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ORIGINAL ARTICLE

The characteristics of mathematical creativity

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Abstract Mathematical creativity ensures the growth of mathematics as a whole. However the source of this growth, the creativity of the mathematician is a relatively unexplored area in mathematics and mathematics education. In order to investigate how mathematicians create mathematics; a qualitative study involving five creative mathematicians was conducted. The mathematicians in this study verbally reflected on the thought processes involved in creating mathematics. Analytic induction was used to analyze the qualitative data in the interview transcripts and to verify the theory driven hypotheses. The results indicate that in general, the mathematicians' creative process followed the four-stage Gestalt model of *preparation–incubation–illumination–verification*. It was found that social interaction, imagery, heuristics, intuition, and proof were the common characteristics of mathematical creativity. In addition contemporary models of creativity from psychology were reviewed and used to interpret the characteristics of mathematical creativity.

Keywords Domain specific creativity · Gestalt psychology · Jacques Hadamard · Systems views of creativity · Theories of creativity · Mathematical creativity

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1 Introduction

Mathematical creativity ensures the growth of the field of mathematics as a whole. The constant increase in the number of journals devoted to mathematical research bears evidence to the growth of mathematics. Yet what lies at the essence of this growth, viz., the creativity of the mathematician has not been the subject of much research. It is usually the case that most mathematicians are uninterested in analyzing the thought processes that result in mathematical creation (Ervynck, 1991). The earliest known attempt to study mathematical creativity was an extensive questionnaire published in the French periodical *L'Enseigement Mathematique* (1902). This questionnaire and a lecture by the renowned 20th century mathematician Henri Poincaré, to the *Societé de Psychologie* on creativity inspired his colleague Jacques Hadamard, another prominent 20th century mathematician to investigate the psychology of mathematical creativity (Hadamard, 1945). Hadamard (1945) undertook an informal inquiry among prominent mathematicians and scientists in America such as George Birkhoff, George Polya, and Albert Einstein, about the mental images used in doing mathematics. Hadamard (1945) influenced by Gestalt psychology of his time theorized that mathematicians' creative process followed the four-stage Gestalt model (Wallas, 1926) of *preparation–incubation–illumination–verification*. The four-stage Gestalt model is a characterization of the mathematician's creative process, which does not define creativity per se. How does one define creativity? In particular what exactly is mathematical creativity? Is it the discovery of a new theorem by a research mathematician? Does student discovery of a hitherto known result also constitute creativity? These are the areas of exploration in this paper.

2 The problem of defining creativity

Mathematical creativity has been simply described as discernment, *choice* (Poincaré, 1948). According to Poincaré (1948), to create consists precisely in not making useless combinations and in making those which are useful and which are only a small minority. This may seem like a vague characterization of mathematical creativity. One can interpret Poincaré's "*choice*" metaphor to mean the ability of the mathematician to choose carefully between questions (or problems) that bear fruition, as opposed to those that lead to nothing new. But this interpretation does not resolve the fact that Poincaré's definition of creativity overlooks the problem of novelty. In other words, characterizing mathematical creativity as the ability to choose between useful and useless combinations is akin to characterizing the art of sculpting as a process of cutting away the unnecessary!

Poincaré's (1948) definition of creativity was a result of the circumstances under which he stumbled upon deep results in Fuchsian functions. The first stage consisted of working hard to get an insight into the problem at hand. Poincaré (1948) called this the preliminary period of conscious work. This is also referred to as the *preparatory stage* (Hadamard, 1945). The second stage is when the problem is put aside for a period of time and the mind is occupied with other problems. This is referred to as the *incubatory stage* (Hadamard, 1945). The third stage is where the solution suddenly appears while perhaps engaged in other unrelated activities. "This appearance of sudden illumination is a manifest sign of long, unconscious prior work" (Poincaré, 1948). Hadamard (1945) referred to this as the *illuminatory stage*. However, the creative process does not end here. There is a fourth and final stage, which consists of expressing the results by language or writing. At this stage one verifies the result, makes it precise, and looks for possible extensions through utilization of the result. The Gestalt model has some shortcomings. First, the model mainly applies to problems that have been posed a priori by mathematicians, thereby ignoring the fascinating process by which the actual questions are arrived at. Secondly, the model attributes a large portion of what "happens" in the *incubatory–illuminatory* phases to subconscious drives. The problem of how questions are arrived at is partially addressed by Ervynck (1991) in his three-stage model.

Ervynck (1991) described mathematical creativity in terms of three stages. The first stage (*Stage 0*) is referred to as the *preliminary technical stage*, which consists of "some kind of technical or practical application of mathematical rules and procedures, without the user having any awareness of the theoretical foundation." (p. 42). The second stage (*Stage 1*) is that of *algorithmic activity*, which consists primarily of performing mathematical techniques, such as explicitly applying an algorithm repeatedly. The third stage (*Stage 2*) is referred to that as *creative (conceptual, constructive) activity*. This is the stage at which true mathematical creativity occurs and consists of non-algorithmic decision-making. 'The decisions that have to be taken may be of a widely divergent nature and always involve a choice." (p. 43). Although Ervynck (1991) tries to describe the process by which a mathematician arrives at the questions through his characterizations of *Stage 0* and *Stage 1*, his description of mathematical creativity is very similar to that of Poincaré and Hadamard. In particular his use of the term "non-algorithmic decision making" is analogous to Poincaré's use of the "choice" metaphor.

The author is unaware of any literature in mathematics education that attempts to explicitly define creativity. There are references made to creativity by the soviet researcher Krutetskii (1976) in the context of student's abilities to abstract and generalize mathematical content. There is however one outstanding example of a mathematician (George Polya) attempting to give heuristics to tackle problems in a manner akin to trained mathematicians. Polya (1954) observed that in "trying to solve a problem, we consider different aspects of it in turn, we roll it over and over in our minds; variation of the problem is essential to our work". Polya (1954) emphasized the use of a variety of heuristics for solving mathematical problems of varying complexity. In examining the plausibility of a mathematical conjecture, mathematicians use a variety of strategies. In looking for conspicuous patterns, mathematicians use a variety of heuristics such as (1) verifying consequences, (2) successively verifying several consequences, (3) verifying an improbable consequence, (4) inferring from analogy, (5) deepening the analogy. Thus, heuristics can be viewed as a decision-making mechanism, which lead the mathematician down a certain path, the outcome of which may or may not be fruitful.

As is evident in the preceding paragraphs, the problem of defining creativity is by no means an easy one.

However psychologists renewed interest in the phenomenon of creativity has resulted in literature that attempts to define and operationalize the word "creativity." Recently psychologists have attempted to link creativity to measures of intelligence (Sternberg, 1979, 1985), the ability to abstract and generalize (Sternberg, 1985), and to complex problem-solving abilities (Frensch & Sternberg, 1992). Sternberg and Lubart (2000) define creativity as the ability to produce unexpected original work, which is useful and adaptive. Mathematicians would raise several arguments with this definition, simply because the results of creative work may not always have implications that are "useful" in terms of applicability in the real world. A recent example that comes to mind is Andrew Wiles' proof of Fermat's Last Theorem. The mathematical community views his work as creative. It was unexpected and original but had no applicability in the sense of what Sternberg and Lubart (2000) suggest. Hence, I think it is sufficient to define creativity as the ability to produce novel or original work, which is compatible with my personal definition of mathematical creativity as the process that results in unusual and insightful solutions to a given problem, irrespective of the level. In the context of this study involving professional mathematicians, mathematical creativity is defined as the publishing of original results in prominent mathematics research journals.

2.1 The motivation for studying creativity

The lack of recent mathematics education literature on creativity was one of the motivations for conducting this study. Fifteen years ago, Muir (1988) invited mathematicians to complete a modified version of the original survey in the *L'Enseigement Mathematique* (1902). The results of this endeavor are of great interest but as yet unknown. The purpose of this study was to gain an insight into the nature of mathematical creativity. This was done by interviewing five accomplished and creative mathematicians, using a modification of the interview protocol that appeared in *L'Enseigement Mathematique* (1902) and Muir (1988). The purpose of using a modified form of this antiquated questionnaire is discussed in the methodology section of the paper. The author was interested in distilling common attributes of the creative process, to see if there were any underlying themes that characterized mathematical creativity. The specific questions of exploration were:

- Is the Gestalt model of mathematical creativity still applicable today?
- What are the characteristics of the creative process in mathematics?
- Does the study of mathematical creativity have any implications for the classroom?

3 Literature review

Any study on the nature of mathematical creativity begs the question as to whether the mathematician discovers or invents mathematics. Therefore the review begins with a brief description of the four popular viewpoints on the nature of mathematics. This is followed by a comprehensive review of contemporary models of creativity from psychology.

3.1 The nature of mathematics

Mathematicians who are actively involved in research have certain beliefs about the ontological status of mathematics, which influences their approach to research (Davis & Hersh, 1981; Sriraman 2004). The Platonist viewpoint is that mathematical objects exist prior to their discovery, and that "any meaningful question about a mathematical object has a definite answer, whether we are able to determine it or not" (Davis & Hersh, 1981). According to this view, mathematicians do not invent or create mathematics—they discover mathematics. Logicists hold that "all concepts of mathematics can ultimately be reduced to logical concepts" which implies that "all mathematical truths can be proved from the axioms and rules of inference and logic alone," (Ernest, 1991). Formalists do not believe that mathematics is discovered; they believe mathematics is simply a game, created by mathematicians, based on strings of symbols which have no meaning (Davis & Hersh, 1981). Constructivism (incorporating Intuitionism) was one of the major schools of thought (besides Platonism, Logicism, and Formalism) that arose due to the contradictions that emerged in the theory of sets and the theory of functions during the early part of the 20th century. Contradictions like Russell's Paradox were a major blow to the absolutist view of mathematical knowledge, for if mathematics is certain and all its theorems are certain, how can contradictions be among its theorems? The early constructivists in mathematics were the intuitionists Brouwer and Heyting. Constructivists claim that both mathematical truths and the existence of mathematical objects must be established by constructivist methods. The constructivist (intuitionist) viewpoint is that "human mathematical activity is fundamental in the creation of new knowledge and that both mathematical truths and the existence of mathematical objects must be established by constructive methods" (Ernest, 1991, p. 29).

The question then is how does a mathematician go about conducting mathematics research? Do the questions appear out of the blue or is there a mode of thinking or inquiry that leads to the meaningful questions and to the methodology for tackling these questions? The author contends that the types of questions asked are determined to a large extent by the culture the mathematician finds herself in. Simply put, it is impossible for an individual to acquire knowledge of the external world without social interaction. According to Ernest (1994) there is no underlying metaphor for the wholly isolated individual mind. Instead the underlying metaphor is that of persons in conversation, comprising persons in meaningful linguistic interaction and dialogue (Ernest, 1994). Language is the shaper of, as well as being the "summative" product of individual minds (Wittgenstein, 1978). The recent literature in psychology acknowledges these social dimensions of human activity as being instrumental in the creative process. This warrants an in-depth review of this literature.

3.2 The notion of creativity in psychology

As stated earlier, research on creativity has been on the fringes of psychology, educational psychology and mathematics education. It is only in the last 25 years that there has been a renewed interest in the phenomenon of creativity in the psychology community. *The Handbook of Creativity* (Sternberg, 2000) which contains a comprehensive review of all research available in the field of creativity suggests that most of the approaches used in the study of creativity can be subsumed under six categories, namely: mystical, pragmatic, psychodynamic, psychometric, cognitive, and social-personality. Each of these approaches are briefly reviewed.

3.2.1 The mystical approach

The mystical approach to studying creativity suggests that creativity is the result of divine inspiration, or is a spiritual process. In the history of mathematics, Blaise Pascal claimed that many of his mathematical insights came directly from God. The renowned 19th century algebraist Leopold Kronecker said that "God made the integers, all the rest is the work of man" (Gallian, 1994). There have been attempts to explore possible relationships between the mathematician's belief on the nature of mathematics and their creativity (Davis and Hersh, 1981; Hadamard, 1945; Poincaré, 1948; Sriraman, 2004). These studies indicate that there is certainly a relationship between a mathematician's belief on the nature of mathematics and creativity. It is commonly believed that the neo-Platonist view is helpful to the research mathematician because of the innate belief that the sought after result/relationship already exists.

3.2.2 The pragmatic approach

The pragmatic approach entails "being concerned primarily with developing creativity" (Sternberg, 2000, p. 5), as opposed to understanding it. Polya's (1954) emphasis on the use of a variety of heuristics for solving mathematical problems of varying complexity is an example of a pragmatic approach. Thus, heuristics can be viewed as a decision-making mechanism, which lead the mathematician down a certain path, the outcome of which may or may not be fruitful. The popular technique of brainstorming used in corporations is another example of inducing creativity by seeking as many ideas or solutions possible in a non-critical setting.

3.2.3 The psychodynamic approach

The psychodynamic approach to studying creativity is based on the idea that creativity arises from the tension between conscious reality and unconscious drives (Hadamard, 1945; Poincaré, 1948, Sternberg, 2000, Wallas, 1926; Wertheimer, 1945). The four-step Gestalt model (preparation–incubation–illumination–verification) is an example of the use of a psychodynamic approach to studying creativity. It should be noted that the Gestalt model has served as the kindling for many contemporary problem-solving models (Polya, 1945; Schoenfeld, 1985; Lester, 1985). Early psychodynamic approaches to creativity were used to construct case studies of eminent creators such as Albert Einstein, but this approach was criticized by the behaviorists because of the difficulty of measuring proposed theoretical constructs.

3.2.4 The psychometric approach

The psychometric approach to studying creativity entails quantifying the notion of creativity with the aid of paper and pencil tasks. An example of this would be the *Torrance Tests of Creative Thinking* developed by Torrance (1974), which are used by many gifted programs in middle and high schools, to identify students who are gifted/creative. This test consists of several verbal and figural tasks that call for problem-solving skills and divergent thinking. The test is scored for fluency, flexibility, originality (the statistical rarity of a response), and elaboration (Sternberg, 2000). Sternberg (2000) states that there are positive and negative sides to the psychometric approach. On the positive side, these tests allow for research with non-eminent people, are easy to administer, and objectively scored. The negative side is that numerical scores fail to capture the concept of creativity because they are based on brief paper and pencil tests. Researchers call for use of more significant productions such as writing samples, drawings, etc to be subjectively evaluated by a panel of experts instead of simply relying on a numerical measure.

3.2.5 The cognitive approach

The cognitive approach to the study of creativity focuses on understanding the "mental representations and processes underlying human thought" (Sternberg, 2000, p. 7). Weisberg (1993) suggests that creativity entails the use of ordinary cognitive processes and results in original and extraordinary products. These products are the result of cognitive processes acting on the knowledge already stored in the memory of the individual. There is a significant amount of literature in the area of information-processing

(Birkhoff, 1969; Minsky, 1985) that attempts to isolate and explain cognitive processes in terms of machine metaphors.

3.2.6 The social-personality approach

The social-personality approach to studying creativity focuses on personality and motivational variables as well as the socio-cultural environment as sources of creativity. Sternberg (2000) states that numerous studies conducted at the societal level, indicate that "eminent levels of creativity over large spans of time are statistically linked to variables such as cultural diversity, war, availability of role models, availability of financial support, and competitors in a domain." (p. 9).

Most of the recent literature on creativity (Csikszentmihalyi, 1988, 2000; Gruber & Wallace, 2000, Sternberg & Lubart, 1996) suggests that creativity is the result of confluence of one or more of the factors from these six aforementioned categories. The "confluence" approach to the study of creativity has gained credibility and the research literature has numerous confluence theories for better understanding the process of creativity. This calls for a review of the most commonly cited confluence theories of creativity. This is followed by a description of the methodology employed for data collection and data analysis in this study.

3.3 Confluence theories of creativity

The three most commonly cited "confluence" approaches to the study of creativity are the "systems approach" (Csikszentmihalyi, 1988, 2000); "the case study as evolving systems approach" (Gruber & Wallace, 2000), and finally the "investment theory approach" (Sternberg & Lubart, 1996).

3.3.1 The systems approach

The systems approach takes into account the social and cultural dimensions of creativity, instead of simply viewing creativity as an individualistic psychological process. The system approach studies the interaction between the individual, domain and field. The field consists of people who have influence over a domain. For example, editors of mathematics research journals would have influence on the domain of mathematics. The domain is in a sense a cultural organism that preserves and transmits creative products to other individuals in the field. The systems model suggests that creativity is a process that is observable at the "intersection where individuals, domains and fields interact" (Csikszentmihalyi, 2000). The three components, namely, individual, domain and field are necessary because the individual operates in a cultural or symbolic (domain) aspect as well as a social (field) aspect.

"The domain is a necessary component of creativity because it is impossible to introduce a variation without reference to an existing pattern. New is meaningful only in reference to the old" (Csikszentmihalyi, 2000). Thus creativity occurs when an individual makes a change in a given domain, and this change is transmitted through time. The personal background of an individual and their position in a domain naturally influence the likelihood of their contribution. For example, a mathematician working immersed in the culture of a research university is more likely to produce research papers because of the time available for "thinking" as well as being immersed in a culture where ideas flourish. It is no coincidence that in the history of science, there are significant contributions from clergymen such as Pascal, and Mendel, to name a few, because they had the means and the leisure to "think." Csikszentmihalyi (2000) then argues that novel ideas that result in significant changes are unlikely to be adopted unless they are sanctioned by a group of experts that decide what gets included in the domain. These "gatekeepers" (experts) constitute the field. For example, in mathematics, the opinion of a very small number of leading researchers was enough to certify the validity of Andrew Wiles' proof to Fermat's Last Theorem.

There are numerous examples within the field of mathematics that fall within the systems model. For instance the Bourbaki, a group of mostly French mathematicians who began meeting in the 1930 s, aimed to write a thorough unified account of all mathematics. The Bourbaki were essentially a group of expert mathematicians that tried to unify all of mathematics and become the gatekeepers of the field so to speak by setting the standard for rigor. Although the Bourbakists failed in their attempt, students of the Bourbakists, who are editors of certain prominent journals to this day demand a very high degree of rigor in submitted articles, thereby serving as gatekeepers of the field.

A different example is that of the role of proof. Proof is the social process through which the mathematical community validates the mathematician's creative work (Hanna, 1991). The Russian logician Manin (1977) said "A proof becomes a proof after the social act of accepting it as a proof. This is true of mathematics as it is of physics, linguistics, and biology."

In summary, the systems model of creativity suggests that for creativity to occur a set of rules and practices must be transmitted from the domain to the individual. The individual then must produce a novel variation in the content of the domain, and this variation must be selected by the field for inclusion in the domain.

3.3.2 Gruber and Wallace's case study as evolving systems approach

In contrast to Csikszentmihalyi's (2000) argument that calls for focus on communities in which creativity manifests, Gruber & Wallace (2000) propose a model that treats each individual as a unique evolving system of creativity and ideas, and therefore each individual's creative work must be studied on their own. This viewpoint of Gruber & Wallace (2000) is a belated victory of sorts for the Gestaltists, who essentially proclaimed the same thing almost a century ago. Gruber & Wallace (2000) use of terminology that jives with current trends in psychology seems to make their ideas more acceptable. They propose a model that calls for "detailed analytic and sometimes narrative descriptions of each case and efforts to understand each case as a unique functioning system (Gruber & Wallace, 2000, p. 93). It is important to note that the emphasis of this model is not to explain the origins of creativity, nor is it the personality of the creative individual, but on "how creative work works?" (p. 94). The questions of concern to Gruber & Wallace are: (1) What do creative people do when they are being creative? and (2) how do creative people deploy available resources to accomplish something unique? In this model creative work is defined as one that is novel and has value. This definition is consistent with that used by current researchers in creativity (Csikszentmihalyi, 2000; Sternberg & Lubart, 2000). Gruber & Wallace (2000) also claim that creative work is always the result of purposeful behavior and that creative work is usually a long undertaking "reckoned in months, years and decades" (p. 94). The author does not agree with the claim that creative work is always the result of purposeful behavior. One counterexample that comes to mind is the discovery of penicillin. The discovery of penicillin could be attributed purely to chance. On the other hand there are numerous examples that support the claim that creative work sometimes entails work that spans years. In mathematical folklore there are numerous examples of such creative work. For example, Kepler's laws of planetary motion were the result of 20 years of numerical calculations. Andrew Wiles' proof of Fermat's Last Theorem was a 7-year undertaking. The Riemann hypothesis states that the roots of the zeta function (complex numbers z, at which the zeta function equals 0) lie on the line parallel to the imaginary axis and half a unit to the right of it. This is perhaps the most outstanding unsolved problem in mathematics with numerous implications. The analyst Levinson undertook a determined calculation on his deathbed that increased the credibility of the Riemann-hypothesis. This is another example of creative work that falls within Gruber & Wallace (2000) model.

The case study as an evolving system has the following components to it. First, it views creative work as multifaceted. So, in constructing a case study of a creative work, one has to distill out the facets that are relevant and construct the case study based on the chosen facets. Some facets that can be used to construct an evolving system case study are: (1) Uniqueness of the work; (2) a narrative of what the creator achieved; (3) systems of belief; (4) multiple time-scales (construct the time-scales involved in the production of the creative work); (5) problem solving; and (6) contextual frame (family, schooling, teacher's influences) (Gruber & Wallace, 2000). So in summary, constructing a case study of a creative work as an evolving system entails incorporating the many facets suggested by Gruber & Wallace (2000). One could also evaluate a case study involving creative work by looking for the above mentioned facets.

3.3.3 The investment theory approach

According to the investment theory model, creative people are like good investors, that is, they buy low and sell high (Sternberg & Lubart, 1996). The context here is naturally in the realm of ideas. Creative people conjure up ideas that are either unpopular or disrespected, but invest considerable time convincing other people about the intrinsic worth of these ideas (Sternberg & Lubart, 1996). They sell high in the sense that they let other people pursue their ideas while they move on to the next idea. Investment theory claims that the convergence of six elements constitutes creativity. The six elements are intelligence, knowledge, thinking styles, personality, motivation, and environment. It is also important that the reader not mistake the word intelligence to an IQ score. On the contrary Sternberg (1985) suggests a triarchic theory of intelligence which consists of synthetic (ability to generate novel, task appropriate ideas), analytic and practical abilities. Knowledge is defined as knowing enough about a particular field to move it forward. Thinking styles are defined as a preference for thinking in original ways of one's choosing, the ability to think globally as well as locally and the ability to distinguish questions of importance from those that are not important. Personality attributes that foster creative functioning are the willingness to take risks, overcome obstacles and tolerate ambiguity. Finally motivation and an environment that is supportive and rewarding are also essential elements of creativity (Sternberg, 1985).

In investment theory creativity involves the interaction between a person, task, and environment. This is in a sense a particular case of the systems model (Csikszentmihalyi, 2000). The implication of viewing creativity as the interaction between person, task, and environment is that what is considered novel or original may vary from one person, task, and environment to another. The investment theory model suggests that creativity is more than a simple sum of the attained level of functioning in each of the six elements. Regardless of the functioning levels in other elements, a certain level or threshold of knowledge is required without which creativity is impossible. High levels of intelligence and motivation can positively enhance creativity, and compensations can occur to counteract weaknesses in other elements. For example, one could be in an environment that is non-supportive of creative efforts, but a high level of motivation would possibly overcome this and pursue creative endeavors.

This concludes the review of three commonly cited prototypical confluence theories of creativity, namely the systems approach (Csikszentmihalyi, 2000), which suggests that creativity is a sociocultural process involving the interaction between the individual, domain, and field; Gruber & Wallace (2000) model that treats each individual case study as a unique evolving system of creativity; and investment theory (Sternberg & Lubart, 1996), which suggests that creativity is the result of the convergence of six elements (intelligence, knowledge, thinking styles, personality, motivation, and environment).

Having reviewed the research literature on creativity, the focus is shifted on the methodology employed for studying mathematical creativity.

4 Methodology

4.1 The interview instrument

The purpose of this study was to gain an insight into the nature of mathematical creativity. The author was interested in distilling common attributes from how mathematicians created mathematics, in order to determine some of the characteristics of the creative process. The author was also interested in testing the applicability of the Gestalt model. The primary method of data collection was through personal interviews. Since the main focus of the study was to ascertain qualitative aspects of creativity, a formal interview methodology was selected. The interview instrument (Appendix) was developed by modifying questions from questionnaires in *L'Enseigement Mathematique* (1902) and Muir (1988). The purpose of using this modified questionnaire was as follows. The questions were general in nature, which allowed the mathematicians to express themselves freely. Secondly the author wanted to somehow test the applicability of the four stage Gestalt model of creativity. Therefore the existing instruments were modified in order to operationalize the Gestalt theory and to allow the natural flow of ideas to form the basis of a thesis that would emerge from this exploration.

4.2 Background of the subjects

Five mathematicians from the mathematical sciences faculty at a large Ph.D. granting mid-western university were selected. These mathematicians were chosen based on their accomplishments and the diversity of the mathematical areas they worked in. This was measured by counting the number of published papers in prominent research journals, as well as the diversity of the mathematical domains in which the mathematicians conducted research. Four of the mathematicians were tenured full professors, who had been professional mathematicians for over 30 years. Only one of the mathematicians was considerably younger, and a tenured associate professor. All the interviews were conducted formally in a closed door setting, in the mathematicians' office. The interviews were taped and transcribed verbatim.

4.3 Data analysis

Since creativity is an extremely complex construct involving a wide range of interacting behaviors, in the author's opinion it should be studied holistically. The principle of analytic induction (Patton, 2002) was applied to the interview transcripts to discover dominant themes, which described the behavior under study. According to Patton (2002) "analytic induction, in contrast to grounded theory, begins with an analyst's deduced propositions or theory-derived hypotheses and 'is a procedure for verifying theories and propositions based on qualitative data' (Taylor and Bogdan, 1984, p. 127)." Following the principles of analytic induction, the data was carefully analyzed in order to extract common strands. These common strands were then compared to theoretical constructs in the existing literature with the explicit purposes of verifying whether the Gestalt model was applicable to this qualitative data, as well as to extract themes that characterized the mathematician's creative process. If an emerging theme could not be classified or named because of being unable to grasp its properties or significance, then theoretical comparisons were made. Corbin and Strauss (1998) state that "using comparisons brings out properties, which in turn can be used to examine the incident or object in the data. The specific incidents, objects, or actions that we use when making theoretical comparisons can be derived from the literature and experience. It is not that we use experience or literature as data "*but rather that we use the properties and dimensions derived from the comparative incidents to examine the data in front of us*." (p. 80). Themes that emerged were social interaction, preparation, use of heuristics, imagery, incubation, illumination, verification, intuition, and proof.

Vignettes that highlight these characteristics are reconstructed in the next section along with commentaries that incorporate the wider conversation, and a continuous discussion of connections to the existing literature.

5 Results, commentaries and discussion

All the mathematicians in this study were tenured professors at a large Ph.D. granting mathematical sciences faculty. Their place of work could be described as academic, with teaching and committee duties. The mathematicians were free to choose their areas of research and the problems that they worked. Four of the five mathematicians had worked and published as individuals with occasional joint ventures. Only one of the mathematicians had done extensive collaborative work. All but one of the mathematicians did not structure their time for research mathematics. The main reasons cited were family commitments and teaching responsibilities during the regular school year. All the mathematicians found it easier to concentrate on research in the summers because of lighter or non-existent teaching responsibilities. Two of the mathematicians showed a pre-disposition toward mathematics at the early secondary school level. The others became interested in mathematics only at a later stage in their university education. None of the mathematicians who participated in this study had any immediate family influence that was of primary importance in their mathematical development. Four of the mathematicians recalled being influenced by particular teachers at various stages of their education. One of the mathematicians was influenced by a textbook. The three mathematicians who worked primarily in analysis strived consciously to obtain a broad overview of mathematics not of immediate relevance to their main interests. The two algebraists expressed interest in other areas of mathematics but were active mainly in their chosen fields.

5.1 Supervision of research and social interaction

As reported earlier, all the mathematicians in this study were tenured professors in a research university. Besides teaching, research, and committee obligations, many mathematicians play a big role in mentoring graduate students interested in their area of research. Research supervision is an aspect of creativity because any interaction between human beings is the ideal setting for exchange of ideas. During this interaction the mathematician is exposed to different perspectives on the subject. All the mathematicians in this study valued the interaction they had with their graduate students. Vignettes of individual responses follow.[1]

5.1.1 Vignette 1

A: I've had only one graduate student per se and she is just finishing up her Ph.D right now and I'd say it has been a very good interaction to see somebody else get interested in the subject and come up with new ideas, and exploring those ideas with her.

B: I have had a couple of students who have sort of started but who haven't continued on to a Ph.D, so I really can't speak to that. But the interaction was positive.

C: Of course, I have a lot of collaborators, these are my former students you know…I am always all the time working with students, this is normal situation.

D: That is difficult to answer (silence)…it is positive because it is good to interact with other people. It is negative because it can take a lot of time. As you get older your brain doesn't work as well as it used to and…younger people by and large their minds are more open, there is less garbage in there already. So, it is exciting to work with younger people who are in their most creative time. When you are older, you have more experience, when you are younger your mind works faster …not as fettered.

E: Oh…it is a positive factor I think, because it continues to stimulate ideas …talking about things and it also reviews things for you in the process, puts things in perspective, and keep the big picture. It is helpful really in your own research to supervise students.

5.1.2 Commentary on vignette 1

The responses of the mathematicians in the preceding vignette are focused on research supervision. However all the mathematicians acknowledged the role of social interaction in general as an important aspect that stimulated creative work. Many of the mathematicians mentioned the advantages of being able to e-mail colleagues, going to research conferences and other professional meetings. This is revealed more in the following section under preparation.

5.2 Preparation and the use of heuristics

When mathematicians are about to investigate a new topic, there is usually a body of existing research in the area of the new topic. One of goals of the study was to find out how creative mathematicians approached a new topic or a problem. Did they try their own approach, or did they first

[1] In all vignettes I = interviewer; A, B, C, D, E = mathematicians.

attempt to assimilate what was already known about that topic? Did the mathematicians make use of computers to gain insight into the problem? What were the various modes of approaching a new topic or problem? The responses indicate that a variety of approaches were used.

5.2.1 Vignette 2

A: Talk to people who have been doing this topic. Learn the types of questions that come up. Then I do basic research on the main ideas. I find that talking to people helps a lot more than reading because you get more of a feel for what the motivation is beneath everything.

B: What might happen for me, is that I may start reading something, and, if feel I can do a better job, then I would strike off on my own. But for the most part I would like to not have to re-invent a lot that is already there. So, a lot of what has motivated my research has been the desire to understand an area. So, if somebody has already laid the groundwork then it's helpful. Still I think a large part of doing research is to read the work that other people have done.

C: It is connected with one thing that simply…my style was that I worked very much and I even work when I could not work. Simply the problems that I solve attract me so much, that the question was who will die first…mathematics or me? It was never clear who would die.

D: Try and find out what is known. I won't say assimilate…try and find out what's known and get an overview, and try and let the problem speak…– mostly by reading because you don't have that much immediate contact with other people in the field. But I find that I get more from listening to talks that other people are giving than reading.

E: Well! I have been taught to be a good scholar. A good scholar attempts to find out what is first known about something or other before they spend their time simply going it on their own. That doesn't mean that I don't simultaneously try to work on something.

5.2.2 Commentary on vignette 2

These responses indicate that the mathematician spends a considerable amount of time researching the context of the problem. This is primarily done by reading the existing literature and by talking to other mathematicians in the new area. This finding is consistent with the systems model, which suggests that creativity is a dynamic process involving the interaction between the individual, domain and field (Csikszentmihalyi, 2000).

At this stage it is reasonable to ask whether a mathematician works on a single problem until a breakthrough occurs or do mathematicians work on several problems concurrently? It was found that each of the mathematicians worked on several problems concurrently, using a back and forth approach.

5.2.3 Vignette 3

A: I work on several different problems for a protracted period of time… there have been times when I have felt, yes, I should be able to prove this result, then I would concentrate on that thing for a while but they tend to be several different things that I thinking about a particular stage.

B: I probably tend to work on several problems at the same time. There are several different questions that I am working on…mm…probably the real question is how often do you change the focus? Do I work on two different problems on the same day? And that is probably up to whatever comes to mind in that particular time frame. I might start working on one rather than the other. But I would tend to focus on one particular problem for a period of weeks, then you switch to something else. Probably what happens is that I work on something and I reach a dead end then I may shift gears and work on a different problem for a while, reach a dead end there and come back to the original problem, so its back and forth.

C: I must simply think on one thing and not switch so much.

D: I find that I probably work on one. There might be a couple of things floating around but I am working on one and if I am not getting anywhere, then I might work on the other and then go back.

E: I usually have couple of things going. When I get stale on one, then I will pick up the other, and bounce back and forth. Usually I have one that is primarily my focus at a given time and I will spend time on it over another, but it is not uncommon for me to have a couple of problems going at a given time. Sometimes when I am looking for an example that is not coming, instead of spending my time beating my head against the wall, looking for that example is not a very good use of time. Working of another helps to generate ideas that I can bring back to the other problem.

5.2.4 Commentary on vignette 3

The preceding vignette indicates that mathematicians tend to work on more than one problem at a given time. Do mathematicians switch back and forth between problems in a completely random manner, or do they employ and exhaust a systematic train of thought about a problem before switching to a different problem? Many of the mathematicians reported using heuristic reasoning, trying to prove something 1 day and disprove it the next day, looking for both examples and counterexamples, the use of "manipulations" (Polya, 1954) to gain an insight into the problem. This indicates that mathematicians do employ some of the heuristics made explicit by Polya. It was unclear whether the mathematicians made use of computers to gain an experimental or computational insight into the problem. The author was also interested in knowing the types of imagery used by mathematicians in their work. The mathematicians in this study were queried about this and the following vignette gives us an insight into this aspect of mathematical creativity.

5.3 Imagery

The mathematicians in this study were asked about the kinds of imagery that they used to think about mathematical objects. This is reported here with the hope that the reader gets a glimpse into how mathematicians think of mathematical objects. The responses also point out the difficulty of explicitly describing imagery.

5.3.1 Vignette 4

A: Yes I do, yes I do, I tend to draw a lot of pictures when I am doing research, I tend to manipulate things in the air, you know to try to figure out how things work. I have a very geometrically based intuition and uhh…so very definitely I do a lot of manipulations.

B: That is a problem because of the particular area I am in. I can't draw any diagrams, things are infinite, so I would love to be able to get some kind of a computer diagram to show the complexity for a particular ring…to have something like the Julia sets or…mmm…fractal images, things which are infinite but you can focus in closer and closer to see possible relationships. I have thought about that with possibilities on the computer. To think about the most basic ring, you would have to think of the ring of integers and all of the relationships for divisibility, so how do you somehow describe this tree of divisibility for integers…it is infinite.

C: Science is language, *you think through language*. But it is language simply; you put together theorems by logic. You first see the theorem in nature…you must see that somewhat is reasonable and then you go and begin and then of course there is big, big, big work to just come to some theorem in non-linear elliptic equations…

D: A lot of mathematics, whether we are teaching or doing, is attaching meaning to what we are doing and this is going back to the earlier question when you talked about how do you do it, what kind of heuristics do you use? What kind of images do you have that you are using. A lot of doing mathematics is creating these abstract images that connect things and then making sense of them but that doesn't appear in proofs either.

E: Pictorial, linguistic, kinesthetic…any of them is the point right! Sometimes you think of one, sometimes another. It really depends on the problem you are looking at, they are very much…often I think of functions as very kinesthetic, moving things from here to there. Other approaches you are talking about is going to vary from problem to problem, or even day to day. Sometimes when I am working on research, I try to view things in a s many different ways as possible, to see what is really happening. So there are a variety of approaches.

5.3.2 Commentary on vignette 4

Besides revealing the difficulty of describing mental imagery, all the mathematicians reported that they did not use computers in their work. This characteristic of the pure mathematician's work is echoed in Poincaré's (1948) use of the "*choice*" metaphor and Ervynck's (1991) use of the term "*non-algorithmic decision making*." The doubts expressed by the mathematicians about the incapability of machines to do their work brings to mind the reported words of Garrett Birkhoff, one of the great, applied mathematicians of our time. In his retiring presidential address to the SIAM, Birkhoff (1969) addressed the role of machines in human creative endeavors. In particular part of this address was devoted to discussing the psychology of the mathematicians (and hence of mathematics). Birkhoff said:

The remarkable recent achievements of computers have partially fulfilled an old dream. These achievements have led some people to speculate that tomorrow's computers will be even more "intelligent" than humans, especially in their powers of mathematical reasoning…the ability of good mathematicians to sense the significant and to avoid undue repetition seems however hard to computerize; without it, the computer has to pursue millions of fruitless paths avoided by experienced human mathematicians. (Birkhoff, 1969, pp. 430–438).

5.4 Incubation and illumination

Having reported on the role of research supervision and social interaction, the use of heuristics and imagery, which can be viewed as aspects of the preparatory stage of mathematical creativity, it is natural to ask what occurs next. As the literature theorizes, after the mathematician works hard to gain insight into a problem, there is usually a transition period (conscious work on the problem ceases and unconscious work begins), where the problem is put aside before the breakthrough occurs. The mathematicians in this study reported experiences, which are consistent with the existing literature (Hadamard, 1945; Poincaré, 1948).

5.4.1 Vignette 5

B: One of the problems is first one does some preparatory work, that has to be the left side, and then you let it sit. I don't think you get ideas out of nowhere, you have to do the groundwork first, okay. This is why people will say, now we have worked on this problem, so let us sleep on it. So you do the preparation, so that the subconscious or intuitive side may work on it and the answer comes back but you can't really tell when. You have to be open to this, lay the groundwork, think about it and then these flashes of intuition come and they represent the other side of the brain communicating with you at whatever odd time.

D: I am not sure you can really separate them because they are somewhat connected. You spend a lot of time working on something and you are not getting anywhere with it…with the deliberate effort, then I think your mind continues to work and organize. And maybe when the pressure is off the idea comes…but the idea comes because of the hard work.

E: Usually they come after I have worked very hard on something or another, but they may come at an odd moment. They may come into my head before I go bed …What do I do at that point? Yes I write it down (laughing). Sometimes when I am walking somewhere, the mind flows back to it (the problem) and says what about that, why don't you try that. That sort of thing happens. One of the best ideas I had was when I was working on my thesis …Saturday night, having worked on it quite a bit, sitting back and saying why don't I think about it again…and ping! There it was…I knew what it was, I could do that. Often ideas are handed to you from the outside, but they don't come until you have worked on it long enough.

5.4.2 Commentary on vignette 5

As is evident in the preceding vignette, three out of the five mathematicians reported experiences consistent with the Gestalt model. Mathematician C attributed his breakthroughs on problems to his unflinching will of never giving up and to divine inspiration, echoing the voice of Pascal in a sense. However Mathematician A attributed breakthroughs to chance. In other words making the appropriate (psychological) connections by pure chance which eventually result in the sought after result. The author thinks that it is necessary to comment about the unusual view of mathematician A. Chance plays an important role in mathematical creativity. Great ideas and insights may be the result of chance such as the discovery of penicillin. Ulam (1976) estimated that there is a yearly output of 200,000 theorems in mathematics. Chance plays a role in what is considered important in mathematical research since only a handful of results and techniques survive out of the volumes of published research. The author wishes to draw a distinction between chance in the "Darwinian" sense (as to what survives), and chance in the psychological sense (which results in discovery/invention). The role of chance is addressed by Muir (1988) as follows.

The act of creation of new entities has two aspects: the generation of new possibilities, for which we might attempt a stochastic description, and the selection of what is valuable from among them. However the importation of biological metaphors to explain cultural evolution is dubious…both creation and selection are acts of design within a social context. (Muir, 1988 p. 33).

Thus, Muir (1988) rejects the Darwinian explanation. On the other hand, Nicolle (1932) in *Biologie de L'Invention* does not acknowledge the role of unconsciously present prior work in the creative process. He attributes breakthroughs to pure chance. "By a streak of lightning, the hitherto obscure problem, which no ordinary feeble lamp would have revealed, is at once flooded in light. It is like a creation. Contrary to progressive acquirements, such an act owes nothing to logic or to reason. The act of discovery is an accident." (Hadamard, 1945, p. 19)

Nicolle's Darwinian explanation was rejected by Hadamard on the grounds that to explain creation occurs by pure chance is equivalent to asserting that there are effects without causes. Hadamard further argued that although Poincaré attributed his particular breakthrough in Fuchsian functions to chance, Poincaré did acknowledge that there was a considerable amount of previous conscious effort, followed by a period of unconscious work. Hadamard (1945) further argued that even if Poincaré's breakthrough was a result of chance alone, chance alone was insufficient to explain the considerable body of creative work credited to Poincaré in almost every area of mathematics. The question then is how does (psychological) chance work?

The author conjectures that the mind throws out fragments (ideas) which are products of past experience.

Some of these random fragments can be juxtaposed and combined in a meaningful way. For example, if one reads a complicated proof consisting of a thousand steps, a thousand random fragments may not be enough to construct a meaningful proof. However the mind chooses relevant fragments from these random fragments and links them into something meaningful. Wedderburn's theorem, that a finite division ring is a field is one instance of a unification of apparently random fragments because the proof involves algebra, complex analysis and number theory.

Polya (1954) addresses the role of chance in a probabilistic sense. It often occurs in mathematics, that a series of mathematical trials (involving computation) throw up numbers which are close to a Platonic ideal. The classic example is Euler's investigation of the infinite series $1 + 1/4 + 1/9 + 1/16 + \cdots + 1/n^2 + \cdots$. Euler obtained an approximate numerical value for the sum of the series using various transformations of the series. The numerical approximation was 1.644934. Euler confidently guessed the sum of the series to be $\pi^2/6$. Although the numerical value obtained by Euler and the value of $\pi^2/6$ coincided up to seven decimal places, such a coincidence could be attributed to chance. However, a simple calculation shows that the probability of seven digits coinciding is one in ten million! Hence Euler did not attribute this coincidence to chance and boldly conjectured that the sum of this series was indeed $\pi^2/6$, not to mention the fact that he later proved his conjecture to be true (Polya, 1954, pp. 95–96).

5.5 Intuition, verification and proof

Once illumination has occurred whether or not through sheer chance or through incubation or through divine intervention, mathematicians usually try to verify that their intuitions were correct, and try to construct a proof. This section of the paper discusses how mathematicians go about verifying their intuitions, and the role of formal proof in the creative process. The mathematicians in this study were asked to describe how they went about forming an intuition about the truth of a proposition. They were asked whether they relied on repeatedly checking a formal proof, or if they used multiple converging partial proofs, or if they first looked for coherence with other results in the area, or if they looked at applications. Most of the mathematicians in this study mentioned that the last thing they looked at was a formal proof. This is consistent with the literature on the role of formal proof in mathematics (Polya, 1954; Usiskin, 1987). Most of the mathematicians mentioned the need for coherence with other results in the area. The mathematician's responses to this question follow.

5.5.1 Vignette 6

B: I think I would go for repeated checking of the formal proof…but I don't think that that is really enough. All of the others have to also be taken into account. I mean, you can believe that something is true although you may not fully understand it. This is the point that was made in the lecture by ___of ___ University on Dirichlet series. He was saying that we have had a formal proof for some time, but that is not to say that it is really understood, and what did he mean by that? Not that the proof wasn't understood, but it was the implications of the result that are not understood, their connections with other results, applications and why things really work? But probably the first thing that I would really want to do is check the formal proof to my satisfaction, so that I believe that it is correct although at that point I really do not understand its implications… it is safe to say that it is my surest guide.

C: First you must see it in the nature, something, first you must see that this theorem corresponds to something in nature, then if you have this impression, it is something relatively reasonable, then you go to proofs…and of course I have also several theorems and proofs that are wrong, but the major amount of proofs and theorems are right.

D: The last thing that comes is the formal proof. I look for analogies with other things… How your results that you think might be true would illuminate other things and would fit in the general structure.

E: Since I work in an area of basic research, it is usually coherence with other things, that is probably more than anything else. Yes, one could go back and check the proof and that sort of thing but usually the applications are yet to come, they aren't there already. Usually what guides the choice of the problem is the potential for application, part of what represents good problems is their potential for use. So, you certainly look to see if it makes sense in the big picture…that is a coherence phenomenon. Among those you've given me, that's probably the most that fits.

5.5.2 Commentary on vignette 6

This vignette indicates that for mathematicians, valid proofs have varied degrees of rigor. "Among mathematicians, rigor varies depending on time and circumstance, and few proofs in mathematics journals meet the criteria used by secondary school geometry teachers (each statement of proof is backed by reasons). Generally one increases rigor only when the result does not seem to be correct" (Usiskin, 1987). Proofs are in most cases the final step in this testing process. "Mathematics in the making resembles any other human knowledge in the making. The result of the mathematician's creative work is demonstrative reasoning, a proof; but the proof is discovered by plausible reasoning, by guessing" (Polya, 1954). How mathematicians approached proof in this study was very different from the logical approach found in proof in most textbooks. The logical approach is an artificial reconstruction of discoveries that are being forced into a deductive system and in this process the intuition that guided the discovery process gets lost.

6 Conclusions and implications

The goal of this study was to gain an insight into mathematical creativity. As suggested by the literature review, the existing literature on mathematical creativity is relatively sparse. In trying to better understand the process of creativity, the author finds that the Gestalt model proposed by Hadamard (1945) is still applicable today. This study has attempted to add some detail to the *preparation–incubation–illumination–verification model* of Hadamard (1945), by taking into account the role of imagery, the role of intuition, the role of social interaction, the use of heuristics, and the necessity of proof in the creative process. The mathematicians worked in a setting that was conducive to prolonged research. There was a convergence of intelligence, knowledge, thinking styles, personality, motivation, and environment that enabled them to work creatively (Sternberg, 2000; Sternberg & Lubart, 1996, 2000). The preparatory stage of mathematical creativity consisted of various approaches used by the mathematician to lay the groundwork. These are, reading the existing literature, talking to other mathematicians in the particular mathematical domain (Csikszentmihalyi, 1988, 2000), trying a variety of heuristics (Polya, 1954), using a back-and-forth approach of plausible guessing. One of the mathematicians said that he first looked to see if the sought after relationships corresponded to natural phenomenon.

All of the mathematicians in this study worked on more than one problem at a given moment. This is consistent within the investment theory view of creativity (Sternberg & Lubart, 1996). The mathematicians invested an optimal amount of time on a given problem, but switched to a different problem if no breakthrough was forthcoming. All the mathematicians in this study considered this as the most important and difficult stage of creativity. The prolonged hard work was followed by a period of incubation where the problem was put aside (and the preparatory stage is repeated for a different problem). Thus, there was a transition in the mind from conscious to unconscious work on the problem. One mathematician said that this is the stage where the "problem begins to talk to you". Another mathematician said that at this stage the intuitive side of the brain began communicating with the logical side, and conjectured that this communication was not possible at a conscious level.

The transition from incubation to illumination occurred when least expected. Many reported the breakthrough occurring as they were going to bed, or walking, or sometimes a result of speaking to someone else about the problem. One mathematician described this as follows. "You talk to somebody and they say just something that might have been very ordinary a month before but if they say it when you are ready for it, and Oh yeah, I can do it that way can't I! But you have to be ready for it. Opportunity knocks but you have to be able to answer the door."

Illumination follows with the mathematician verifying the occurred idea by constructing a proof. In this study, most of the mathematicians looked for coherence of the result with other existing results in the area of research. If the result cohered with other results, and fit the general structure of the area, only then did the mathematician try to construct a formal proof. In terms of the mathematician's beliefs about the nature of mathematics and its influence on their research the study revealed that four of the mathematicians leaned toward platonism, running contrary to the popular notion that platonism is an exception today. A detailed discussion of this aspect of the research is beyond the scope of this paper. However it was found that beliefs regarding the nature of mathematics influenced how these mathematicians' conducted research and were deeply connected to their theological beliefs (Sriraman, 2004).

The mathematicians hoped that the results of their creative work would be sanctioned by a group of experts in order to get the work included in the domain (Csikszentmihalyi, 1988, 2000) primarily in the form of a publication in a prominent journal. However, the acceptance of a mathematical result, the end product of creation, does not ensure its survival in the Darwinian sense (Muir, 1988). The mathematical result, may or may not be picked up by other mathematicians. If the mathematical community picks it up as a viable result, then it is likely to undergo mutations and lead to new mathematics. This however is determined by chance!

6.1 Implications

It is in the best interest of the field of mathematics education that we identify and nurture creative talent in the mathematics classroom. "Between the work of a student who tries to solve a difficult problem in mathematics and a work of invention (creation)…there is only a difference of degree" (Polya, 1954). Creativity as a feature of mathematical thinking is not a patent of the mathematician! (Krutetskii, 1976). Most studies on creativity have focussed on eminent individuals (Arnheim, 1962; Gardner, 1993, 1997; Gruber, 1981). The author is suggesting that contemporary models from creativity research can be adapted for studying non-eminent samples such as high school students. Such studies would reveal more to the mathematics education research community about creativity in the classroom. Educators can ask themselves (1) Does mathematical creativity manifest in the school classroom? (2) How can the teacher identify creative work? One plausible way to answer these questions is by reconstructing and evaluating student work as a unique evolving system of creativity (Gruber & Wallace, 2000) by incorporating some of the facets suggested by Gruber & Wallace (2000). This necessitates the need to find suitable problems for the appropriate levels that stimulate student creativity. A common trait among mathematicians is to rely on particular cases, isomorphic re-formulations, or analogous problems that simulate the original problem situations in their search for a solution (Polya, 1954; Skemp, 1986). It is also the case that creating original mathematics requires a very high level of motivation, persistence and reflection, all of which are considered indicators of creativity (Amabile, 1983; Policastro & Gardner, 2000; Gardner, 1993). The literature suggests that most creative individuals tend to be attracted to complexity, which most school math curricula has very little to offer. Classroom practices and math curricula rarely use problems with an underlying mathematical structure and allow students a prolonged period of engagement and independence to work on such problems. The author conjectures that in order for mathematical creativity to manifest in the school classroom, students should be given the opportunity to tackle non-routine problems with complexity and structure, which require not only motivation and persistence but also considerable reflection. This implies that educators should recognize the value of allowing students to reflect on previously solved problems and draw comparisons between various isomorphic problems (English, 1991, 1993; Hung, 2000; Maher & Kiczek, 2000; Maher & Martino, 1996; Maher & Speiser, 1997; Sriraman, 2003, Sriraman 2004b). In addition, encouraging students to look for similarities in a class of problems also fosters "mathematical" behavior (Polya, 1954), leading some students to discover fairly sophisticated mathematical structures and principles in a manner akin to creative mathematicians.

Appendix: Interview protocol

(The interview instrument was developed by modifying questions from questionnaires in *L'Enseigement Mathematique* (1902) and Muir (1988))

1. Describe your place of work and your role within it?
2. Are you free to choose the mathematical problems you tackle or are they determined by your work place?
3. Do you work and publish mainly as an individual or as part of a group?
4. Is supervision of Research a positive or negative factor in your work?
5. Do you structure your time for mathematics?
6. What are your favorite leisure activities apart from mathematics?
7. Do you recall any immediate family influences, teachers, colleagues or texts, of primary importance in your mathematical development?
8. In which areas were you initially self-educated? In which areas do you work now? If different, what have been the reasons for changing?
9. Do you strive to obtain a broad overview of mathematics, not of immediate relevance to your area of research?
10. Do you make a distinction between thought processes in learning and research?
11. When you are about to begin a new topic, do you prefer to assimilate what is known first or do you try your own approach?
12. Do you concentrate on one problem for a protracted period of time or on several problems at the same time?
13. Have your best ideas been the result of prolonged deliberate effort, or have they occurred when you were engaged in other unrelated tasks?
14. How do you form an intuition about the truth of a proposition?
15. Do computers play a role in your creative work (mathematical thinking)?
16. What types of mental imagery do you use when thinking about mathematical objects?

Questions regarding foundational and theological issues have been omitted in this protocol. The discussion resulting from these questions are reported in Sriraman, B. (2004). The influence of Platonism on mathematics research and theological beliefs. Theology and Science, 2 (1), 131–147.

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