond with each other. In the following we therefore do not always verbalize both aspects and use the abbreviations B_{pm} and G_{pm} to distinguish the beliefs and goals attached to the same idea. This system is supported by the statements of the teacher in the interview. When articulating e.g. a goal, she almost always implies that the underlying belief is undisputable and therefore does not need to be explicitly mentioned.

Before the lesson was videoed, the teacher had visited a teacher training course on the use of the computer in school. Thus, the central question for the teacher on how the lesson should be designed comes as no surprise: *How shall I do it: with or without the computer?* Under the impression of the recently experienced teacher training course, her positive decision to employ the computer appeared close at hand. This is rather independent of the content, as it underlies: *You can do things in geometry with the computer*. The following goal on the basis of the positive assessment of the computer – abbreviated as G_{pm} 1 – is explicitly expressed as follows:

G_{pm} 1: Whenever possible, I employ the computer in mathematics lessons.

This goal is complemented by beliefs concerning the formal goal G_{formal} 2:

B_{pm} 2: The contents of the theme 'Linear Functions' can be mediated with the computer.

The teacher views the suitable mediation of this theme by employing the table calculation software Excel, i.e. the belief is expressed as follows:

B_{pm} 3: Excel is suitable for dealing with linear functions.

This teacher's goal is recognized in students' comments and is, for example, formulated as follows: *I think we should try and find out whether we can solve tasks with the aid of the computer and computer* *programs.* An important prerequisite for the teacher is that Excel offers the required possibilities, out of which a detailed, mathematics-specific goal is formulated:

G_{pm} 4: The students are to draw graphs.

Again a student reflects this goal when he or she articulates: *[...] that we are to do these graphs correctly.* However, Excel was not primarily designed as lesson software but as an office program. For example, diagrams for linear functions are produced in standardized formats and thus often look uniform; differences in slope values are very quickly blurred or lost. In the words of the teacher: *Excel fools you.* The teacher transforms such possibly occurring confusions positively by formulating from this circumstance a further pedagogical goal concerning the use of the computer:

G_{pm} 5: The use of the computer has to be accompanied by a critical discussion.

A further aspect emphasizing the relevance of the goal G_{pm} 1 can be formulated as follows:

B_{pm} 6: The computer is a modern, progressive medium.

Implicitly formulated as a goal:

G_{pm} 6: School lessons should use modern media.

Complementing this assessment that the computer is a progressive medium, the teacher also sees other educational advantages in employing this medium.

B_{pm} 7: The computer supports learning through discovery.

Independent of the computer, she formulates more generally:

G_{pm} 8: Mathematics lessons should offer students free space for discovery.

Following this, she articulates more pointedly her belief B_{pm} 3 about the suitability of Excel by linking it to G_{pm} 8:

B_{pm} 9: You can discover a lot with Excel.

This goal naturally demands circumstantial conditions concerning lesson organization by the teacher:

B_{pm} 10: Mathematics lessons have to be designed open.

Open lesson organization by the teacher and free space for students to discover are reciprocal. She completely fulfilled this requirement in her lesson planning and realized this approach consequently in the first half of the lesson up to the mentioned turning point. She explicates further:

B_{pm} 11: Open questions have to be prepared.

Complementary to the fundamentally positive approach, namely to design lessons open and with a discovery bias, she is well aware of the relevance of creating a suitable motivation disposition in the students. She wishes to fulfill this by stating clearly the goal:

G_{pm} 12: Mathematics lessons have to be motivating.

She justifies this goal from her point of view once again with the use of the computer when she states:

B_{pm} 13: The use of the computer can be motivating, in particular, in mathematically weak classes.

One can generally assume that the widespread claim that school lessons with contents taken from everyday life are particularly motivating is true. Her further going beliefs related to basic orientation in mathematics lessons can be formulated as follows:

B_{pm} 14: Mathematics lessons should have a link to the student's reality.

B_{pm} 15: Mathematics lessons have to be meaningful for the students.

In spite of all these good intentions, the way the lesson turned out did, for many reasons, not meet the teacher's expectations. She deserves credit without reservation for wanting to give innovative and successful lessons. This is also documented through her participation in further professional development activities. Therefore one can relate the following belief to her:

B_{pm} 16: Teacher training or professional development activities encourage new, progressive concepts for the improvement of lessons.

Incorporating such concepts in her lessons is an intention documented by the initial lesson plan. After the lesson the teacher therefore reflects in detail on the reasons for its deviation from the original plan and justifies her sudden modification of it by the following statement:

B_{pm} 17: Students can more easily handle concrete directives than open questions.

At a first glance the beliefs B_{pm} 17 and B_{pm} 10 about the difficulty and the necessity of open lessons respectively seem to contradict each other. The teacher is well aware of this contradiction and reconciles it with the thought that open questions also have to be prepared. In her words: *Open questions have to be drilled. You cannot simply throw an open question at the students and then say: Okay, start!* The phenomenon of contradictory beliefs is often mentioned in the literature and is dealt with by the hypothesis that beliefs need only be locally consistent, whereby their contradictory nature can then be coped with by the person holding them (Green, 1971).

It is also typical that she explicitly presents her belief in open questions as anchored to another individual and by this means delegates the question of responsibility to that person when she laughingly points out that B. encouraged her to try out the new approach: *B. is to blame again with her open question.* Abelson (1979) has presented the anchoring of beliefs to persons as a typical characteristic of beliefs.

The beliefs identified here once again demonstrate that it is appropriate to speak of ‚belief bundles', sometimes also called factors. In the following list, we focus on explicitly mentioned beliefs and the beliefs that are derived from goals. For example, G_{pm} 8 mentioned above appears here as B_{pm} 8. We can identify five such belief bundles:

(A) Beliefs about the computer:

B_{pm} 1: The computer und mathematics lessons belong together

B_{pm} 2: Linear functions can be dealt with using the computer

B_{pm} 3: Excel is the choice tool for linear functions

B_{pm} 5: Critical reflection is necessary when using the computer

B_{pm} 6: The computer is a modern and progressive medium

B_{pm} 7: The computer is an adequate tool for learning by discovering

(B) Beliefs about discovery-oriented lessons:

B_{pm} 8: Mathematics lessons have to be discovery lessons

B_{pm} 7: The computer is an adequate tool for learning by discovering

B_{pm} 9: Excel is a useful tool for discovery lessons

(C) Beliefs about open lessons:

B_{pm} 10: Mathematics lessons are to be openly designed

B_{pm} 11: Open questions have to be prepared

(D) Beliefs about motivational mechanisms:

B_{pm} 12: Mathematics lessons have to be motivating

B_{pm} 13: Employing the computer enhances motivation in weak classes

(E) Beliefs about reality-related lessons

B_{pm} 14: Mathematics lessons have to be reality-related

B_{pm} 15: Mathematics lessons have to address the sense of the lesson

Belief bundle (A) is characterized by a very positive assessment of the role of the computer; bundle (B) focuses on discovery-oriented lessons, whereas bundle (C) is concerned with open lessons and (D) with motivation in lessons. Bundle (A) is central and relates to the "corner" bundles (B), (C) and (D) with different intensity. Bundle (E) is concerned with the role of reality-related tasks for learning mathematics. The teacher appears to link this factor more strongly to (D) and less to (A); at least one cannot find any explicit clue pointing from (E) directly to (A).

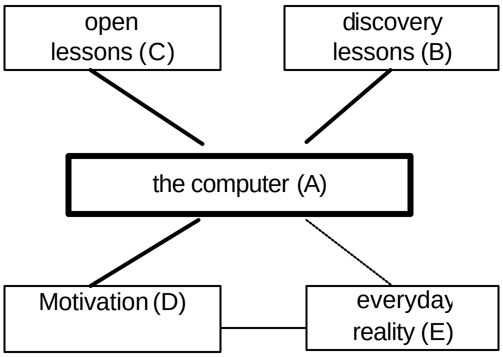

Diagram 1: Networking the belief aspects

Diagram 1 sketches the relations between the factors. We would like to point out that it is a drastic simplification which does not describe all the subtle connections between the bundles and even leaves out some other beliefs that played a role in the lesson, for example:

B_{pm} 16: Teacher training or professional development activities encourage new, progressive concepts for the improvement of lessons.

B_{pm} 17: Students can more easily handle concrete directives than open questions.

4.3 Internal structure of the content goals and beliefs

The content or content-specific beliefs and the goals derived from them are not only rooted in mathematics but also in beliefs about the curriculum. On the whole they create the impression of forming a stable network whose core is independent of pedagogical or methodological factors and is unaffected by the 'shortcomings' in the concrete realization of the lesson plan. These beliefs/goals will be labeled by the subscript mc.

B_{mc} 1: The term function is a central term of mathematics.

The teacher's diction emphatically points to the importance she gives to the theme: *I think the **term** function is infinitely important* where her stress on the word *term* is significant. This leads to a curricular goal whose manifold facets are discussed in many authoritative books on mathematical didactics:

G_{mc} 2: Dealing with functions is a central issue in mathematics lessons.

If this is so, one has to agree with the following judgment of the teacher:

B_{mc} 3: Linear functions are an elementary but important subclass of functions and are suitable for grade 8.

This leads to the following strengthened formulation as a goal:

G_{mc} 4: The treatment of linear functions is to be given more attention in grade 8.

The now following belief could be presented in various slightly different perspectives but its essential point is:

B_{mc} 5: Linear functions are defined by their slopes; the slope of a linear function is its most important characteristic.

The teacher reveals another relevant aspect by referring to the needs of future studies:

B_{mc} 6: Functions are important for “Analysis” (Calculus) in grade 12.

Extending the content of B_{mc} 5, the teacher underlines the following belief:

B_{mc} 7: The central term to be mediated in the context of linear functions is the concept of slope, which prepares students for the concept of derivative.

From this results the specific mathematical goal as a kind of output directive in connection with the formal und functional goals:

G_{mc} 8: The term slope must be mentioned in this lesson.

It appears that the goals G_{mc} 2 ... G_{mc} 8 describe an implication structure: firstly in the sense of a content hierarchy 2 $\rightarrow$ 4 $\rightarrow$ 8; secondly belief B_{mc} 6 and a similar content goal G_{mc} 4 are understood as propaedeutical arguments that in the last instance unavoidably characterize goal G_{mc} 8.

It seems that this implication structure is never put into question – as could be observed during the lesson – in spite of the consequences it has for the realization of the lesson under adherence to the goal G_{mc} 8.

In the interview, the teacher stated that being fixated on reaching this goal in that hour (!) proved to be a mistake. However, she did not question the importance of this fundamental goal, which probably could have been reached in the next lesson.

4.3 Structuring within the belief and goal bundles

The previous discussion has made clear that diverse goals cannot be simply understood as a list one can pull together according to certain overriding categories. Although it may make sense to bundle them together according to general characteristics, it should also be admitted that these are not the only relations between the goals. In any case one can ascertain a deductive structure (overriding goals – derived goals) influenced by mutual correlations. This is, however, not the place to analyze in detail beliefs in the sense of Green’s categorization. It seems that the prioritizations/assessments changed in the course of the lesson.

Besides, we see another mechanism of having greater relevance, namely the uncoupling, on a metalevel, of the pedagogical-methodological factors mentioned above. Concerning the pedagogical-methodological goals an interesting phenomenon appears as shown above in Diagram 1. Beliefs centred on the role of the computer are dominant, produce all the other connections, and are central for the conception of the lesson although a reciprocal correlation between the computer and the ‘everyday reality’ aspect could not be fully established. The use of the computer becomes problematic at the turning point in the lesson when it loses its central role, i.e., is simply switched off.

For this and other reasons not discussed here, the factors ‘open lesson’, ‘discovering lesson’ and ‘motivation’ play only marginal roles after the switching off; from this point onwards, global content and formal goals dominate the lesson activities to reach the one goal: the term slope must be mentioned. In other words, all pedagogical-methodological goals lose their rather positive value and step down to make room for content-related goals.

In a deliberately provocative formulation, content-related goals might be called ‘hard’ and pedagogical-methodological goals ‘soft’. In the words of a teacher who participated in a discussion of this lesson: *When the house is on fire, who will then worry about pedagogy? Then you can rely only on the systematic nature of the content.* Pedagogy then loses out in the game ‘pedagogy’ versus ‘content’ (comp. Wilson / Cooney (2002))

5. Evaluations

It was not the objective of this paper to conduct an analysis of this lesson with equal attention to all aspects. Thus one could continue with further explanations for the events in the lesson. However, the scope of this paper limits us here.

It has become clear, however, that Schoenfeld´s KGB-approach is of convincing explanatory power and has enabled us to illuminate central focal points. The dominance of the computer in the lesson plan is both its strength and its weakness and thus presents a risk factor for a successful unfolding of the lesson. Switching off the computer after 20 minutes rendered its mediating function obsolete (Noss & Hoyles, 1996). This regressive action decomposed and separated the well-intended pedagogical-methodological beliefs and made room for an approach dominated by systematic content.

We have only marginally mentioned the linking of beliefs and goals to the available teacher's knowledge, e.g. to the teacher's available action scripts. Here we find another reason for the turning point in the lesson: the teacher had not hitherto developed a sufficiently solid repertoire in employing Excel for the introduction of the concepts concerning linear functions, but she possesses sufficient experience for an introduction by a reliable and robust traditional approach.

A further deficit has also become apparent to the authors while dealing with this theme: there are no papers or research dealing with topological characteristics and the interweaving of these networks of beliefs and goals. Also, the gradation of beliefs according to their valences - for example in the sense of Green's categorization - is still a research field hardly touched.

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