CHAPTER 2

THE EXISTENTIAL VOID IN LEARNING

Juxtaposing Mathematics and Literature

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ABSTRACT

The term "existential" is normally used in the context of the human search to give meaning to existence. This metaphor could be used in the context of learning. A student, whose learning experiences in school lacks moral or ethical guidelines which does not have the power to give meaning to her world, experiences the existential void. Even the brightest students experience this void. On the other hand many are not aware of this because they are caught up in their world of rock and roll. The teachers role is to create learning experiences that help students fill the void. Mathematics has symmetry or a totality that blends the parts to the whole. It is a continuum as opposed to pieces of a disjointed puzzle. Mathematics can allow students to experience the exhilaration of discovery as well as see its connections to areas in the arts, business, and sciences. Similarly, the study of literature through the prism

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of critical thinking can also allow the student to experience its cohesiveness to life. Literature can be practical, inspirational, appealing, stimulating and educational if approached with this purpose in mind.

Ideally the goal of learning is to extend vision, to broaden perspective, and to bring out coherence and unity among the disciplines. The general goal of this paper is to demonstrate how education can fill the existential void felt by students, and to give education purpose and viability. In particular, the authors will use one classical mathematical problem and one contemporary literary work to show how teachers can fill the existential void in learning.

INTRODUCTION

The term *existential* may be used in three ways. To refer to (1) existence itself; (2) the meaning of existence; and (3) the striving to find a concrete meaning in personal existence, that is to say, the will to meaning (Frankl, 2000). Applying the third way "the will to meaning" to education is the question for exploration in this chapter.

Ideally the goal of learning is to extend vision, to broaden perspective, and to bring out coherence and unity among the disciplines. A student whose learning experiences in school involve the mindless regurgitation of facts, figures and formulae, which lack meaning nor add beauty to her world, experiences the existential void. The standard rationale that education means fulfillment begs the question: how can education fill the existential void felt by students? The answer may seem simplistic and/or idealistic but the onus should be placed on the teacher. It is the teachers role to create learning experiences that help students fill the void and to give education purpose and viability. The authors will draw on their experiences as teachers in the field of literature and mathematics to illustrate how a subject matter can be used to initiate the search for meaning.

The study of literature can be practical, inspirational, appealing, stimulating and educational if approached through critical thinking, which in turn can allow the student to experience its connections to life. Similarly mathematics can allow students to experience the cohesiveness of an idea from the classroom, to numerous applications in the various sciences, as well as connections to underlying patterns in nature, art and music. For example, a particular branch of mathematics called number theory has profound applications to modern day life, in the guise of cryptography and computer science.

USING CONJECTURE–PROOF–REFUTATION TO FILL THE EXISTENTIAL VOID

Most high school students in the United States view mathematics as consisting of immutable truths. For example, in geometry students believe that

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remembering theorems, facts and rules is the primary goal and very few understand the relationship between definitions, postulates, axioms and theorems (Fawcett, 1938; Senk, 1985, Usiskin, 1987). Within mathematical philosophy, mathematical knowledge is espoused as either absolutist or fallibilist. The absolutist view suggests that mathematical knowledge consists of certain and unchallengable truths (Ernest, 1991). The fallibilist view of knowledge asserts that mathematical knowledge is "fallible and corrigible, and can never be regarded as beyond revision and correction" (Ernest, 1991, p. 18). The philosopher Lakatos (1976) viewed mathematics as an ongoing process of conjecture, proof, and refutation, and this is the view that the authors subscribe to. In other words the authors subscribe to a fallibilist epistemology and favor instructional practices, which are more open to the influence of students.

In the United States in most traditional high school curricula, students do not experience the process of establishing a mathematical truth until they encounter geometry. In fact when students first study geometry, they are immediately introduced to a deductive method of proof, which deprives them of the process of discovery of the mathematical truth. Inductive reasoning plays a major role in the real world. However, it is only in mathematics that one comes across the notion of a proof, whose sole purpose is to establish the truth (or falsity) of a given statement. The reasoning that one normally comes across in many mathematics textbooks is crisp and deductive, with one statement flowing from another until the desired outcome is reached. An artificially reconstructed logical proof conveys little or no insight to the student about the processes and the motivations for constructing the proof.

The last decade has shown some change to this traditional method of studying geometry with the introduction of dynamic computer software such as the Geometer's sketchpad and Cabri Geometry. Although such software has great capabilities, most of the anticipatory questions would have to be carefully designed by teachers if they wish to guide students into the discovery of a mathematical truth. However many rural and less affluent school districts lack the technological and human resources that would enable students to study geometry using an inductive approach on a medium such as the Geometer's Sketchpad.

Aside from geometry, students in their study of two years of high school algebra encounter very little experience with proof. In fact most traditional algebra curricula takes the properties of the real number system as a priori truths and focuses on analytic representations of geometric figures on the Cartesian co-ordinate system with subsequent manipulations of polynomial equations and inequalities. A substantial amount of time is also spent on solving contrived "word problems" under the guise of "applications" in order to somehow justify the topics that have been covered. There is very little room for 'play' or exploration within such an approach. This approach also

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does not convey to the student that mathematics is a process of ongoing conjecture, proof and refutation (Lakatos, 1976).

USING A CLASSICAL DIOPHANTINE PROBLEM TO INITIATE CONJECTURE–PROOF–REFUTATION

The Greek mathematician Diophantus (200 AD) is renowned for his work on solving equations with rational number solutions. Number theorists relish tackling diophantine equations with integer solutions. The beauty of many diophantine equations lies in the fact that they are easy to understand, yet very difficult to solve. Fermat's Last Theorem is a notorious example to illustrate this point. Elementary number theoretic concepts such as prime numbers, and tests for divisibility are introduced in most middle school curricula. However at the high school level, the curriculum offers students very little opportunity to tackle number theoretic problems. For example, many questions in number theory may be posed as diophantine equations—equations to be solved in integers. Catalan's conjecture was whether 8 and 9 were the only consecutive powers? This conjecture asks for the solution to $x^a - y^b = 1$ in integers. The Four Squares Theorem states that every natural number is the sum of four integer squares. In other words, it asserts that $x^2 + y^2 + z^2 + w^2 = n$ is solvable for all n. But the attempt to solve these equations requires rather powerful tools from elsewhere in mathematics to shed light on the structure of the problem (Oystein, 1988).

This led the author to incorporate diophantine equations as the teacher of a beginning algebra course. Students in this course were introduced to elementary diophantine equations under the guise of recreational journal problems. The problem chosen for investigation was the classic n-tuple diophantine problem posed by Diophantus himself. Simply put, a diophantine n-tuple is a set of n positive integers such that the product of any two is one less than a square integer. It was the authors' hope that a very elementary version of the problem would kindle student interest and eventually result in an attempt to tackle the as yet unsolved 5-tuple problem in integers. Does there exist a diophantine 5-tuple? Many mathematicians propose this problem as the successor to Fermat's last theorem.

The author initiated this problem by simply mentioning in class off hand the 3-tuple problem, if one considers the integers 1, 3, and 8, then it is always the case that the product of any two is always one less than a perfect square. Indeed $1 \times 3 = 2^2 - 1$; $1 \times 8 = 3^2 - 1$; and $3 \times 8 = 5^2 - 1$. This remark led students to wonder if other such 3-tuples existed? This problem was then assigned as a recreational journal problem. This simple question eventually led to an investigation of the unsolved 5-tuple problem over the course of the school year (see Figure 2.1).

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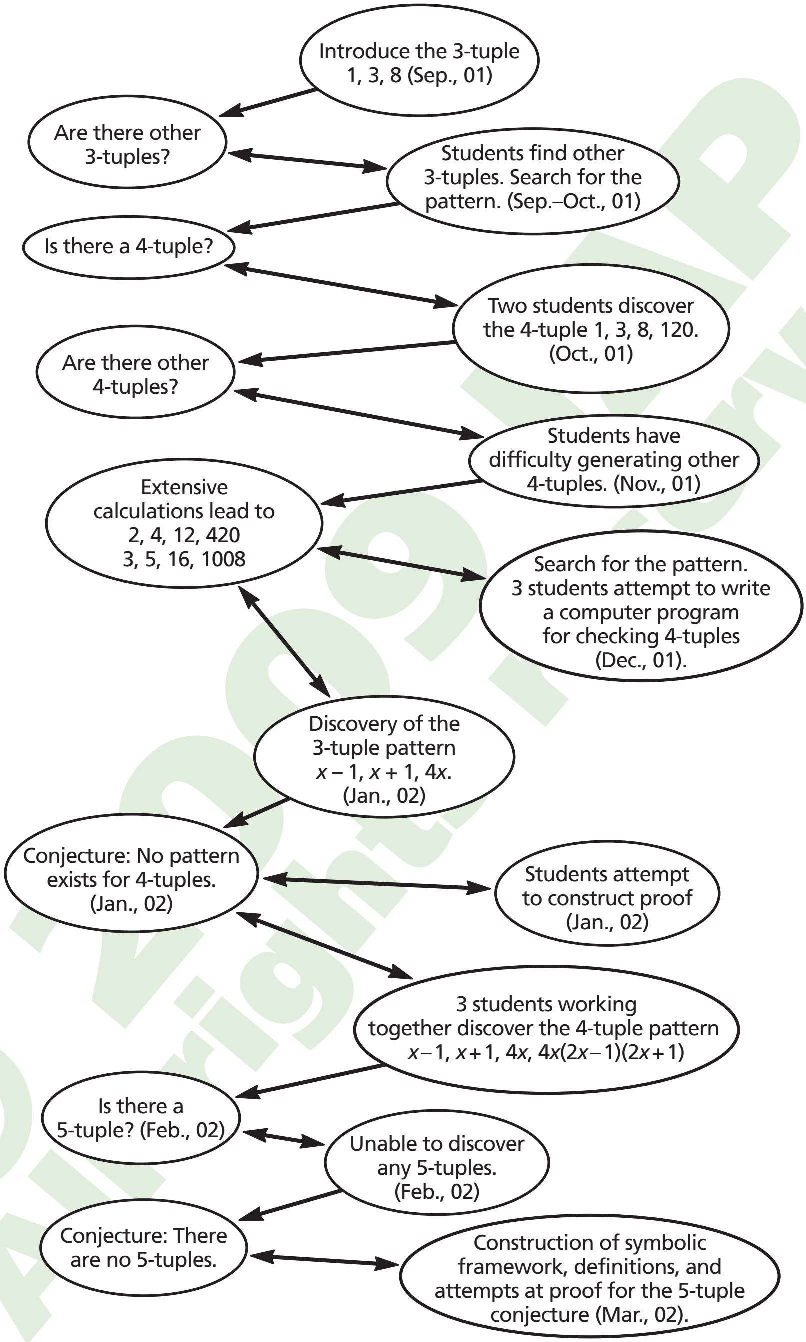

Figure 2.1 Conjecture, proof, and refutation in the 5-tuple Diophantine problem.

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As indicated in Figure 2.1, the students attempted to solve the open 5-tuple diophantine problem being immersed in the process of conjecture, proof and refutation. Once a week, the teacher utilized one class period to allow students to present their work to the other students. The teacher acted as a moderator in this process to facilitate the discussion. Many of the students were surprised at the difficulty of solving these seemingly easy problems. The mathematical experiences of several students also led them into writing algorithms and computer programs to check for integer solutions. Usually when other students in the class became convinced that multiple integer solutions to a particular problem existed, they formed conjectures on what the underlying pattern was and tried to express them symbolically. This in turn allowed the teacher to use student conjectures as a starting point to initiate the process of mathematically proving or disproving a conjecture. It is important that the reader note that the progress of the problem depended completely on the "will" of the students. There were some weeks when students did not attempt the problem, especially before or after school breaks and during examination weeks. When this occurred, the author continued with the regular algebra curriculum. It was crucial that students initiated this process of conjecture, proof and refutation out of their need to resolve the unimagined difficulties that arose from a seemingly easy problem. It was noteworthy that all thirteen students in this class willingly engaged in trying to solve one of the unresolved conjectures of our time over a seven month time period through the process of conjecture, proof and refutation. The interested reader can find the actual mathematics created by the students in Appendix A.

USING CRITICAL THINKING IN LITERATURE TO FILL THE EXISTENTIAL VOID

The process of conjecture, refutation and proof does have a counterpart in the realm of literature. The process of critical thinking could be viewed as the use of reasoning in the pursuit of "truth." Critical thinking makes implicit use of logic in order to draw inferences and/or make comparisons. Critical thinking enables the student to understand the cultural and instructional influences on accepted thought. Because of this "second sight"-so to speak, the student can adjust or trick the mind into a new view of the issue. True critical thinking is a matter of adjustment to tune the process, so that bias no longer controls thought or action.

In the previous section, the authors used the diophantine n-tuple problem, in order to kindle the pursuit of a mathematical truth by the process

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of conjecture, proof and refutation. In a similar spirit, the authors will now use a contemporary novel in order to demonstrate how a simple story can be used to initiate students into the process of critical thinking and into making inferences on "truths" about society and life. The "truths" that we infer are naturally influenced by our social, economic, cultural, religious backgrounds, and value systems.

The English philosopher Francis Bacon warned about blind observance to so called "truths" in his *Novum Organum*, using the metaphor of "the idols of the mind" (Bacon, 1994). There are four classes of idols which beset men's mind, namely, the idols of tribe, the idols of cave, the idols of marketplace, and idols of theater. Titus (1994) describes Francis Bacon's "idols" metaphor in the following words.

> Bacon has given us a classic statement of the errors of thinking. These are first, the idols of the tribe. We are apt to recognize evidence and incidents favorable to our own side or group (tribe or nation). Second, there are the idols of the cave. We tend to see ourselves as the center of the world and to stress our own limited outlook. Third, the idols of the marketplace cause us to be influenced by the words and names with which we are familiar in everyday discourse. We are led astray by emotionally toned words-for example, in our society, such words as communist or liberal. Finally, the idols of theater arise from our attachment to parties, creeds, and cults. These fads, fashions, and schools of thought are like stage plays in the sense that they lead us into imaginary worlds; ultimately, the idols of theater lead us to biased conclusions. (Titus, 1994, p. 171)

Using a Contemporary Novel to Initiate Critical Thinking

Grisham's (2001) *Skipping Christmas* is a retelling of Charles Dickens' *A Christmas Story* with contemporary characters facing contemporary problems. Using life in suburbia as his backdrop, Grisham (2001) weaves his tale of Luther "Scrooge" Krank and his dream of skipping Christmas and all the baggage this holiday carries.

Luther is a character devoid apparently of any Christmas spirit. Tradition is abhorrent to him and his tirades against the idea of Christmas are a litany of all the negative clichés about the commercialization of Christmas. From tipsy office partygoers, thoughtless gifts, crass symbols, to dollars spent, Luther condemns Christmas to the "bah humbug" status of the typical Scrooge. He decides, and then he cons Nora, his wife, into going on a

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Caribbean cruise starting on December 25. This is Luther's ruse to miss out on Christmas and all that goes with the season.

Grisham (2001) using all the usual traditions as his canvas then paints the hazardous tale of Luther and Nora's decision. He veers from his usual deep, dark sub-plots although sub-plots abound. These sub-plots involve traditional value systems, which have become part of the American canon or code. Family, love, co-operative spirits, concern and love of neighbor, respect for the beliefs of others, equality of birth, sharing and finally the rights of all of us, life, liberty and the pursuit of happiness are just some of these traditional values touched upon.

In a sense the novel seems to be an appeal to past values. In the world of Luther values have become passé signifying our modern world of "me-centric" existence. In some cases the youth of today have become deprived of an ethical inheritance. No thing, no person, no ideal, no code has come to fill the existential void. The events of September 11, 2001 jolted many Americans into remembrance of time past. That was then, this is now has been turned around by this horrendous event. Americans, especially the young do seek the values and security of the past. *Skipping Christmas* is Grisham's (2001) attempt to review and renew some of those values.

The literature teacher through discussion of this short novel has the opportunity to review the basic values of our democracy. The destruction of the twin towers of honesty and truth by the political and business leaders of the past few years has deprived our youth of a continuity of values. Thus, the deadly existential void has become apparent.

Using *Skipping Christmas* as a critical thinking didactic tool, literature teachers can provide opportunities for students to fill the existential void they may be experiencing. The role of the teacher in this process can be thought of as that of an eye specialist rather than that of a painter. A painter tries to convey to us a picture of the world as he sees it; the eye doctor tries to enable us to see the world as it really is. It is crucial that teachers not be propagandists or try to indoctrinate students. The teachers role through critical thought extends the visual field of the student, so that the whole spectrum of potential meaning becomes visible to him/her (Frankl, 2000). With this philosophy of interpretation in mind, students can be exposed to the novel. There are various strategies that teachers can use to kindle critical thinking in the students. The teacher can use critical thinking questions to set up a forum for discussion. For example, comparisons can be made to Dicken's timeless fable *A Christmas Carol*. Dicken's novel is about a character Scrooge and his all consuming love of money. The spirit of Christmas is a contrast to his selfishness and lack of love, joy and care for others. It is the story of the haunting of Ebenezer Scrooge by ghosts of Christmas past,

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present and yet to come. These ghosts drive Scrooge to reconsider his life and construct a new mode of living. *Skipping Christmas* is also about just such a character, Luther Krank. Even his last name "Krank" seems to depict his personality.

An English teacher can also use an anticipatory questionnaire using student answers for discussion in a manner akin to conjecture, proof and refutation. The anticipatory questions must be designed not to set a cause effect pattern of thought. This can range from particular questions to general views about the subject matter. One needs almost to upset the natural biased way of interpretation. What results then, is a personal pattern rather than an algorithmic type of answer. In literature various interpretations lead to critical thought actually arrived at by the student, rather than "I have found what you-the teacher are thinking." The student's thought as well as teacher's thought combined reaches a valued conclusion. This of course must be tested in the "market place of ideas." The authors will now present the results of an experiment in critical thinking using *Skipping Christmas* as a didactic tool. Eight high school seniors who read the novel were invited to discuss the book. The story was used to frame five general anticipatory questions (see Appendix B), which kindled a discussion of personal value systems and values in general. In other words, the book was simply a didactic tool to initiate critical thinking about traditions and value systems and used by the teacher to provide a framework for the discussion of the five questions.

The authors will now show the progression of thought as the novel is delineated by the use of the Socratic method of discovery. In an effort to show that a continuity of thought exists from question to answer to subsequent conclusion, student responses from the discussion are presented in the form of a schematic analogous to conjecture-proof-refutation (see Figures 2.2–2.5).

As the discussion schemes outlined in Figures 2.2–2.5 reveal, daily class discussion using anticipatory questions and the answers of students as guidelines or starting points is a powerful method of instruction This method allows the teacher to introduce historical and philosophical perspectives to moral questions. These perspectives combined with the student's views leads students to examine codes of behavior and biases. The teachers role is to create a classroom atmosphere that nurtures and kindles critical thinking in students, so that students begin to examine their biases and have the opportunity to discuss their value systems and their perspectives.

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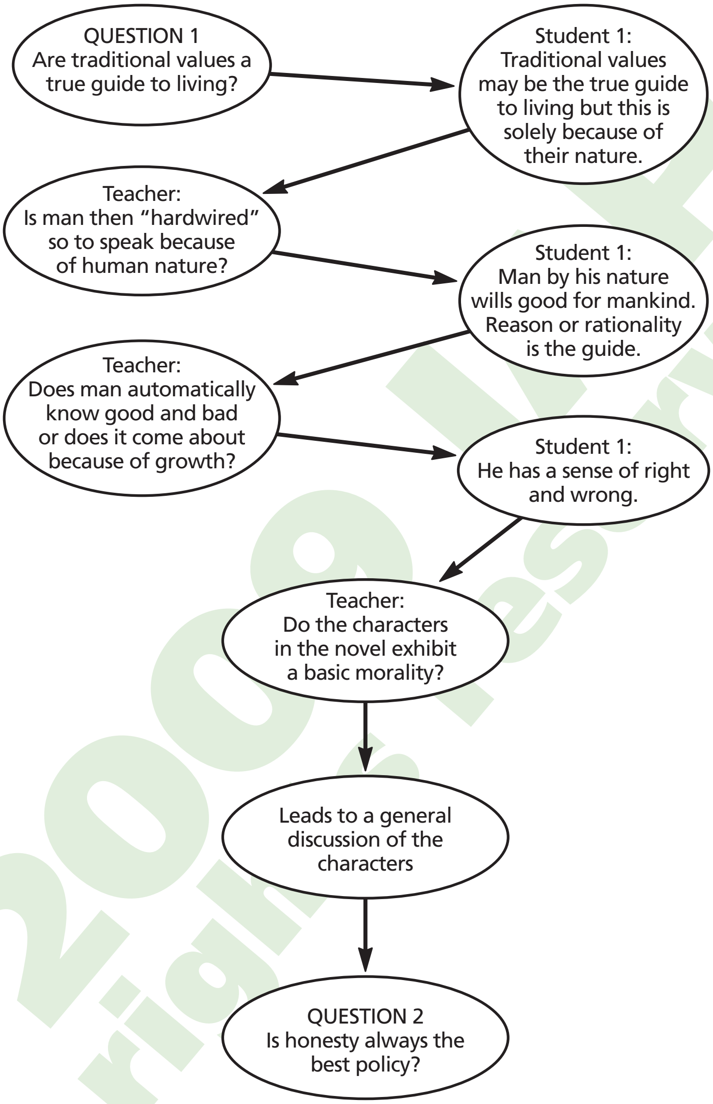

Figure 2.2 Discussion leading Question 1 and Question 2.

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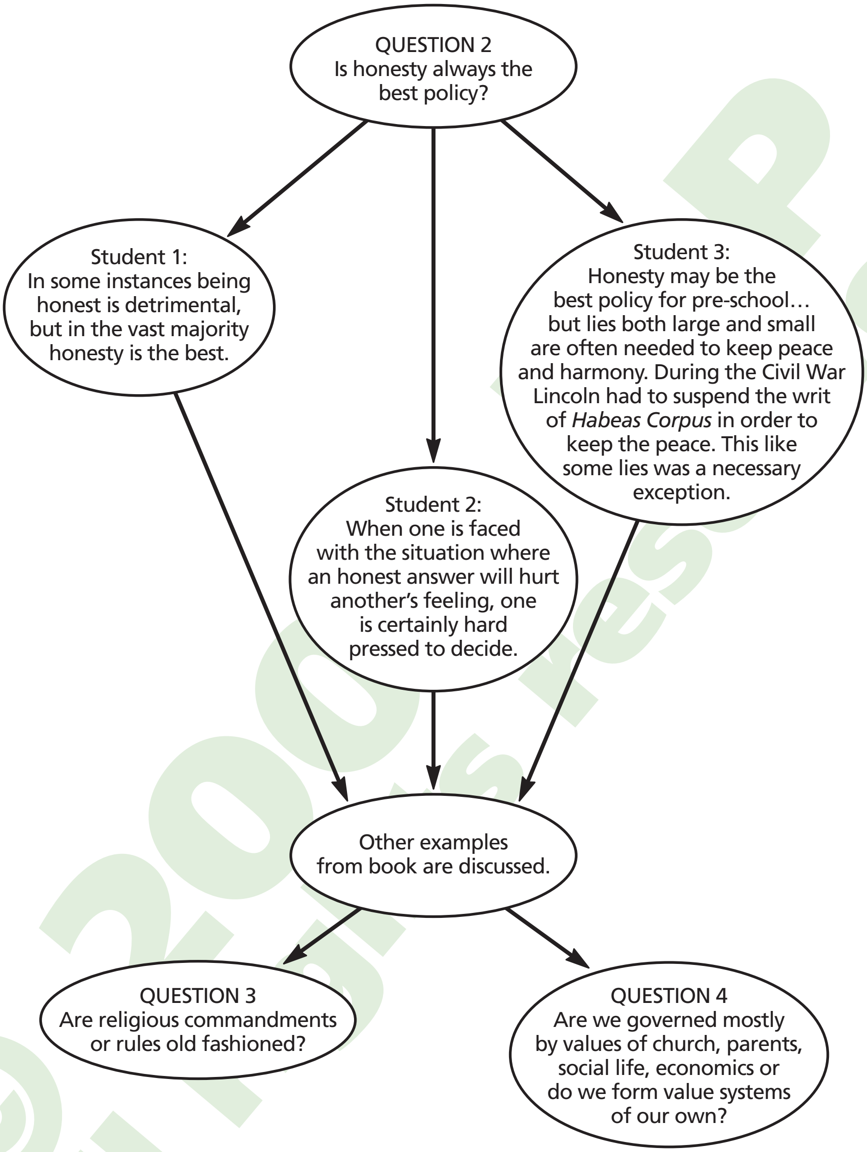

Figure 2.3 Discussion leading Question 2 to Question 3 and 4.

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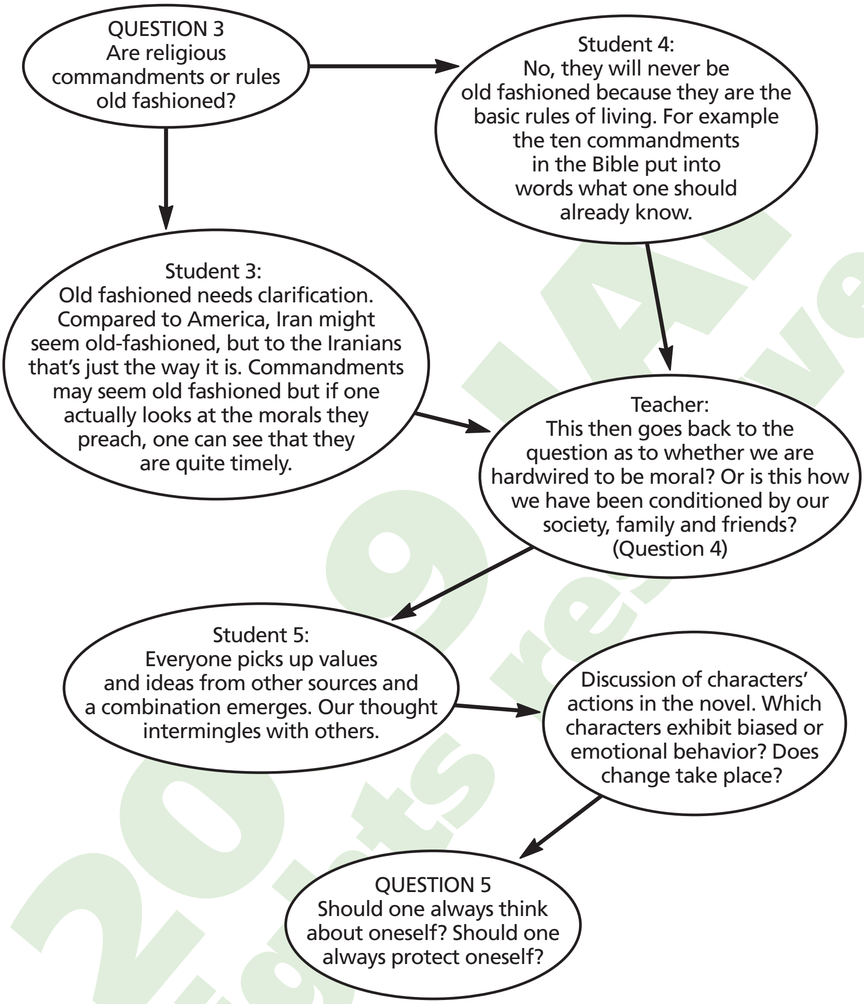

Figure 2.4 Discussion leading Question 4 and 4 to Question 5.

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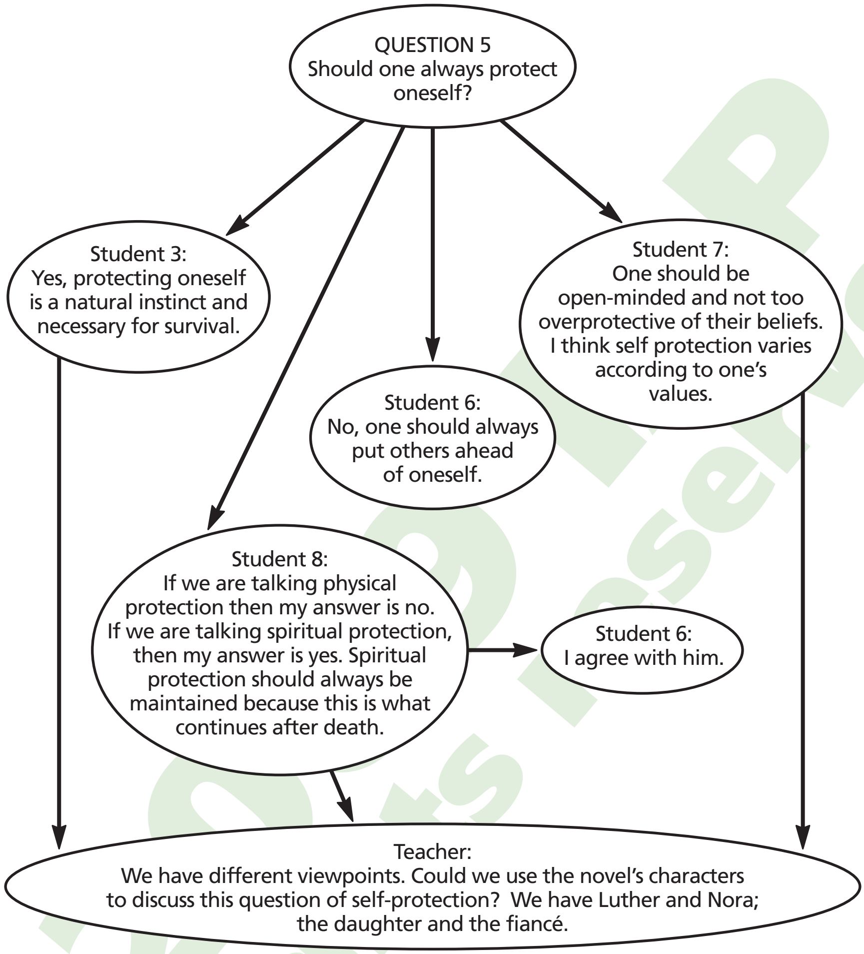

Figure 2.5 Discussion of Question 5.

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CONCLUSIONS

The authors hope that they have conveyed to the reader the value of the use of conjecture-proof-refutation in the mathematics classroom and the use of critical thinking in the literature classroom in order to create meaningful learning experiences. The mathematics created by the students in trying to solve the classic 5-tuple diophantine problem clearly indicates that students are capable of original thought that goes beyond the mimicking and application of procedures taught in the classroom. Similarly, the critical thinking demonstrated by the students in the discussion of *Skipping Christmas* indicates that students are willing to discuss questions of belief, morality and values. The classic problem and the contemporary novel were tools used by the authors to sow the seed that allowed students to create mathematics and examine value systems. This method of teaching and learning adds personal meaning to the students schooling experiences, thus filling the existential void. As Bertrand Russell once said, "It should be one of the functions of a teacher to open vistas before his pupils, showing them the possibility of activities that will be as delightful as they are useful."

REFERENCES

Bacon, F. (1994). *Novum organum.* J. Gibson and P. Urbach (Eds.). Open Court Publishing Company.

Ernest, P. (1991). *The philosophy of mathematics education*, Briston, PA: The Falmer Press.

Fawcett, H. P. (1938) *The nature of proof.* New York: Teachers College, Columbia University.

Frankl, V. E. (2000). *Man's search for meaning.* Beacon Press.

Grisham, J. (2001). *Skipping Christmas.* New York: Doubleday Publishing Company.

Lakatos, I. (1976). *Proofs and refutations.* Cambridge, UK: Cambridge University Press.

Oystein,O. (1988). *Number theory and its history.* New York: Dover Publications.

Senk, S. (1985). How well do students write geometry proofs? *The Mathematics Teacher, 78,* 419–486.

Titus, H. (1994). *Living issues in philosophy.* Oxford: Oxford University Press.

Usiskin, Z. P. (1987). Resolving the continuing dilemmas in school geometry. In M. M. Lindquist, & A. P. Shulte (Eds.) *Learning and teaching geometry, K–12: 1987 Yearbook* (pp. 17–31). Reston, VA: National Council of Teachers of Mathematics.

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APPENDIX A

Mathematics created by students via Conjecture-Proof-Refutation in the solution of the 5-tuple diophantine problem over a 7 month period.

Definition 1: A set of 3 positive integers $\{a_1, a_2, a_3\}$ is called a Diophantine 3-tuple if $a_i a_j + 1$ is a perfect square for all 1 $i < j \leq 3$.

Conjecture 1: There exist infinitely many 3-tuples $\{a_1, a_2, a_3\}$.

Proof: Let $a_1 = x - 1$; $a_2 = x + 1$ and $a_3 = 4x$

Then $(x-1)(x+1) + 1 = x^2$

Also, $4x(x-1) + 1 = 4x^2 - 4x + 1 = (2x-1)^2$, and

$4x(x+1) + 1 = 4x^2 + 4x + 1 = (2x+1)^2$

Conjecture 2: There exist 4 positive integers $\{a_1, a_2, a_3, a_4\}$ called a Diophantine 4-tuple such that *at least one* $a_i a_j + 1$ is a perfect square for 1 $i < j \leq 4$.

Laborious calculations yield: {1, 3, 8, 120}, {2, 4, 12, 420}, {3, 5, 16, 1008}

Conjecture 3: There exist infinitely many 4-tuples of the form $\{a_1, a_2, a_3, a_4\}$. The fourth integer a_4, must be some combination of a_1, a_2, and a_3 as found in Conjecture 1.

Proof: Let $a_1 = x - 1$; $a_2 = x + 1$; $a_3 = 4x$; and $a_4 = 4x(2x-1)(2x+1)$.

Then $a_i a_j + 1$ is a perfect square for 1 $i < j \leq 3$.

For $j = 4$, $a_1 a_4 + 1 = (x-1)\ 4x(2x-1)(2x+1) = 4x(x-1)$

$(4x^2 - 1) + 1$ = Hard to factor into a perfect square!

Students reach a dead end. This resulted in numerical calculations using the sets {1, 3, 8, 120}, and {2, 4, 12, 420}, in order to somehow factor $a_1 a_4 + 1$ into a perfect square. One month later a new proof is attempted, based on student discovery that $120 = 1 + 3 + 8 + 2\ (1)\ (3)\ (8) + 2\sqrt{4}\sqrt{9}\sqrt{25}$, where $4 = (1)(3) + 1$; $9 = (1)(8) + 1$ and $25 = (3)(8) + 1$.

New Proof: Let $\{a_1, a_2, a_3\}$ be a Diophantine triple and $a_1 a_2 + 1 = r^2$, $a_1 a_3 + 1 = s^2$, $a_2 a_3 + 1 = t^2$, where r, s, t are positive integers.

Let $a_4 = a_1 + a_2 + a_3 + 2\ a_1 a_2 a_3 + 2rst$.

Then $\{a_1, a_2, a_3, a_4\}$ is a Diophantine quadruple, because:

$a_1 a_4 + 1 = (a_1 t + rs)^2$,

$a_2 a_4 + 1 = (a_2 s + rt)^2$,

$a_3 a_4 + 1 = (a_3 r + st)^2$.

Numerous 4-tuples are verified and fit the pattern described in the new proof.

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Conjecture 4: There exist 5 positive integers $\{a_1, a_2, a_3, a_4\ a_5\}$ called a Diophantine 5-tuple such that *at least one* $a_i a_j + 1$ is a perfect square for 1 $i < j \leq 5$.

Calculations last for over a month. Students use the sets {1, 3, 8, 120}, {2, 4, 12, 420}, {3, 5, 16, 1008} but are unable to find a_5, such that all products + 1 yield a perfect square.

Conjecture 5: There does not exist a Diophantine 5-tuple $\{a_1, a_2, a_3, a_4\ a_5\}$, such that $a_i a_j + 1$ is a perfect square for 1 $i < j \leq 5$.

Students are unable to construct a general argument. This changes their conjecture to.

Conjecture 6: The set {1, 3, 8, 120} cannot be extended into a Diophantine 5-tuple.

Proof: Suppose a_5 is the 5th number. Then the following equations must all be true at the same time:
$a_5 + 1 = r^2$; $3a_5 + 1 = s^2$; $8a_5 + 1 = t^2$; $120\ a_5 + 1 = u^2$, where r, s, t, and u are integers.
This means there exist perfect squares of the form:
$r^2 - 1$; $(s^2 - 1)/3$; $(t^2 - 1)/8$; and $(u^2 - 1)/120$.
Consider $r = 2, 3, 4, 5, 6, 7, 8, 9, 10, \ldots$
Then $r^2 - 1 = 3, 8, 15, 24, 35, 48, 63, 80, 99, \ldots$
It is impossible for $r^2 - 1$ to be a perfect square. This is also true for s, t, and u. Therefore a_5 does not exist.

Note: The proof created by the students for conjecture 6 does not resolve the 5-tuple problem by any means because one still has to check infinitely many other possibilities. However, the reader might appreciate the fact that 9th graders worked their way up to try and resolve a problem similar in difficulty to Fermat's Last Theorem.

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APPENDIX B

The five anticipatory questions used in the discussion of the novel.

1. Do you think traditional values are the true guide to living?
2. Is honesty always the best policy?
3. Are religious commandments or rules old fashioned?
4. Are we governed mostly by values of church, parents, social life, economic conditions, political views etc., or do we form value systems of our own?
5. Should one always protect one-self?

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