



The Second Edition of The Handbook of International Research in Mathematics Education continues the mission of bringing together important new mathematics education research that makes a difference in both theory and practice. It updates and extends the Handbook's original key themes and issues for international research in mathematics education for the 21st century, namely:

- priorities in international mathematics education research
- lifelong democratic access to powerful mathematical ideas
- advances in research methodologies
- influences of advanced technologies

Each of these themes is examined in terms of learners, teachers, and learning contexts, with theory development being an important component of all these aspects. This edition also examines other catalysts that have gained increased import in recent years including a stronger focus on the teacher and teacher practice, a renewed interest in theory development, an increased focus on the mathematics needed in work place settings, and a proliferation of research designs and methodologies that have provided unprecedented opportunities for investigating (and ultimately improving) mathematical teaching and learning. This edition includes ten totally new chapters; all other chapters are thoroughly revised and updated.

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Advancing the discipline by presenting dynamic, future-oriented works that address new and emerging priorities in mathematics education research, the Second Edition includes ten totally new chapters; all other chapters are thoroughly revised and updated. This is a must-have volume for scholars, professors, and graduate students in the field of mathematics education and in related areas such as educational psychology and educational research.

**Lyn D. English** is currently professor of mathematics education at the Queensland University of Technology, Australia. She is an elected Fellow of The Academy of the Social Sciences in Australia, and founding editor of *Mathematical Thinking and Learning: An International Journal*. Publications include *Mathematical and Analogical Reasoning of Young Learners*, the first edition of the *Handbook of International Research in Mathematics Education*, *Classroom Research in Mathematics: A Critical Examination of Methodology* (with Simon Goodchild), *Mathematical Reasoning: Analogies, Metaphors, and Images*, and *Mathematics Education: Models and Processes* (with Graeme Halford), and numerous book chapters, journal articles, conference papers, and special journal issues.



# Handbook of International Research in Mathematics Education

Second Edition

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 **Routledge**  
Taylor & Francis Group  
NEW YORK AND LONDON

First published 2008  
by Routledge  
270 Madison Ave, New York, NY 10016

Simultaneously published in the UK  
by Routledge  
2 Park Square, Milton Park, Abingdon, Oxon OX14 4RN

Routledge is an imprint of the Taylor & Francis Group, an informa business

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Typeset in Galliard by EvS Communication Network, Inc.  
Printed and bound in the United States of America on acid-free paper by

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Library of Congress Cataloging in Publication Data  
Handbook of international research in mathematics education / edited by Lyn D. English ; associate editors, Maria Bartolini Bussi ... [et al.]. — 2nd ed.  
p. cm.  
1. Mathematics--Study and teaching—Research. I. English, Lyn D. II. Bartolini Bussi, Maria G. (Maria Giuseppina)  
QA11.2.H36 2008  
510.71—dc22  
2007041882

British Library Cataloguing in Publication Data  
A catalogue record for this book is available from the British Library

ISBN 10: 0-8058-5875-X (hbk)  
ISBN 10: 0-8058-5876-8 (pbk)  
ISBN 10: 0-203-93023-1 (ebk)

ISBN 13: 978-0-8058-5875-4 (hbk)  
ISBN 13: 978-0-8058-5876-1 (pbk)  
ISBN 13: 978-0-203-93023-6 (ebk)

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# Preface

This second edition of the *Handbook of International Research in Mathematics Education* is intended for those interested in international developments and future directions in educational research, in particular, mathematics education research. The original edition (English, 2002) was prepared in response to a number of major global catalysts for change, including the impact of national and international mathematics comparative assessment studies; the social, cultural, economic, and political influences on mathematics education and research; the influence of enhanced sophistication and availability of technology; and the increased globalization of mathematics education and research.

Although our discipline has made considerable progress in the past decade, with significant theoretical and practical advances in many areas, the foregoing catalysts for change are still pertinent today, but with varying degrees of impact. Other catalysts have gained greater import in recent years including an increased focus on the teacher and teacher practice, a renewed interest in theory development in our discipline (e.g., semiotic mediation, which is the focus of a few chapters here), an increased focus on the mathematics needed in work place settings, and a proliferation of research designs and methodologies that have provided us with unprecedented opportunities for investigating (and ultimately improving) mathematical teaching and learning.

This second edition addresses the original priority themes and issues for international research in mathematics education for the 21st century, namely, life-long democratic access to powerful mathematical ideas, advances in research methodologies, and influences of advanced technologies. Each of these themes is examined in terms of learners, teachers, and learning contexts, with theory development being an important component of all these aspects. Some of the original chapters have been removed from this second edition and new chapters have been added (namely, chapters 4, 11, 18, 20, 25, 27, 28). Remaining original chapters have been updated, many quite substantially.

The volume comprises four sections. The first section, *Priorities in International Mathematics Education Research*, provides important background information on the key themes of the book, and also introduces new and emerging research trends in the field. Following my introductory chapter, Carol Malloy (chapter 2) explores democratic access to mathematics through democratic education, while Dylan Wiliam and Frank Lester (chapter 3) address the purpose of mathematics education research, and Cliff Konold and Richard Lehrer explore technology and mathematics education.

Section 2 focuses on *lifelong democratic access to powerful mathematical ideas* from the perspective of (a) learning and teaching, and (b) learning contexts and policy issues. With respect to learning and teaching, consideration is given to students' learning during the preschool and beginning school years (Bob Perry & Sue Dockett, chapter 5), the elementary and middle school years (Cynthia Langrall, Edward Mooney, Steven Nisbet, and Graham Jones, chapter 6), the secondary school years (Teresa Rojano, chapter 7), and finally, the advanced levels of mathematics education (Joanna Mamona-Downs & Martin Downs, chapter 8). Issues pertaining to representation

in mathematical learning and problem solving are addressed by Gerald Goldin in chapter 9. Research in preservice and inservice teacher education is explored in chapter 10 by Ruhama Even and Dina Tirosh, and in chapter 11 by João Pedro da Ponte and Olive Chapman.

The second component of section 2, namely, *learning contexts and policy issues*, covers a range of globally significant topics such as access and opportunity within the political and social context of mathematics education (Celia Rousseau and William Tate, chapter 13), democratic access to mathematical learning in a developing country (Luis Moreno-Armella and Manual Santos-Trigo, chapter 14), and a cultural psychology perspective on mathematical learning in out-of-school contexts (Guida de Gabreu, chapter 15). The complexities of change in mathematics education reform are addressed by Miriam Amit and Michael Fried in chapter 16, while Ole Skovsmose's and Paolo Valero's analysis of democratic access to powerful mathematical ideas completes the section.

In section 3, the chapters focus on *advances in research methodologies*. In chapter 18 Alan Schoenfeld explores the numerous past, current, and possible future trends in conceptual frameworks and paradigms used in mathematics education research. Margret Hjalmarson and Richard Lesh consider design research with a focus on engineering, systems, products, and processes for innovation. The importance of linking research with practice is also emphasized in this section, in particular, in the chapters by Nicolina Malara and Rosetta Zan (chapter 20), Kenneth Ruthven and Simon (chapter 21), and Douglas Clements (chapter 22). In chapter 25, Fulvia Furinghetti and Luis Radford discuss how the pedagogical use of the history of mathematics can serve as a means to transform teaching. The section concludes with Bharath Sriraman's and Günter Törner's survey and analysis of different traditions in mathematics education research within Europe particularly in Germany, France, and Italy (chapter 25).

In the final section, the *influences of advanced technologies on mathematical learning and teaching are investigated*. These chapters also include substantial theoretical development in relation to technology and mathematics education. Chapter 26, by Jim Kaput, Richard Noss, and Celia Hoyles looks at developments of new notations for mathematics learning in the computational era. This chapter has not been altered from its original version, given the tragic passing of Jim Kaput in 2005. The ideas presented in the chapter are as relevant, powerful, and future-oriented as they were in the first edition. New chapters in this section include those by Ferdinando Arzarello and Ornella Robutti (chapter 27), who address an embodied mind approach within a multimodal paradigm, and Maria Bartolini Bussi and Maria Alessandra Mariotti, (chapter 28), who provide insights into semiotic mediation in the mathematics classroom. In the remaining chapters, Michal Tabach, Rina Hershkowitz, Tommy Dreyfus, and Abraham Arcavi (chapter 29) present a research-design view of computerized environments in mathematics classrooms, Michal Yerushalmy and Danile Chazan (chapter 30) look at technology and curriculum design with a focus on the ordering of discontinuities in school algebra, and Rosa Bottino and Giampaolo Chiappini (chapter 31) explore advanced technology and learning environments with a focus on their relationships within the arithmetic problem-solving domain.

The concluding chapter (32) addresses some of the key 21st century issues in the advancement of mathematics education and mathematics education research. These include, among others, interdisciplinary debates on the powerful mathematical ideas students need to succeed in today's world, calls for research to support more equitable mathematics curriculum and learning access for all students and to find more effective ways of creating learning environments that can increase such learning access, and the need to improve teacher education and development to achieve our goal of powerful mathematics for all. The concluding chapter also reviews the broadening of research designs and methodologies in our discipline, which are providing strong bases for advancing the learning and teaching of mathematics.

#### Acknowledgments

This second edition of the *Handbook of International Research in Mathematics Education* would not have been possible without the unwavering support of many people. First, I wish to

extend my sincere thanks to all the authors—without their contribution the *Handbook* would not exist. Second, I convey my heartfelt thanks to the associate editors for their continued support, in particular, I wish to note my appreciation of the contribution of the new associate editor, Bharath Sriraman, for his insightful reviews of many of the chapters. Third, I sincerely thank Jo Macri, who has been a wonderful support to me in finalizing the *Handbook*. Her dedication, efficiency, and keen eye for detail have been superb.

Last, but not least, I wish to thank Larry Erlbaum, Naomi Silverman, and Erica Kica from Lawrence Erlbaum Associates for providing me this opportunity to produce a second edition of the *Handbook* and for their continued support throughout this process. The more recent support I have received from Taylor & Francis, in particular, Mary Hillemeier, is also gratefully acknowledged.



