Preface to Chapter 2 Ernest's Reflections on Theories of Learning

Lakatos-Hersh-Ernest: Triangulating Philosophy-Mathematics-Mathematics Education

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Philosophy has always maintained an intricate relationship with mathematics. It was also implicitly accepted that the philosophical positions of a bearer influence his/her view on mathematics and its teaching (Törner & Sriraman, 2007), which leads us into the domain of beliefs theory. However the centrality of philosophy and its intricate relationship to theory development in mathematics education only came about two decades ago when Paul Ernest and Hans-Georg Steiner (1987) each independently became aware of the importance of epistemological issues that impact the teaching and learning of mathematics. Sierpinska and Lerman (1996) state:

> *Epistemology as a branch of philosophy concerned with scientific knowledge poses fundamental questions such as: 'What are the origins of scientific knowledge?' (Empirical? Rational?); 'What are the criteria of validity of scientific knowledge?' (Able to predict actual events? Logical consistency?); 'What is the character of the process of development of scientific knowledge?' (Accumulation and continuity? Periods of normal*

science, scientific revolutions and discontinuity? Shifts and refinement in scientific programs?).

The question of what is mathematics, for teaching and learning considerations brings into relevance the need to develop a philosophy of mathematics compatible with mathematics education. In order to answer this question for mathematics education, several theorists have played a role directly or indirectly. In this preface to chapter 2, we briefly summarize the role that Lakatos, Hersh and Ernest have played. Reuben Hersh began to popularize Lakatos' book *Proofs and Refutations* to the mathematics community in a paper titled, "Introducing Imre Lakatos" (1978) and called for the community of mathematicians to take an interest in re-examining the philosophy of mathematics. Hersh (1979) defined the "philosophy of mathematics" as the working philosophy of the professional mathematician, the philosophical attitude to his work that is assumed by the researcher, teacher, or user of mathematics and especially the central issue – the analysis of truth and meaning in mathematical discourse. Much later, Hersh (1991), wrote

> *Compared to "backstage" mathematics, "front" mathematics is formal, precise, ordered and abstract. It is separated clearly into definitions, theorems, and remarks. To every question there is an answer or at least, a conspicuous label: "open question". The goal is stated at the beginning of each chapter, and attained at the end. Compared to "front" mathematics, mathematics "in back is fragmentary, informal, intuitive, tentative. We try this or that, we say "maybe" or "it looks like".*

So, it seems that Hersh is not too concerned with dry ontological concerns about the nature of mathematics and mathematical objects, but is more concerned with the methodology of doing mathematics, which makes it a human activity. In 1978 Paul Ernest published a review of *Proofs and Refutations* in *Mathematical Reviews*, and subsequently wrote reviews of the works of Lakatos and Wittgenstein (see Ernest 1979 a,b; 1980). This coupled with his doctoral dissertation became the basis on which Ernest formulated a Philosophy of Mathematics Education (Ernest, 1991) and Social constructivism as a philosophy of mathematics (Ernest, 1998).

Pimm, Beisiegel and Meglis (2008) summarize Ernest's "extension" of Lakatos' philosophical position as follows:

> *Ernest (1991) claimed that the fallibilist philosophy and social construction of mathematics presented by Lakatos not only had educational implications, but that Lakatos was even aware of these implications (p. 208). Ernest argued that school mathematics should take on the socially constructed nature presented by Lakatos, and also that teacher and students should engage in ways identical to those in his dialogue, specifically posing and solving problems, articulating and confronting assumptions, and participating in genuine discussion*.

As a philosophy of mathematics, social constructivism, as defined by Ernest (1991), views mathematics as a social construction. It is based on conventionalism, which acknowledges that "human language, rules and agreement play a role in establishing and justifying the truths of mathematics" (p. 42). Ernest gives three grounds for this philosophy. The first is that linguistic knowledge, conventions and rules form the basis for mathematical knowledge. The second is that interpersonal social processes are needed to turn an individual's subjective mathematical knowledge into accepted objective knowledge. The last is that objectivity is understood to be social. A key part of what separates social constructivism from other philosophies of mathematics is that it takes into account the interplay between subjective and objective knowledge. When a discovery is made by an individual, this subjective knowledge later becomes knowledge accepted by the community – thus becoming objective. Then, as this knowledge is further spread to others, they internalize it and it becomes subjective again.

However the philosophy is not without its critics. Gold (1999) raises several objections. The first is that this philosophy fails to account for the usefulness of mathematics in the world. Social constructivism does fine when explaining how mathematics can be created to solve practical problems. However, it does nothing to explain mathematics created long before application. Social constructivism also fails to account for cases like that of Ramanujan, who developed his results through interaction with mathematical objects and not a mathematical community.Gold's main critique, however, is the failure of social constructivism to distinguish between mathematical knowledge and mathematics itself. Mathematical knowledge is what is socially created and/or discovered. She repeatedly draws on physics as an illustration. "(P)hysical objects either are or are not made up of atoms, and it is not the community of physicists that makes that true or false" (Gold, 1999, p. 377). While our knowledge of something may change over time, the reality of it does not. If mathematics is a human creation, can the same not be said for the quarks? Social constructivists would point to the fallibility of proofs as evidence that mathematics is a social construct and therefore lacks certainty. If the verification of mathematical facts can turn out to be false, then mathematical facts are subject to question as well. Gold points out, though, that proofs are among the activities that concern human knowledge. As such, they are subject to revision, as are theories in the physical sciences that mean to explain some physical phenomenon. The revision of explanatory theory, however, does not change the physical phenomenon. Hence, the social constructivist philosophy of mathematics is not a philosophy of mathematics education per se, but it does have educational implications. Social constructivism as a philosophy of mathematics can serve as a basis for developing a theory of learning, such as constructivism (Sriraman & English, this volume).

What implications if any does such a philosophy have for the necessity, teaching and learning of proof. Ontologically speaking, social constructivism would say that a mathematical proof becomes one when it is accepted by the community, and given the status that "result x, y, z, etc exist". In other words, the burden of mathematical proof is that it must convince others. The largest implication this has on mathematics education is that students need to learn that this is what a proof is meant to do (versus the idea that proof is a logical deduction from known facts). The philosophy of social constructivism also has an epistemological implication for the teaching and learning of proof. This is the realization that mathematical proof has its origin in human activity and is therefore is in a sense fallible and dynamic. They also need to be made aware that the burden of proof has changed at different times, depending on the rigor demanded by certain mathematical communities. In this way, they will realize that they need be sensitive to what is considered proof in their community. While a mathematician needs to be aware of what will constitute a proof within his or her community, students need to be taught it. Mathematicians make use and are aware of the methods that are recognized as valid in the community and students need to be taught those methods. As a philosophy of mathematics, social constructivism aims to describe what mathematics truly is and what is done by those in the field. On the other hand, as a philosophy of mathematics education, its aim is to train students in a way that is reflective of this view of mathematics as a whole.

Simon Goodchild points out in his commentary on Ernest's chapter *Reflections on theories of learning*, the words philosophies and theories often get used interchangeably by him, when the former is what is intended since the latter bears a much higher burden of testability in order to garner acceptance. Ernest is well aware of this distinction as his chapter unfolds into the different strains of constructivism and their relevance for learning. The second commentary to to the chapter, is Ernest's own reflections to his previous chapter Reflections on theories of learning. This lends the metacognitive spin that Simon Goodchild lamented was lacking in the original chapter, albeit this meta-cognition is being engaged in strictly at a theoretical level!

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