PME-NA- Atlanta Working Group

Research Advances in Theories of Mathematics Education

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Purpose of Working Group

This working group revolves around the launch of a new book series entitled *Advances in Mathematics Education* by Springer Science, Heidelberg, and in particular on the first book in the series which focuses on Theories of Mathematics Education. This edited book in turn is based on a research forum on Theories of Mathematics Education at PME 29 in Melbourne, 2005, which resulted in two ZDM special issues on theories of mathematics education(issue 6/2005 and issue 1/2006). Since the research forum in Melbourne, numerous advances have taken place in the area of theory development in mathematics education in Europe and in North America. The purpose of this working group on research advances in theories of mathematics education is to integrate, synthesize and present a coherent picture on the state of the art. The working group will attempt to be both summative as well as forward looking by highlighting theories from psychology, philosophy and social sciences that continue to influence theory building, as well as provide participants insights into new developments in feminist, critical and political theories of mathematics education.

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Theory and its role in mathematics education

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Theories are like toothbrushes…everyone has their own and no one wants to use anyone else's."(Campbell,2006)

Abstract: The increased recognition of the theory in mathematics education is evident in numerous handbooks, journal articles, and other publications. For example, Silver and Herbst (2007) examined "Theory in Mathematics Education Scholarship" in the *Second Handbook of Research on Mathematics Teaching and Learning* (Lester, 2007) while Cobb (2007) addressed "Putting Philosophy to Work: Coping with Multiple Theoretical Perspectives" in the same handbook. And a central component of both the first and second editions of the *Handbook of International Research in Mathematics Education* (English, 2002; 2008) was "advances in theory development." Needless to say, the comprehensive second edition of the *Handbook of Educational Psychology* (Alexander & Winne, 2006) abounds with analyses of theoretical developments across a variety of disciplines and contexts. Numerous definitions of "theory" appear in the literature (e.g., see Silver & Herbst, in Lester, 2007). It is not our intention to provide a "one-size-fits-all" definition of theory per se as applied to our discipline; rather we consider multiple perspectives on theory and its many roles in improving the teaching and learning of mathematics in varied contexts.

At the 2008 *International Congress on Mathematical Education*, Assude, Boero, Herbst, Lerman, and Radford (2008) referred to theory in mathematics education research as dealing with the teaching and learning of mathematics from two perspectives: a *structural* and a *functional* perspective. From a structural point of view, theory is "an organized and coherent system of concepts and notions in the mathematics education field." The "functional" perspective considers theory as "a system of tools that permit a 'speculation' about some reality." When theory is used as a *tool*, it can serve to: (a) conceive of ways to improve the teaching/learning environment including the curriculum, (b) develop methodology, (c) describe, interpret, explain, and justify classroom observations of student and teacher activity, (d) transform practical problems into research problems, (e) define different steps in the study of a research problem, and (f) generate knowledge. When theory functions as an *object*, one of its goals can be the advancement of theory itself. This can include testing a theory or some ideas or relations in the theory (e.g., in another context or) as a means to produce new theoretical developments.

Silver and Herbst (2007) identified similar roles but proposed the notion of theory as a mediator between problems, practices, and research. For example, as a mediator between research and problems, theory is involved in, among others, generating a researchable problem, interpreting the results, analysing the data, and producing and explaining the research findings. As a mediator between research and practice, theory can provide a norm against which to evaluate classroom practices as well as serve as a tool for research to understand (describe and explain) these practices. Theory that mediates connections between practice and problems can enable the identification of practices that pose problems, facilitate the development of researchable problems, help propose a solution to these problems, and provide critique on solutions proposed by others. Such theory can also play an important role in the development of new practices, such as technology enhanced learning environments.

What we need to do now is explore more ways to effectively harmonize theory, research, and practice (Silver & Herbst, 2007) in a coherent manner so as to push the field forward. This leads to an examination of the extant theoretical paradigms and changes that have occurred over the last two decades. This was briefly discussed at the outset of this chapter.

Changes in Theoretical Paradigms

As several scholars have noted over the years, we have a history of shifting frequently our dominant paradigms (Berliner, Calfee, in Alexander & Winne, 2006). Like the broad field of psychology, our discipline "can be perceived through a veil of 'isms'" (Alexander & Winne, 2006, p.982). We have witnessed, among others, shifts from behaviourism, through to stage and level theories, to various forms of constructivism, to situated and distributed cognitions, and more recently, to complexity theories and neuroscience. For the first couple of decades of its life, mathematics education as a discipline drew heavily on theories and methodologies from psychology. According to Lerman (2000), the switch to research on the social dimensions of mathematical learning towards the end of the 1980s resulted in theories that emphasized a view of mathematics as a social product. Social constructivism, which draws on the seminal work of Vygotsky and Wittgenstein (Ernest, 1994) has been a dominant research paradigm for many years. Evidence of the social turn can be found in Lerman's analysis of articles published from 1990 to 2001 in *Educational Studies in Mathematics* (ESM), *Journal for Research in Mathematics Education* (JRME), and the *Proceedings of the International Group for the Psychology of Mathematics Education* (PME), revealed that, while the predominant theories used during this period were traditional psychological and mathematics theories, an expanding range from other fields was evident especially in PME and ESM. Psycho-social theories, including re-emerging ones, increased in ESM and JRME. Likewise, papers drawing on sociological and socio-cultural theories also increased in all three publications together with more papers utilizing linguistics, social linguistics, and semiotics. Lerman's analysis revealed very few papers capitalizing on broader fields of educational theory and research and on neighboring disciplines such as science education and general curriculum studies. This situation appears to be changing in recent years, with interdisciplinary studies emerging in the literature (e.g., see English, 2008); and papers that address the nascent field of neuroscience in mathematics education

Numerous scholars have questioned the reasons behind these paradigm shifts. Is it just the power of fads? Does it only occur in the United States? Is it primarily academic competitiveness (new ideas as more publishable)? One plausible explanation is the diverging, epistemological perspectives about what constitutes mathematical knowledge. Another possible explanation is that mathematics education, unlike "pure" disciplines in the sciences, is heavily influenced by unpredictable cultural, social, and political forces (e.g., Sriraman, 2007). A critical question, however, that has been posed by scholars now and in previous decades is whether our paradigm shifts are genuine. That is, are we replacing one particular theoretical perspective with another that is more valid or more sophisticated for addressing the hard core issues we confront (Kuhn, 1966; Alexander & Winne, 2006)? Or, as Alexander and Winne ask, is it more the case that theoretical perspectives move in and out of favour as they go through various transformations and updates? If so, is it the voice that speaks the loudest that gets heard? Who gets suppressed? The rise of constructivism in its various forms is an example of a paradigm that appeared to drown out many other theoretical voices during the 1990s (Goldin, 2003). In essence, the question we need to consider is whether we are advancing professionally in our theory development.

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Appreciating *Scientificity* in Qualitative Research

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Abstract: This paper is situated within an educational paradigm that is concerned with the education of itself, its peers and its students. From here, we acknowledge the necessity for knowledge and that *in learning* we discover knowledge either through ourselves, through our peers or through synthesizing a dialectic between the governing bodies of knowledge and an educational system. We might understand that we discover knowledge in an educational setting by processes that are akin to scientific discovery. I propose that we establish knowledge in this very way and in reflecting on our *constructing-knowledge* enterprise, we endeavor to adhere to a *meta-constructionist phenomenology*, which draws upon the learning theory of constructionism (Papert & Harel, 1991) whereby we establish a construction built on a faithful establishment of education and assess the mechanics of the constructed phenomenon through reflexivity and interactivity with the field.

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Mathematics Education as a design science

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Abstract: We propose re-conceptualizing the field of mathematics education research as that of a design science akin to engineering and other emerging interdisciplinary fields which involve the interaction of "subjects", conceptual systems and technology influenced by social constraints and affordances. Numerous examples from the history and philosophy of science and mathematics and ongoing findings of M&M research are drawn to illustrate our notion of mathematics education research as a design science. Our ideas are intended as a framework and do not constitute a, "grand" theory (see Lester. 2005, this issue). That is, we provide a framework (a system of thinking together with accompanying concepts, language, methodologies, tools, and so on) that provides structure to help mathematics education researchers develop both models and theories, which encourage diversity and emphasize Darwinian processes such as: (a) selection (rigorous testing), (b) communication (so that productive ways of thinking spread throughout relevant communities), and (c) accumulation (so that productive ways of thinking are not lost and get integrated into future developments)

Teaching Mathematics through Problem Solving:What We Know and Where We Are Going

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Abstract: Problem solving has a long history in school mathematics. In the past several decades, there have been significant advances in the understanding of the complex processes involved in problem solving. There also has been considerable discussion about teaching mathematics with a focus on problem solving. However, teaching mathematics *through* problem solving is a relatively new idea in the history of problem solving in the mathematics curriculum. In fact, because teaching mathematics through problem solving is a rather new conception, it has not been the subject of much research.

Contemporary discussions of goals for mathematics education emphasize the importance of thinking, understanding, reasoning, and problem solving, with an emphasis on connections, applications, and communication. This view stands in contrast to a more conventional view of mathematics, involving the memorization and recitation of facts, rules, and procedures, with an emphasis on the application of well-rehearsed procedures to solve routine problems. Because teaching mathematics through problem solving has been considered an instructional approach better aligned with the contemporary views of school mathematics, it is receiving increasingly strong support from researchers, educators, and teachers. Although less is known about the actual mechanisms students use to learn and make sense of mathematics through problem solving, there is widespread agreement that teaching through problem solving holds the promise of fostering student learning.

While there is no universal agreement about what teaching mathematics through problem solving should really look like, there are some commonly accepted features of teaching mathematics through problem solving. Teaching through problem solving starts with a problem. Students learn and understand important aspects of a mathematical concept or idea by exploring the problem situation. The problems tend to be open-ended and allow for multiple correct answers and multiple solution approaches. Students play a very active role in their learning—exploring problem situations with teacher guidance and “inventing” their own solution strategies. In fact, the students’ own exploration of the problem is an essential component in teaching with this method. In students’ problem solving, they can use any approach they can think of, draw on any piece of knowledge they have learned, and justify any of their ideas that they feel are convincing. While students work on the problem individually, teachers talk to individual students in order to understand their progress and provide individual guidance. After students have used at least one strategy to solve the problem or have attempted to use a strategy to solve the problem, students are given opportunities to share their various strategies with each other. Thus, students’ learning and understanding of mathematics can be enhanced by considering one another’s ideas and debating the validity of alternative approaches. During the process of discussing and comparing alternative solutions, the students’ original solutions are supported, challenged, and discussed. Students listen to the ideas of other students and compare other students’ thoughts with their own. Such interactions help students clarify their ideas and acquire different perspectives of the concept or idea they are learning. In other words, students have ownership of the knowledge because they devise their own strategies to construct the solutions. At the end, teachers make concise summaries and lead students to understand key aspects of the concept based on the problem and its multiple solutions.

Theoretically, this approach makes sense. Empirically, there is lacking of data confirming the promise of teaching through problem solving. In particular, we need to seek answers to a number of important research questions, such as, (1) Does classroom instruction using a problem-solving approach have any positive impact on students' learning of mathematics? If so, what is the magnitude of the impact? (2) How does classroom instruction using this approach impact students' learning of mathematics? (3) What actually happens inside the classroom when a problem-solving approach is used effectively or ineffectively? (4) What do the findings from research suggest about the feasibility of teaching mathematics *through* problem solving in classroom?

In this paper, I will explore these research questions through reviewing two lines of research. The FIRST line of research includes those recently conducted studies on *NSF-funded* curricular programs that teach mathematics through problem solving and that have been implemented by teachers in classroom. The *NSF-funded* curricula are problem-based curricula, and the intent is to teach mathematics and to build students' understanding of important mathematical ideas through explorations of real-world situations and problems.

The SECOND line of research includes studies based on innovative materials developed by researchers in specific content areas. Unlike the first line of research, in this second line, researchers usually focus on teaching grade-specific mathematical topics using a problem-solving approach. These studies are important because they provide insights into the ways teachers teach specific content topics through problem solving in classroom.

Networking strategies for connecting theoretical approaches

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Abstract: One of the characteristics of the research community in mathematics education seems to be the large diversity of different theories, research paradigms and theoretical frameworks. This diversity has become an important issue to discuss at many conferences and in many publications. Is diversity a problem or a resource or a barrier for further research? How shall the scientific community deal with this diversity? Internationally different approaches have been developed to cope with this diversity (see Sriraman / English in monograph 1). Under a European perspective the approach of networking strategies, which aim to connect different theoretical approaches using several strategies, has been developed. This perspective bases its work on the assumption that the variety of different theoretical approaches and perspectives in mathematics education research is a *rich resource* upon which the scientific community should build more consequently. This perspectives calls for the connections of different theories and

rejects isolationistic tendencies of separating different theoretical approaches. This approach does not intend to develop one grand unified theory, but intends to network local theories, which deal with background theories but use diverging conceptual systems for describing the same phenomena.

Different networking strategies are presented in a landscape, linearly ordered according to their degree of integration. These networking strategies such as comparing or contrasting, combining or coordinating can contribute to the development of theories and their connectivity and offer hence an interesting research strategy for the didactics of mathematics as scientific discipline.

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Feminist perspectives and mathematics education

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Abstract: Feminism has many faces. There is, however, a unifying dimension to all the theoretical shades of feminism. Feminism is considered a movement for attaining the right of women to be equal to men in all aspects of life – social, political, legal, and educational. The impact of feminist thinking on mathematics education research is the focus of this presentation.

In her article entitled *Feminist pedagogy and mathematics,* which is to be reproduced in the first monograph in the Springer series, *Advances in mathematics education*, Judith Jacobs (1994) concluded that:

> ... previous research and intervention programs designed to promote females … have been based on the assumption of male as the norm, the model of the successful mathematics student or mathematician who is to be emulated if the non-successful are to succeed. Little research and work has begun from the assumption that females have strengths, experiences and learning styles that can succeed in mathematics.

Jacobs (1994) provided a theoretical framework for a feminist pedagogy for which the assumption was “that being a woman is the norm for females” (p. 16), and the teacher had the responsibility “to capitalize on females’ strengths and interests in order to facilitate their success in mathematics” (p. 16). Jacobs believed that all students would benefit from this approach which, she claimed, “in no way denies the power or beauty of mathematics” (p. 16).

Leder (in press) has provided a commentary on Jacobs’ (1994) chapter which is also to be included in the Springer monograph). Leder summarised the main points raised by Jacobs (1994) including the caution not to essentialise women as a group in response to research generalisations about differences between women and men.

Drawing on a number of earlier as well as more contemporary sources, Leder also discussed a range of feminist perspectives and their influence on research into gender issues in mathematics education. The feminist links evident in chapters found in the influential edited collection by Rogers and Kaiser (1995) are highlighted and include:

- Kaiser and Rogers' (1995) five stages of the mathematics curriculum beginning with "womanless" mathematics" and ending with "mathematics reconstructed"; and
- Becker's (1995) chapter in which Belenky et al.'s (1986) 'women's ways of knowing' are extended to the knowing of mathematics.

Whilst liberal feminism receives much criticism from many feminist theorists, it would appear to underpin and dominate many research endeavours, particularly those which do not specifically identify with any feminism. Many researchers continue to call for the monitoring of all large scale studies involving achievement and/or participation data for gender differences, and caution not to ignore gender as a factor in smaller, more focussed studies. As is evident in Australia today, educational disadvantages once considered to have been addressed can resurface. Recent data revealing the re-opening of gender gaps favouring males will be presented.

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Problem Solving Heuristics, Affect, Representations and Discrete Mathematics

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Abstract: It has been suggested that activities in discrete mathematics allow a kind of new beginning for students and teachers. Students who have been "turned off" by traditional school mathematics, and teachers who have long ago routinized their instruction, can find in the domain of discrete mathematics opportunities for mathematical discovery and interesting, non-routine problem solving. Sometimes formerly low-achieving students demonstrate mathematical abilities their teachers did not know they had. To take maximum advantage of these possibilities, it is important to know what kinds of thinking during problem solving can be naturally evoked by discrete mathematical situations—so that in developing a curriculum, the objectives can include pathways to desired mathematical reasoning processes. This article discusses some of these ways of thinking, with special attention to the idea of "modeling the general on the particular." Some comments are also offered on the global ideas of Moreno-Armella & Sriraman (2005) pertaining to the development of representational systems. The discussion focuses on the co-evolution of

symbols and their referents, and the shared interpretation of mathematical symbols in a community of practice. Some future directions are suggested.

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