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Commentary on
Theorizing in mathematics education research: differences in modes and quality

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The authors confront a major and troubling issue for the field of mathematics education, namely the "bewildering array of theories, theoretical models, or theoretical frameworks" abundantly found in the literature that characterizes research today. This commentary is spurred by the provocative nature of the said article and having recently compiled and edited a major book on theories of mathematics education (Sriraman & English, 2010) whose research and development brought to the foreground many of the core issues eloquently and critically addressed by Jablonka & Bergsten. I will briefly spell out the salient points made by the authors in need of attention and consideration by the community within the larger framework of post-modernism.

In the Introductory section of their article a critical allusion is made to *what was accepted* in the canon of literature in the past, namely introspective articles, as well as articles that relied on quotes as a means of data presentation and analysis. Yet several treatments of the so-called state of the art of mathematics education are often made by mathematicians and even mathematics educators who have done little or no research in the classroom or other settings to substantiate claims. This malady is even more evident when pieces of legislation or reports from advisory panels of government bodies become the basis of curricular changes and research programs. For instance the call of the National Mathematics Advisory Panel in the United States to use Algebra as a panacea for curricular ills as well as the stipulation of psychometric aptitude-treatment-interaction based clinical studies as the only genre of acceptable research worthy of funding has led a cadre of mathematics education researchers jumping on the funding gravy train and blindly following this mode of research without regard for the fundamental problems of the field (see Greer, 2008). This swerving of research focus based on political tides does not bode well for mathematics education if it claims to be a field of research. One of the main points made by Jablonka & Bergsten is to carefully build theoretical bases for the field based on ongoing research as well as establish a rigorous framework for theorizing according to a specific linguistic grammar such as Basil Bernstein's internal/external languages of description. Once this is achieved and a sort of coherence pervades the objects of theoretical discussion, there will be a natural and much needed end to cyclically justifying the existence of mathematics education as "research field" every decade or so! (e.g., Sierpinska & Kilpatrick,1998; Sriraman & English, 2010).

In order to argue their case Jablonka & Bergsten critique the strengths and weaknesses of four ways of theorizing and illustrate what they mean by modes and qualities of theorizing. The four modes of theorizing selected from the literature and presented as examples are diverse enough to cover the spectrum of existing "theoretical" trends. The first example is the PISA framework in

which the ill defined notion of "mathematization" has become a major constituent of mathematical literacy and despite the weak operationalization of basic notions and disregard and criticisms of the cross cultural validity of PISA test items, the framework has dangerously mutated into a basis for curricular reform in many countries of the world when questions about the covert corporate nature of the implementation and fiscal aspects of this test abound (see Sriraman, 2008). Jablonka & Bergsten's article correctly points to the complete lack of a theoretical foundation of the PISA framework. The second example is the theory of authentic task situations taken from a "Nordic" country for the purposes of being self critical, in this case a framework developed in Sweden. The arbitrariness of categories (or aspects) chosen in the operational framework is evaluated and an argument made to show that relationships between categories appear vague as well as empirically tenuous. The APOS theory is next used as the third example of a theory that deals with conceptual development in mathematics. This neo-Piagetian theory is appraised as having a relatively strong internal consistency in its grammar and specific theorizing in terms of actions, processes and objects within schemas. Finally the Anthropological Theory of Didactics (ATD) is used to illustrate an example of a theory that uses a specialized language and develops hierarchical relationships between praxeologies. The last theory chosen by the authors, namely ATD, which is also elaborated more in their paper is of particular interest to the community given its ecological nature and the wideness of its applicability. Simply put ATD is the extension of Brousseau's ideas from within the institutional setting to the wider "**Institutional**" setting [I use capital I]. Artigue (2002) clarifies this subtlety by saying that:

The anthropological approach shares with "socio-cultural" approaches in the educational field (Sierpinska and Lerman, 1996) the vision that mathematics is seen as the product of a human activity. Mathematical productions and thinking modes are thus seen as dependent on the social and cultural contexts where they develop. As a consequence, mathematical objects are not absolute objects, but are entities which arise from the practices of given institutions. The word "institution" has to be understood in this theory in a very broad sense…[a]ny social or cultural practice takes place within an institution. Didactic institutions are those devoted to the intentional apprenticeship of specific contents of knowledge. As regards the objects of knowledge it takes in charge, any didactic institution develops specific practices, and this results in specific norms and visions as regards the meaning of knowing or understanding such or such object (p.245).

The motivation of Chevallard for proposing a theory much larger in scope than TDS was to move beyond the cognitive program of mathematics education research, namely classical concerns (Gascon, 2003) such as the cognitive activity of an individual explained independently of the larger institutional mechanisms at work which affect the individuals learning. Chevellard's (1985,1992a,b; 1999a,b) writings essentially contend that a paradigm shift is necessary within mathematics education, one that begins within the assumptions of Brousseau's work, but shifts its focus on the very origins of mathematical activity occurring in schools, namely the institutions which produce the knowledge (K) in the first place. The notion of didactical transposition (Chevellard, 1985) is developed to study the changes that K goes through in its passage from scholars/mathematicians→curriculum/policymakers→teachers→students. In other words, Chevellard's ATD is an "epistemological program" which attempts to move away from

the reductionism inherent in the cognitive program (Gascon, 2003). Bosch, Chevellard & Gascon (2005) clarify the desired outcomes of such a program of research:

"ATD takes mathematical activity institutionally conceived as its primary object of research. It thus must explicitly specify what kind of general model is being used to describe mathematical knowledge and mathematical activities, including the production and diffusion of mathematical knowledge. The general epistemological model provided by the ATD proposes a description of mathematical knowledge in terms of mathematical praxeologies whose main components are types of tasks (or problems), techniques, technologies, and theories." (pp.4-5).

It is noteworthy that the use of ATD as a theoretical framework by a large body of researchers in Spain, France and South America resulted in the inception of an International Congress on the Anthropological Theory of Didactics, held biennially since 2005. The aim of this particular Congress and future congresses is to propose a cross-national research agenda and identify research questions which can be systematically investigated with the use of ATD as a framework. In Sriraman & Toerner (2008) several focal points were isolated via historical analysis to suggest ways in which the theoretical differences between the German, French and Italian schools of thought can be bridged (or networked) and made to interact in the present and future. In outlining the differences and similarities between the various positions and schools of thought in these three countries it became apparent that researchers were often entrenched in "ideological" perspectives. Although Jablonka & Bergsten do not get into a discussion of ideologies creeping into the scientificity of a field, it is something that one needs to be aware of. Lerman (2000) explained that these ideological tendencies are a result of the field adopting theoretical frameworks via a process of recontextualization (Bernstein, 1996). In this process "different theories become adapted and applied, allowing space for the play of ideologies" (Lerman, 2000, p.19). However, Jablonka & Bergsten by using Dowling's sociological framework, clarify and elaborate on different modes of classification, modeling and theorizing with respect to relational densities among basic concepts within a theory, as well as levels of discursive saturation (or lack of it) in the four examples of theorizing in mathematics education chosen. One upshot of their article is the preponderance of homegrown theories, the lack of high relational density and intertextuality in the current modes of theorizing. More importantly they point to the tendency of researchers in our field borrowing from terms and concepts fields such as sociology, social anthropology, linguistics etc, without committing to the deeper levels of theorizing that occurs in those fields. Their analysis and observation resonates the observations and critique of Fredric Jameson[1] on postmodernism and consumer society in which "pastiche" and "schizophrenia" pervade ways in which art or other creative endeavors have developed. This metaphor is applicable to the research scene in mathematics education. As Jameson (1983) puts it "pastiche is, like parody, the imitation of a peculiar of unique style, the wearing of a stylistic mask, speech in a dead language…[p]astiche is like blank parody"(p. 114). In mathematics education, too many researchers simply adopt a methodology or a theoretical framework without paying attention to the deeper meanings and layers of connections to previous research or theoretical traditions. This results in what Jameson (1983) calls schizophrenia, namely not relating the present to the past, and simply being content in having "a far more intense experience of any given present of the world" (p.119) without connecting it to meaning derived in the past from similar experiences. Much of the present research in mathematics education

[1] The author would like to thank Claire Payne for bringing Jameson's work to my attention.

appears to be isolationist and following the modes of pastiche and schizophrenia, which does not bode well for any field that claims to have its own identity. The article by Jablonka & Bergsten drives home this point well and warns of the dangers of the distortion caused by the cut-and-paste mentality in borrowing theories and methodologies without adherence to consistency, quality and coherence with established bodies of existing research. . In spite of my criticism of post-modernism rampant within our field in general by using Jameson's words, there are solid efforts in our research community to consolidate existing structures within strong intertextual discursive frameworks to build theory (e.g., Walshaw, 2010). It is up to gatekeepers of our field to uphold standards of research and consolidate efforts at theory building if we are to evolve as a scientific field.

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