**Let Lakatos Be! - A Commentary to "Would the real Lakatos please stand up"**

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Abstract: *In this commentary, some remarks are offered on David Pimm, Mary Beisiegel and Irene Meglis' article "Would the real Lakatos please stand up". The commentary focuses on relatively recent developments in the philosophy of mathematics based on the work of Lakatos; on theory development in mathematics education; and offers critique on whether Lakatos' Proofs and Refutations can be directly implicated in mathematics education.*
Key words: mathematical methodology, philosophy of mathematics, mathematics education, philosophy of mathematics education, classroom discussion, school mathematics, Imre Lakatos, *Proofs and Refutations*,

*"Nature and Nature's laws lay hid in night; God said, Let Newton be! and all was light."*
*-Alexander Pope*

***Prelude***

Keeping in spirit with David Pimm et al's tongue-in-cheek title "*Would the real Lakatos please stand up*", which people familiar with hip hop culture will realize is a play on Eminem's famous song "(Will) *The Real Slim Shady...please stand up*", the title of this commentary is a play on Pope's famous couplet with a more serious intent, namely to examine the arguments made by Pimm et al., as well as reflect on my own changing position on the significance of Lakatos' work for mathematics education.

***What is Proofs and Refutations about?***

At the very outset, let me clarify that *Proofs and Refutations* is a work situated within the philosophy of science and clearly not intended for, nor advocates a didactic position on the teaching and learning of mathematics. Pimm et al., point out that the mathematics education community has not only embraced the work but has also used it to put forth positions on the nature of mathematics (Ernest, 1991), its teaching and learning (Ernest, 1994; Lampert, 1990; Sriraman, 2006). They further state:

> We are concerned about the proliferating Lakatos personas that seem to exist, including a growing range of self-styled 'reform' or 'progressive' educational practices get attributed to him.

This is a serious concern, one that I sympathize with but one that the community of mathematics educators has not addressed. So I welcome this opportunity to comment on and hopefully complement the arguments put forth by Pimm et al. Generally speaking *Proofs and Refutations* addresses the importance of role of history and the need to consider the historical development of mathematical concepts x,y,z in advocating any philosophy of mathematics. In other words, the book attempts to bridge the worlds of historians and philosophers. As one of the early reviews of the book points out:

> "His *(Lakatos')* aim is to show while the history of mathematics without the philosophy of mathematics is blind, the philosophy of mathematics without the history of mathematics is empty" (Lenoir, 1981, p. 100). (italics added)

Anyone that has read *Proofs and Refutations* and tried to find other mathematical "cases" such as the development of the Euler-Descartes theorem for polyhedra , will know that the so called "generic" case presented by Lakatos also happens to be one of the few special instances in the history of mathematics which reveals the rich world of actually *doing* mathematics, the world of the working mathematician, of informal mathematics, characterized by conjectures, failed proofs, thought experiments, examples and counter examples etc. The book in sense reveals the inner world of the working mathematician and Pimm et al., state "his work as a study of 'mathematical methodology', and his argument with philosophers of mathematics".

Reuben Hersh began to popularize this book to the mathematics community in a paper titled, "*Introducing Imre Lakatos*" (Hersh, 1978) and called for the community of mathematicians to take an interest in re-examining the philosophy of mathematics. Nearly three decades later, Hersh (2006) attributes *Proofs and Refutations* as being instrumental in a revival of the philosophy of mathematics informed by scholars from numerous domains outside of mathematical philosophy, "in a much needed and welcome change from the foundationist ping-pong in the ancient style of Rudolf Carnap or Willard van Ormond Quine" (p.vii). An interest in this book among the community of philosophers grew as a result of Lakatos' untimely death, as well as a favourable review of the book given by W.V.Quine himself in 1977 in the *British Journal for the Philosophy of Science.* The book can be viewed as a challenge for philosophers of mathematics, but resulted in those outside this community to take interest and contribute to its development (Hersh, 2006).

According to Lerman (2000), current interest in the mathematics education community in the philosophy of mathematics is also traceable to the very same book *Proofs and Refutations*. The work of Lakatos has influenced mathematics education as seen in the social constructivists' preference for the "Lakatosian" conception of mathematical certainty as being subject to revision over time, in addition to the language games a là Wittgenstein "in establishing and justifying the truths of mathematics"(Ernest, 1991, p .42) to put forth a fallible and non-Platonist viewpoint about mathematics. This position is in contrast to the Platonist viewpoint, which views mathematics as a unified body of knowledge with an ontological certainty and an infallible underlying structure. The emergence of social constructivism as a philosophy of mathematics education (Ernest, 1991), the well documented debates between radical constructivists and social constructivists (Steffe et al.,1998; von Glasersfeld, 1984, 1987), the recent interest in mathematics semiotics, an increased focus on the cultural nature of mathematics, and a call for the teaching and learning of mathematics as a humanistic, quasi-empirical activity subject to fallibility as fertile common ground for debate among mathematics educators, mathematicians, cognitive scientists, linguists, sociologists, anthropologists, and last but not least the philosophers! This leads to the central question, which Pimm et al., address, have *we* distorted or misused or misappropriated the intent of Lakatos' book in the mathematics education community? My answer is yes and no.

As stated earlier, *Proofs and Refutations* was intended for philosophers of mathematics to be cognizant of the historical development of ideas. Yet, its popularization by Reuben Hersh (and Philip Davis) gradually led to the development of the so called "maverick" traditions in the philosophy of mathematics, culminating in the release of Reuben Hersh's book *18 Unconventional essays on the nature of mathematics*, a delightful collection of essays written by mathematicians, philosophers, sociologists, an anthropologist, a cognitive scientist and a computer scientist. These essays are scattered "across time" in the fact that Hersh collected various essays written over the last 60 years that support the "maverick" viewpoint. His book questions what constitutes a philosophy of mathematics and re-examines foundational questions without getting into Kantian, Quinean or Wittgensteinian linguistic quagmires. In a similar vein the work of Paul Ernest can be viewed as an attempt to develop a maverick philosophy, namely social constructivist philosophy of mathematics (education). I have put the word education in parentheses because Ernest does not make any explicit argument for an associated pedagogy (Steffe, 1992).

***Embracing Unconventionalism: Hersh's 18 unconventional essays***

The essays in Hersh's (2006) recent book corroborate many of Lakatos' and Ernest's positions. For instance in the chapter, entitled *Introduction to Filosofia e Matematica*, Carlo Celluci shreds to pieces 13 dominant views/assumptions about mathematics. According to Celluci, mathematics is a messy human endeavor, and not the playground for purely speculative and abstract philosophical questions and solutions alà Frege. Therefore the view of the *philosophy of mathematics* as a specialized area of philosophy in which problems occur in a "pure or especially simplified form" (p.17) marginalizes the scope of philosophical problems that occur in mathematics. Celluci also gives convincing arguments to the messy and social nature of proof, far removed from the dominant view that deduction is the holy grail of mathematical thinking. In another chapter, William Timothy Gowers (the 1998 fields medal winner) poses the question, "Does mathematics needs a philosophy?" One of the main arguments made in this chapter is that mathematics would proceed pretty much the way it always has irrespective of any brand new –isms developed in philosophy. One interesting theme in Gower's chapter is that what professional philosophers consider important is too far removed from the actual business of doing mathematics. In a similar vein, Gian-Carlo Rota, a distinguished applied mathematician and a philosopher, writes that while philosophy can never truly answer its fundamental questions definitively, these "answers" are more connected with problems of our existence. Rota provokes the field of philosophy by saying that the fundamental questions in philosophy have been the same starting with the Greeks. So the answers posed by one generation of philosophers end up being revised or rejected by the next generation. Thus, there are no definitive answers per se. Rota then argues that the field of mathematics is the envy of philosophy because mathematicians are able to give definitive answers to the problems that occur within mathematics. This in turn has influenced philosophy to adopt mathematical standards of rigor for the study of philosophical problems. Logic, which was always a part of philosophy, has mutated into mathematical logic and consequently become subsumed as one of the many areas of mathematics. Does philosophy want to meet the same fate? This is Rota's provocation to philosophers. In this book, Eduard Glas argues that Popper's contributions to the philosophy of science are both relevant and adaptable to a good working philosophy of mathematics without the entrapments of the current –isms in use. Hersh succeeds in knitting together a masterful collection of essays that reduce commonly held prejudices about mathematics as well as the "tower" of ideas constituting the philosophy of mathematics. The fact that scholars from domains outside of mathematics can better inform mathematics on the building blocks for a philosophy of mathematics has some implications for mathematics education.

***Lakatos and Mathematics Education: Indirect Implications for research and practice***

Does Lakatos' work have any direct significance for mathematics education? Pimm et al., have given an excellent exposition on the "Lakatosian" revolution in mathematics education starting with Joseph Agassi, onto Paul Ernest, Magdalene Lampert, and referred to my work. It is therefore unnecessary for me to give my own summary of this "revolution". My angle on the relevance of Lakatos for mathematics education comes more from the view of doing research and being a practitioner, both of which have to rest on an underlying philosophy and an associated theory of learning.

The present diversity in the number of new theories used in mathematics education from domains like cognitive science, sociology, anthropology and neuro-sciences are both natural and necessary given the complexity in teaching and learning processes/situations in mathematics. However, theory development is essential for any field and mathematics education has often been accused of "faltering" in theories. The development of "universal" theoretical frameworks has been problematic for mathematics education. A research forum on this topic was organized by me, with Lyn English at the 29th Conference of the International Group of Psychology of Mathematics Education in Melbourne (Sriraman & English, 2005) . In one of the extended papers coming out from this research forum, Lester elaborated on the effect of one's philosophical stance in research. Lester (2005) wrote:

> "Cobb puts philosophy to work by drawing on the analyses of a number of thinkers who have grappled with the thorny problem of making reasoned decisions about competing theoretical perspectives." He uses the work of noted philosophers such as (alphabetically) John Dewey, Paul Feyerabend, Thomas Kuhn, Imre Lakatos, Stephen Pepper, Michael Polanyi, Karl Popper, Hilary Putnam, W. V. Quine, Richard Rorty, Ernst von Glasersfeld, and several others to build a convincing case for considering the various theoretical perspectives being used today "as sources of ideas to be appropriated and adapted to our purposes as mathematics educators." (p.461)

Can Lakatos' *Proofs and Refutations* be directly implicated for the teaching and learning of mathematics? My position today is no. However *Proofs and Refutations* may very well serve as a basis for a philosophy of mathematics, such as a social constructivist philosophy of mathematics (note, I do not say mathematics education here), which in turn can be used a basis to develop a theory of learning such as Constructivism. This is a position that Steffe advocated, that has gone unheeded. Les Steffe in his review of Ernest's (1991) *The Philosophy of Mathematics* Education wrote:

> Constructivism is sufficient because the principles of the brand of constructivism that is currently called "radical"(von Glasersfeld,1 989) should be simply accepted as the principles of what I believe should go by the name Constructivism. It seems to me that the radical constructivism of von Glasersfeld and the social constructivism of Ernest are categorically two different levels of the same theory. Constructivism (radical), as an epistemology, forms the hard core of social constructivism, which is a model in what Lakatos (1970) calls its protective belt. Likewise, psychological constructivism is but a model in the protective belt of the hard-core principles of Constructivism. These models continually modify the hard-core principles, and that is how a progressive research program that has interaction as a principle in its hard core should make progress. It is a lot easier to integrate models in the protective belt of a research program that has been established to serve certain purposes than it is to integrate epistemological hard cores (Steffe, 1992, p.184).

***Caveat Emptor: Some Instances of experimenting with Proofs and Refutations***

My encounter with *Proofs and Refutations* began as a result of reading *The Mathematical Experience* (Davis & Hersh, 1981) and the expository writings of Reuben Hersh in *The Mathematical Intelligencer* which drew inspiration from Lakatos. In my first reading of the *Proofs and Refutations* many of the objections raised by some of the imaginary students reminded me of Proclus, a commentator on Euclid's Elements. Using the literary latitude allowed to neo-Lakatosians (pun intended), I will offer the unfamiliar reader a digress into the life of Proclus[1]. Proclus (410-485 AD), was a philosopher, part time-mathematician, born in Byzantium, studied law in Alexandria. He was also educated in Aristotelian logic and mathematics with Heron, who indoctrinated him into a newer blend of Platonism. Proclus went to Athens in 430 and studied Aristotle and Plato. For fifty years Proclus taught and preached Neo-Platonism, and devoted a lot of time writing commentaries on Plato's *Timaeous* and on Euclid's *Elements*, a majority of which were objections, philosophical digressions, and alternative proofs. Many of the objections and digressions indulged in by Proclus were not his own but those of the mathematicians before him (similar to the exercise of Imre Lakatos). In most instances, it is not Proclus' original thoughts that are written but instead those of mathematicians before him. Proclus attributed the creation of geometry to the Egyptians, brought to Greece by Thales, where it was further developed by Pythagoras and his followers. By incorporating the work, imaginary objections and alternative proofs of others involved in the creation of geometry in his commentaries, Proclus in a sense came up with a historical pre-cursor to *Proof and Refutations*.

About nine years ago as a classroom teacher, I tried to experiment with using conjecture-proof-refutations with high school students in relatively small classrooms. Some of these classrooms consisted of students labeled as extremely high academic achievers, who were open to lessons that had a dose of ambiguity and which often did not terminate in a Platonic formula. I have reported on some of these experiments with high school students (Sriraman, 2003), individualized thought-experiments with pre-service elementary teachers (Sriraman & Daniels, preprint) and also idealized on the pedagogical possibilities of mathematical discourse (Sriraman, 2006) inspired by Lakatos' imaginative fashion. However they do not in any way represent any "exemplary" instance of exactly replicating Lakatos' discourse.

As Pimm et al., point out the role of the teacher in *Proofs and Refutations* was that of a literary device who "had to mediate a fictionalized discussion", and I will add in strong terms, a teacher thoroughly versed in the historical development of, and the mathematics associated with that particular problem. There are certainly possibilities of using the book as an inspiration for classroom discourse and conjecture-proof-refutation on a particular problem but this would entail careful planning and an environment where the "students" are given unlimited time to pursue/delve into the historical details of the problem under investigation (a priori). Even under such conditions, it is impossible to predict whether the resulting discourse will be mathematically fruitful. A better pedagogical "application" for mathematics education may be the introduction of historical instances of thought experiments in the mathematics engaged in by Galileo, Newton and others, or using problems that bring in ambiguity (Sriraman & Knott, 2006). There is also some value to teaching the "methodology" of doing mathematics. In this vein, a recent study

[1] The research done by Nicole Crouch and Sarah Olson in my Euclidean and Non-Euclidean Geometry course, aided in this biographical compilation of Proclus. I wish to thank them for their diligence.

done in Korea explored "how the constructions of mathematically gifted fifth and sixth grade students using Euler's polyhedron theorem compared to those of (historical) mathematicians discussed by Lakatos" (Yim, Song, Kim, 2008, p. 125). In this structured study, "some of the students suggested using the monster-barring method, in addition to two new types of conjectures to resolve the conflicts between counterexamples and the theorem, the exception-baring method and the monster-adjustment method and their constructions resembled those presented by mathematicians as discussed by Lakatos" (see Yim, Song, Kim, 2008, p.125). Again these are all experimentations inspired by Lakatos' book done under very different circumstances than that intended by *Proofs of Refutations.*

Pimm, Beisiegel and Meglis have started an important and necessary discussion on the question of "appropriating" ideas from the philosophy of science into mathematics education, particularly those of Lakatos. I will close by using a slightly mischievous turn of phrase- Having experimented and used "Lakatos" in the classroom, and reflected on the outcomes (positive and negative) of such an approach, I find Pimm et al's Coda worth heeding. My own coda for the education community is "*Let Lakatos Be!*"

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