Editorial Manager(tm) for Educational Studies in Mathematics
Manuscript Draft

Manuscript Number:

Title: Creativity and Mathematical Problem Posing: An Analysis of High School Students' Mathematical Problem Posing in China and the United States

Article Type: Original Research

Keywords: advanced high school students; cross cultural thinking; geometry; mathematical creativity; novelty; problem posing; problem solving; U.S and Chinese students; rural and urban Chinese students

Corresponding Author: Xianwei Yuan Van Harpen, Ph.D

Corresponding Author's Institution:

First Author: Xianwei Yuan Van Harpen, Ph.D

Order of Authors: Xianwei Yuan Van Harpen, Ph.D;Bharath Sriraman, Ph.D

Abstract: In the literature, problem posing abilities are reported to be an important aspect/indicator of creativity in mathematics. The importance of problem posing activities in mathematics is emphasized in educational documents in many countries, including the United States and China. This study was aimed at exploring high school students' creativity in mathematics by analyzing their abilities in posing problems in geometric scenarios. The participants in this study were from one location in the United States and two locations in China. All participants were enrolled in advanced mathematical courses in the local high school. Differences in the problems posed by the three groups are discussed in terms of quality as well as quantity. The analysis of the data indicated that even mathematically advanced high school students had trouble posing good quality and/or novel mathematical problems. We discuss our findings in terms of the culture and curricula of the respective school systems and suggest implications for directions in problem posing research within mathematics education.

Creativity and Mathematical Problem Posing: *An Analysis of High School Students' Mathematical Problem Posing in China and the United States*

Xianwei Y. Van Harpen

Illinois State University

Bharath Sriraman

The University of Montana

Abstract: In the literature, problem posing abilities are reported to be an important aspect/indicator of creativity in mathematics. The importance of problem posing activities in mathematics is emphasized in educational documents in many countries, including the United States and China. This study was aimed at exploring high school students' creativity in mathematics by analyzing their abilities in posing problems in geometric scenarios. The participants in this study were from one location in the United States and two locations in China. All participants were enrolled in advanced mathematical courses in the local high school. Differences in the problems posed by the three groups are discussed in terms of quality as well as quantity. The analysis of the data indicated that even mathematically advanced high school students had trouble posing good quality and/or novel mathematical problems. We discuss our findings in terms of the culture and curricula of the respective school systems and suggest implications for directions in problem posing research within mathematics education.

Keywords: advanced high school students; cross cultural thinking; geometry; mathematical creativity; novelty; problem posing; problem solving; U.S and Chinese students; rural and urban Chinese students

1. Introduction

Creativity is a buzz word in the 21st century often invoked by policy makers, scientists, industry, funding bodies, and last but not least systems of education worldwide. In fact the vision and/ or mission statements of most school districts in the U.S., and Canada include the word "creativity" in it. Until recently, the last decade of published research includes only a handful of articles focused specifically on mathematical creativity (Leikin, Berman, Koichu, 2010). This is even more amplified within the domain of mathematics education research in their scarcity in articles that tackle giftedness and/or creativity. For instance in *Educational Studies in Mathematics (ESM)*, one of the oldest journals in mathematics education, there are 6 articles that report on studies related to giftedness (high ability) and creativity in the last 40 years starting with Presmeg (1986). In 2010 two papers focused on creativity were published in ESM. Shriki (2010) tried to move beyond creativity as process versus product dichotomy in a study involving 17-prospective mathematics teachers participating in a series of creativity awareness developing activities. This study relied on teacher reflections as a way to understand how creativity awareness can be fostered among teachers. Bolden, Harries & Newton (2010) used questionnaires and semi structured interviews with pre-service teachers in the U.K., to resolve differences between “teaching creatively” versus “teaching for creativity”, the latter of which required a deeper understanding of mathematical conceptual knowledge. Both these papers targeted prospective mathematics teachers. Other than the studies reported by Sriraman (2003, 2004, 2005, 2008, 2009, 2011), there are very few attempts to understand the nature of mathematical creativity in high school students when confronted with novel mathematical tasks. The present article continues this sequence of studies but from a cross cultural viewpoint involving high school students in China and the U.S.

2. The State of the Art- Creativity Research: 1945-2010

In general, there has been an over reliance on the writings of eminent mathematicians as research literature as the norm for those interested in mathematical creativity (Brinkmann & Sriraman, 2009; Sriraman, 2005). Several prominent 20th century mathematicians like Jacques Hadamard, George Polya and Garrett Birkhoff attempted to demystify the mathematician’s craft and explain the mystery of “mathematical” creation (Sriraman, 2005). Hadamard (1945), influenced by Gestalt psychology of his time described the creative process as that of *preparation-incubation-illumination and verification*. Hadamard, like Poincaré (1948) attributed a large part of the creative process to unconscious drives that occurred during the incubatory period before any insight (or the Aha! moment) occurred. This description is generic in a sense and *does explain* the Gestalt or the whole of the creative process in any field per se but is also vague because it offers no insight specifically into the mathematician’s mind. However a number of studies since have specifically examined the role of an incubation period in creative problem solving. Sio & Ormerod (2007) conducted a meta-analytic [1] review of empirical studies that investigated incubation effects on problem solving, and found that incubation is crucial is fostering insightful thinking. Psychologists term this the fatigue hypothesis, i.e., the mind after a period of frenzied and intense activity requires a period of rest to overcome fatigue, and the relaxation during the period of rest results in new insights. According to this report and others similar to it (Vul & Pashler, 2008), understanding the role of the incubatory period may allow us to make use of it more efficiently in task designs to foster creativity in problem solving, classroom learning, and working environments. Mathematics educators try to incorporate incubation periods in classroom activity in temporal pauses during classroom discourse (Barnes, 2000) or extended time periods for problem based learning (Sriraman 2003), and positive incubation results in positive effects in promoting students’ creativity (Sriraman 2004, Sriraman 2005) and this seems to be self evident for mathematicians (Kaufman & Sternberg, 2006). There are recommendations based on this line of research that students should be encouraged to engage in challenging problems and experience this aspect of problem solving (Sriraman, 2008, 2009; Sriraman & Lee, 2011; Stillman et al., 2010).

According to the U.S. Department of Education (1998):

[1] There were 117 studies included in this meta-analysis that most of them support the existence of incubation effects on problem solving.

> Advances in science and technology are playing a greater role in shaping the future of our nation and our world, it is useful to look beyond the general levels of science and mathematics general knowledge and focus on the advanced levels of knowledge of those who are likely to become our next generation of professionals in fields related to mathematics and science. (p. 41)

In Usiskin's (2000) eight-tiered hierarchy of mathematical talent, students who are gifted[2] and/or creative in mathematics have the potential of moving up into the professional realm with appropriate affective and instructional scaffolding as they progress beyond the K–12 realm into the university setting (Sriraman, 2005). Therefore, gifted and/or creative students in mathematics have been of special interest to many researchers in the field of mathematics education. Hadamard (1945) posited the ability to pose key research questions as an indicator of exceptional talent in the domain of mathematics. Krutetskii (1976) and Ellerton (1986) contrasted the problem posing of subjects with different ability levels in mathematics. In Krutetskii's study of mathematical "giftedness", he used a problem-posing task in which there was an unstated question (e.g., "A pupil bought $2x$ notebooks in one store, and in another bought 1.5 times as many."), for which the student was required to pose and then answer a question on the basis of the given information. Krutetskii argued that there was a problem that "naturally followed" from the given information, and he found that high ability students were able to "see" this problem and pose it directly; whereas, students of lesser ability either required hints or were unable to pose the question. In Ellerton's (1986) study, students were asked to pose a mathematics problem that would be difficult for a friend to solve. She found that the "more able" students posed problems of greater computational difficulty (i.e., more complex numbers and requiring more operations for solution) than did their "less able" peers.

According to Jay and Perkins (1997), "the act of finding and formulating a problem is a key aspect of creative thinking and creative performance in many fields, an act that is distinct from and perhaps more important than problem solving" (p. 257). Silver (1997) claimed that inquiry-oriented mathematics instruction which includes problem-solving and problem-posing tasks and activities can assist students to develop more creative approaches to mathematics. It is claimed that through the use of such tasks and activities, teachers can increase their students' capacity with respect to the core dimensions of creativity, namely, fluency, flexibility, and originality (e.g., Presmeg, 1981; Torrance, 1988)

During the past four decades, a large number of international evaluation studies of school mathematics have been conducted. In most of these studies, U.S. students were outperformed by students in many other countries, especially students in East Asian countries. In most cross-national studies involving Chinese and U.S. students' mathematics performance that have been reported (e.g., Husen, 1967; Robitaille & Garden, 1989), Chinese students outperformed their U.S. counterparts. However, mathematics classes in China are often described as not conducive to effective learning (Wong, 2004). In order to understand more fully factors contributing to the outstanding performances of Chinese students, many comparative studies have been conducted

---

[2] We do not enter into a discussion of the definition of mathematical giftedness in this paper. This is a well defined term in the research literature in gifted education. In this paper, the participants by virtue of their enrollment in the advanced mathematical courses were among the high achievers in their respective schools and included students of varying mathematical abilities.

involving U.S. and Chinese students (e.g., Cai, 1995, 1997, 1998; Ma, 1999; Stevenson, 1993; Stevenson & Stigler, 1992; Vital, Lummis, & Stevenson, 1988). But at the same time, it is widely accepted in China that U.S. students are more creative in mathematics than Chinese students (e.g., National Center for Education Development, 2000; Yang, 2007). There are studies showing that U.S. students are better than Chinese students in solving open-ended problems (e.g., Cai & Hwang, 2002) and in posing problems in mathematics (e.g., Cai, 1997, 1998). Therefore, more and more researchers have started looking at the strengths of U.S. students' mathematics learning other than merely focusing on computational skills and routine problem solving.

The purpose of this study was to investigate mathematically advanced high school students' abilities in posing mathematical problems. Participants were junior or senior students (16-18 year olds) in high school. As stated before, very few studies have specifically focused on high school students as opposed to pre-service teachers. By focusing on these age levels, we aim to reveal the students' problem posing abilities at their end of K-12 school education and, therefore, shed light on the students' creativity in mathematics after their K-12 school education. This study reports part of the results from a dissertation study (Yuan, 2009; Yuan & Sriraman, 2011). Among the three tasks in the problem posing test, only one is discussed and reported in detail in this paper. The study is also different to previous studies in the sense that we focus on problem posing as an important but overlooked and least understood aspect of mathematical creativity. In the history of mathematics, there are numerous papers considered as seminal not because they proved a long standing theorem, but because they opened up entirely new areas of mathematical inquiry such as Hewitt's (1948) paper on rings of continuous functions, in addition to Hilbert's (1900) famous 23 problems that shaped the 20th century of mathematics.

2.1 Problem Posing as Creativity

The topic of problem posing has been of interest to the research community in the past decades, however, there is a lack of theory concerning problem posing. In 1982, Dillon claimed that no theory of problem finding had been constructed and that there are several different terms such as problem sensing, problem formulating, creative problem-discovering, problematizing (Allender, 1969; Bunge, 1967; Taylor, 1972). Similarly, Stoyanova and Ellerton (1996) proposed that research into the potential of problem posing as an important strategy for the development of students' understanding of mathematics had been hindered by the absence of a framework which links problem solving, problem posing and mathematics curricula. Building on Guillford's (1950) structure of the intellect, the framework proposed by Stoyanova and Ellerton (1996), classified a problem-posing situation as free, semi-structured or structured. According to this framework, a problem-posing situation is referred to as free when students are asked to generate a problem from a given, contrived or naturalistic situation (see Example 1 below). A problem-posing situation is referred to as semi-structured when students are given an open situation and are invited to explore the structure of that situation, and to complete it by applying knowledge, skills, concepts and relationships from their previous mathematical experiences (see Example 2 below). A problem-posing situation is referred to as structured when problem-posing activities are based on a specific problem (see Example 3 below). All three examples below are taken from Stoyanova (1998). In this study we make use of problem posing activities to study mathematical creativity in advanced high school mathematics students, and compared to existing studies that report on either students identified as gifted, or prospective mathematics teachers, our focus is on groups of students with variations in high mathematical ability.

Example 1: Make up some problems which relate to the right angled triangle. (p. 64)

Example 2: Last night there was a party and the host's doorbell rang 10 times. The first time the doorbell rang only one guest arrived. Each time the doorbell rang after that, three more guests arrived than had arrived on the previous ring. Ask as many questions as you can. Try to put them in a suitable order. (p. 66)

Example 3: Some integers are arranged in the way shown below:

1

2 3 4

5 6 7 8 9

10 11 12 13 14 15 16

17…………………………………… 25

(a) What would be the third number from the left of the 89th row of the accompanying triangular number pattern?

(b) State other meaningful questions. (p. 70)

3. Methodology

3.1 Participants

According to Peverly (2005), even within the one country, different locations in China can vary greatly in terms of culture. Thus this study selected students from two locations in China: Shanghai —an economically well developed city in the south of China and Jiaozhou —a small city that is considered as having strong historical roots in Confucian culture in the north of China. Students from the United States were from Normal[3], Illinois — a mid-western town in the United States. The U.S. students in this study were from two advanced placement Calculus classes and two Pre-Calculus classes. Those students were in the 11th or 12th grade.

In China, high school students usually are divided into two strands, namely, a science strand and an art strand. After the first semester in high school, students choose a strand and are assigned to different classes. Science strand students take more advanced mathematics courses in high

[3] The reader may be surprised to learn that the term "normal" schools for teachers colleges comes from the first such school in Normal, Illinois.

school than arts strand students. In the school in this study in Jiaozhou, in each grade, there are two art strand classes and ten science strand classes, two of which are “express” or accelerated science strand classes. Students in these two express science strand classes were admitted according to their achievement (total score of five subjects, namely, mathematics, Chinese literature, English, physics, and chemistry) in the high school entrance examination of the city, which they took after the 9th grade immediately before they entered the high school. The class in this study is one of the two 12th grade express science strand classes. Similarly to the Jiaozhou participants, the Shanghai participants in this study came from two 11th grade science strand classes and the two classes were also the top two among the ten 11th grade classes in the high school. Therefore, the Chinese participants can be considered as advanced in mathematics.

Although participants in this study were from three very different locations, by choosing students from advanced classes in high schools in each of the three locations, the researchers managed to focus on mathematically advanced high school students in each of the three locations. Initially, 68 Jiaozhou students, 73 Shanghai students, and 77 U.S. students agreed to participate in this study. However, since some students had to miss one or two of the three tests, not all the participants’ test papers were analyzed. In the end, 55 Jiaozhou participants, 44 Shanghai participants, and 30 U.S. participants were present for all the tests. Among the 30 U.S. students, 17 were female and 13 were male; 17 were from Advanced Placement Calculus Course students and 13 were from Pre-Calculus Course students. Among the 44 Shanghai students, 19 were female and 25 were male; all of the Shanghai students were in the 11th grade. Among the 55 Jiaozhou students, 18 were female and 37 were male; all of the Jiaozhou students were in the 12th grade.

3.2 Measures and instrumentation

The measures and instrumentation in this study include a mathematics content test and a mathematical problem-posing test. Both of the two tests were translated into Chinese for the participants in China. Several pilot tests were conducted before it was used for the study.

3.2.1 The mathematics content test: The purpose of the mathematics content test in this study is to measure the participants’ basic mathematical knowledge and skills. Instead of developing a test for this study, the researchers adapted the National Assessment of Educational Progress (NAEP) 12th grade Mathematics Assessment as the mathematics content test because this assessment fits the purpose of this study very well. NAEP is the only nationally representative and continuing assessment of what America's students know and can do in various subject areas (National Center for Education Statistics, 2009). The 2005 mathematics framework focuses on two dimensions: mathematical content and cognitive demand. By considering these two dimensions for each item in the assessment, the framework ensures that NAEP assesses an appropriate balance of content along with a variety of ways of knowing and doing mathematics. The 2005 framework describes four mathematics content areas in high school: number properties and operations, geometry, data analysis and probability, and algebra.

3.2.2 The Mathematical problem-Posing Test. Using Stoyanova and Ellerton’s (1996) framework of mathematical problem posing, three situations were included in the mathematical problem-posing test, namely, free situation, semi-structured situation, and structured situation. To encourage participants to try their best in posing mathematical problems, the following scenario was added in the beginning of the problem posing test.

> Imagine that your school is participating in a problem posing competition in mathematics among all the high schools in town. The schools that generate the most problems or/and the best quality problems will be rewarded. In addition, the students who pose the most number of problems or/and the best quality problems will be rewarded. Last week, student Jenny from another high school created 5 really good problems for each of the three situations below. Jenny also bragged that no one else could do better than she did. Now, try to prove her wrong by making up as many problems as you can. Do not limit yourself to the problems you have seen or heard of - try to think of as many possible and challenging mathematical problems as you can.

The following is the semi-structured problem posing situation. Since this paper only reports data on the semi-structured problem posing situation, only one task will be discussed here.

> In the picture below, there is a triangle and its inscribed circle. Make up as many problems as you can that are **in some way** related to this picture. The problems could also be real-life problems. Again, do not limit yourself to the problems you have seen or heard of - try to think of as many possible and challenging mathematical problems as you can.

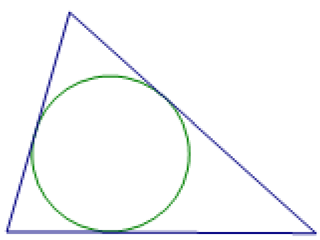

Figure 1. Semi-structured problem posing situation.

4. Data Analysis and Results

Since this paper only reports part of the data from a dissertation (Yuan, 2009), only part of the results is discussed in this section. Since the semi-structured problem posing situation task is the focus of this paper, and this task is about geometry, for the mathematics content test, only scores on geometry tasks are reported.

4.1 The mathematics content test

Among the 50 items in the mathematics content test, 16 are about geometry. The averages of the three groups are 14.8 for Jiaozhou group, 11.4 for Shanghai group, and 12.5 for U.S. group. These results indicate that Jiaozhou students are stronger in geometry than the other two groups.

4.2 The Mathematical problem-Posing Test

The problems posed by the participants in the mathematical problem-posing test were first judged as to their viability. Responses that are not viable were eliminated from further consideration. For example, responses such as “Find the area of the circle” without any other additional information were eliminated. The remaining responses that are viable were scored according to the rubrics in terms of their fluency and flexibility. The rubrics were developed by the researcher following these steps:

1. Typed all the responses into a Microsoft Word document and recorded the frequency with which each of the responses occurred. The responses generated by the three different groups of students were separated so that the researcher could see the differences among the groups.

2. Categorized the responses. The three groups of students’ responses to the mathematical problem-posing test were categorized. It turned out that the categories are not the same for the three samples. For example, Jiaozhou students have a category of dilation but U.S. students and Shanghai students do not have this category. After the responses generated by each group of students were categorized, all the categories were combined to make a common rubric for all the three groups. The total number of viable problems generated by a student is defined as his/her fluency score. The total number of categories that a student’s viable problems involve is defined as his/her flexibility score, and is not necessarily the same as the fluency score.

In scoring the responses generated by the students in this study, two researchers scored the same six copies of test papers and compared the scores. Inter-rater reliabilities were calculated as shown in Table 1.

Table 1
Inter-Rater Correlations on the Mathematical Problem-Posing Test

	Fluency score	Flexibility score
Spearman Correlation	1.00	1.00

A comparison of the means and medians of the students’ scores on the mathematical problem-posing test showed that Jiaozhou students and U.S. students’ fluency and flexibility are similar and are both higher than those of the Shanghai students (See Table 2 and Table 3).

Table 2
Comparison of students’ fluency scores

	U.S. students	Shanghai students	Jiaozhou students
Mean	4.6	2.0	4.9
Median	4	1.5	5

Table 3
Comparison of students' flexibility scores

	U.S. students	Shanghai students	Jiaozhou students
Mean	3.9	1.6	4.1
Median	4	1	4

4.2.1 Viable problems versus nonviable problems. As discussed above, in analyzing the problems generated by the students, problems that lack the information needed to determine solution were excluded from further analysis. Those problems were considered as nonviable problems. Since the numbers of students in each of the three groups were different, the average percentage of non-viable problems generated by the students in each group was calculated with the following division:

$$\frac{\text{Number of nonviable problems}}{\text{Number of nonviable problems + Number of viable problems}}$$

It should be pointed out that the criteria classifying problems as viable or nonviable were based on the researcher's judgment. 31% of the U.S. students' problems, 42% of the Shanghai students' problems, and only 15% of the Jiaozhou students' problems were non-viable problems. Many students posed problems such as "what is the area of the circle" or "what is the area of the triangle" without giving the measures of the radius or the sides. Notice that Jiaozhou students posed the least percentage of non-viable problems, which means that Jiaozhou students tended to give necessary information for the problems to be solvable.

4.2.2 Trivial problems versus nontrivial problems. After the non-viable problems were eliminated, the problems were analyzed for their triviality. For example, the following problem is considered as a trivial problem.

If the diameter of the circle is 32, what is the circumference?

The percentages were calculated by doing the following division:

$$\frac{\text{Number of trivial problems}}{\text{Number of viable problems}}$$

9% of the U.S. students' viable problems, 8% of the Shanghai students' viable problems, and 6% of the Jiaozhou students' viable problems were trivial problems.

4.2.3 Distribution of the categories. In counting the number of problems generated by the students in each group, the same problems generated by the same group of students were counted once. For example, the following two problems were counted as one problem and were categorized as "Given the three sides of the triangle, find the area of the inscribed circle".

Problem 1: Given that the three sides of the triangle are 3, 4, and 5, find the area of its inscribed circle.

Problem 2: Given that the three sides of the triangle are 5, 6, and 7, find the area of the circle.

Table 4 and Figure 2 show the distribution of the different categories posed by different groups of students. Consistently, for the three groups, the biggest two categories are Length and Area. For U.S. students and Jiaozhou students, the Area category is the largest one and the Length category is the second one. For Shanghai students, the Length category is the first and the Area category is the second.

Table 4

Distribution of the categories of the three groups' viable problems

Groups	1	2	3	4	5	6	7	8	9	10	Total
U.S.	1	39	44	3	0	3	6	3	0	7	106
(%)	(0.9)	(36.8)	(42)	(2.8)	(0)	(2.8)	(5.7)	(2.8)	(0)	(6.7)	
Shanghai	0	27	22	2	0	10	1	0	0	9	71
(%)	(0)	(38)	(31)	(2.8)	(0)	(14.1)	(1.4)	(0)	(0)	(12.7)	
Jiaozhou	11	48	61	8	1	24	14	8	10	15	200
(%)	(5.5)	(24)	(30.5)	(4)	(0.5)	(12)	(7)	(4)	(5)	(7.5)	

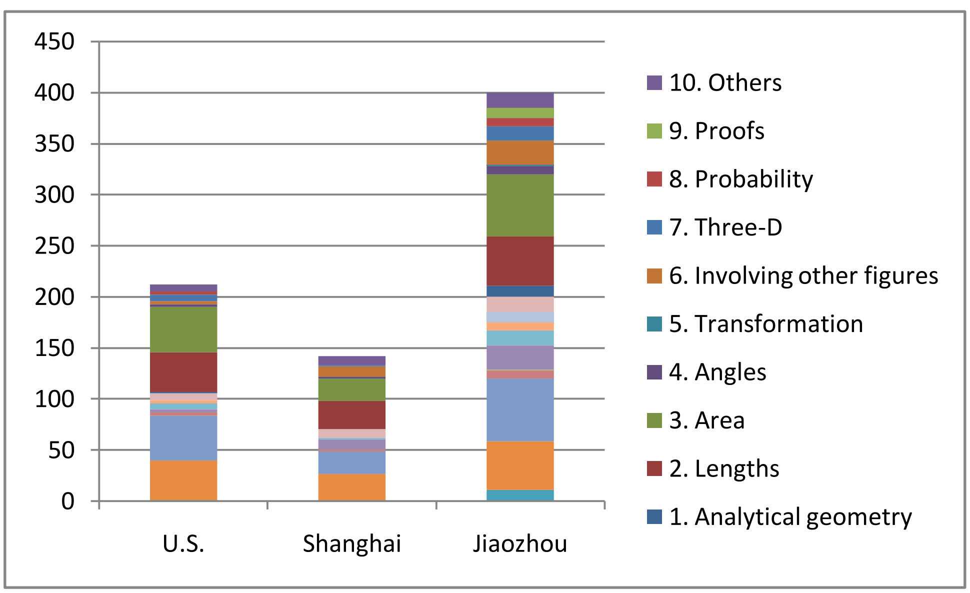

Figure 2. Distribution of the categories of the three groups' viable problems.

However, not all the three groups posed problems for all 10 categories. The U.S. students did not pose problem involving categories 5 and 9, which are Transformation and Proof. The Shanghai students did not pose problems involving categories 1, 5, 8, and 9, which are

Analytical geometry, Transformation, Probability, and Proof. The Jiaozhou students posed problems that covered all the 10 categories.

As to Category 1, Analytical geometry, only 0.9% of the U.S. students' problems were in this category and none of the Shanghai students posed problem of this category. Jiaozhou students, different from the other two groups, posed 11 problems of Analytical geometry category. See the following problem for an Analytical geometry example.

Point B and C are fixed. Point A is movable. |BC|=4 and |AC|-|AB|=2. Find the locus of A.

Another observation is that both Shanghai students and Jiaozhou students posed more problems that involve auxiliary figures (14.1% and 12%); while not many problems of that category were posed by the U.S. students (2.8%). For example,

a) Adding lines: Draw a tangent line of the circle and intercept the triangle at D and E. The vertex of the triangle between D and E is M. Find the range of MD/ME.
b) Adding triangles: If there is an inscribed triangle similar to the original one, find out the ratio of the area of the two triangles.
c) Adding circles: If the triangle is inscribed in another circle, find the ratio of the area of the two circles.
d) Adding quadrilaterals: AB, BC, and AC are given. Build a rectangle in the circle. Find the rectangle with the largest area.

Jiaozhou students posed ten problems of Category 9, Proof, while none of the U.S. students or the Shanghai students did. See the following problem for example.

Given triangle ABC, D, E, and F are the midpoints of AB, BC, and CA. Prove that AD=AF and |AB-AC| = |BE-EC|.

Distribution of sub-categories. Some of the categories are subdivided into subcategories. A closer look at those categories shows that within the subcategories, the distribution is very different, too. For example, within the Lengths category and Area category, seven subcategories appear in each (as shown in Figure 3 and Figure 4). Therefore, although Lengths and Area are the top two categories for all the three groups, the distribution of the sub-categories varies greatly.

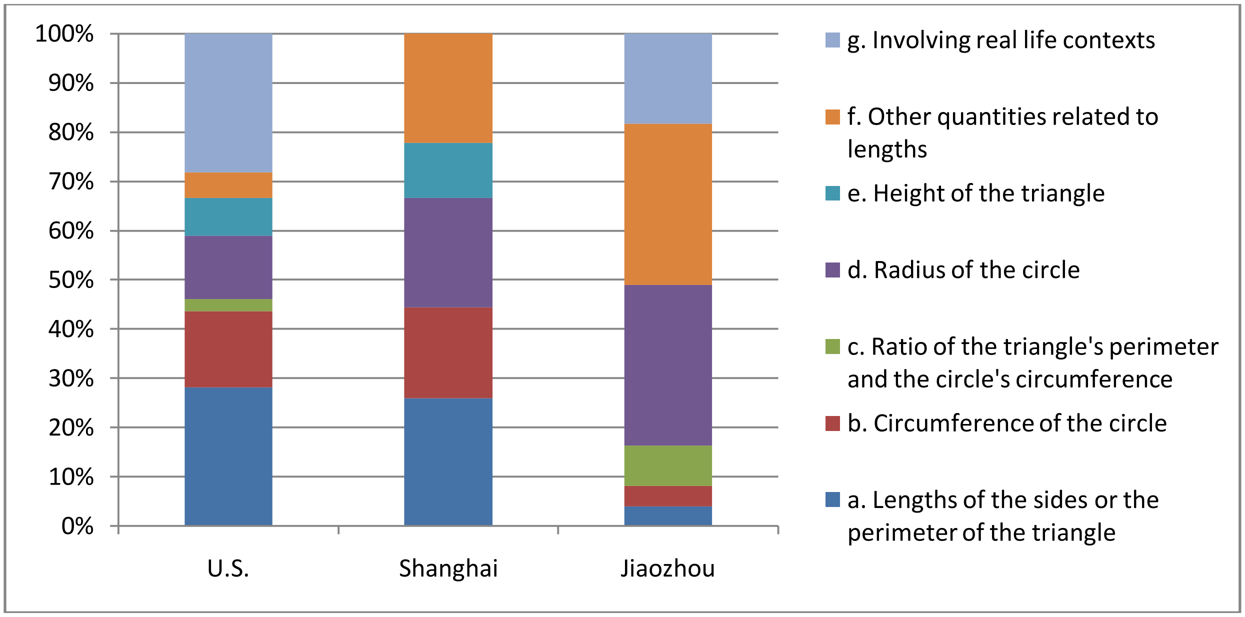

Figure 3. Distribution of subcategories of Category 1—Length.

Figure 3 shows that Jiaozhou students posed much fewer problems of category a, b, and e, which involve finding the lengths of the sides of the triangle, the height and perimeter of the triangle, and the circumference of the circle. Those are more "straight forward" problems. Instead, Jiaozhou students seemed to focus more on category d, f, and g, which involve finding the radius or the circle, other quantities related to lengths, and problems that involve real life contexts. Another finding is that Shanghai students did not pose problems that involve real life contexts. That might indicate the preference in their mathematics instruction.

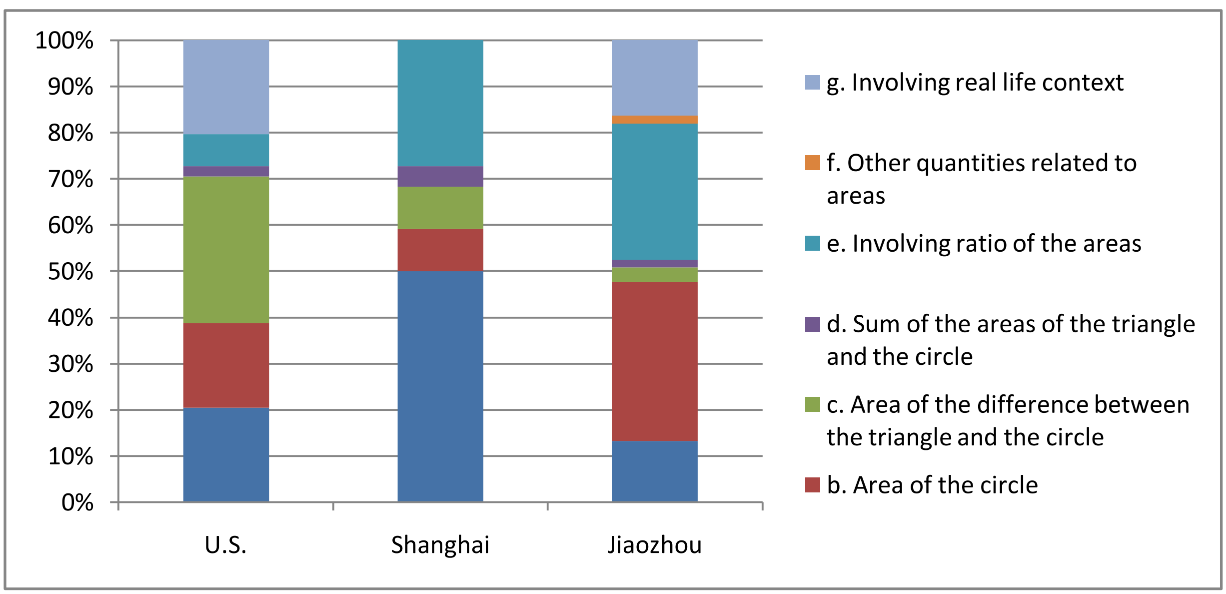

Figure 4. Distribution of subcategories of Category 2—Area.

Figure 4 shows that both Shanghai students and Jiaozhou students posed more than 25% of their problems of category e—problems involving ratio of the two areas; while U.S. students posed more than 30% of their problems of category c—problems involving the difference between the two areas. Again, Shanghai students did not pose problems involving real life contexts.

5. Discussion and Concluding Points

In the problem posing test, despite the fact that students were told that "Do not limit yourself to the problems you have seen or heard of - try to think of as many possible and challenging mathematical problems as you can", students from the three groups all posed problems that are non-viable or trivial. These findings suggest that, despite the emphasis placed on this topic by the educators and governors in the United States and China (e.g., NCTM, 1989, 2000; Mathematics Curriculum Development Group of Basic Education of Education Department, 2002), problem posing is not an established element in instruction in the classrooms yet. The participants in this study were considered as advanced in mathematics based on their school achievement; however, many of them posed non-viable problems or trivial problems. This suggests that students who are good at solving routine mathematical problems or taking routine mathematical tests might not be good at posing mathematical problems. Furthermore, it also indicates that mathematically gifted students are not necessarily mathematically creative ones, and supports the model proposed by Sriraman (2005) that giftedness does not necessarily imply creativity in mathematics, and additional scaffolding is needed to cultivate creativity in the mathematics classroom.

5.1 Influence of Curricula and Culture: In this study, Jiaozhou students posed significantly more problems of the "Analytical Geometry" category than Shanghai students and U.S. students; and Jiaozhou students and Shanghai students posed significantly more problems that involve auxiliary figures than U.S. students. The differences in the distribution of the categories of posed problems suggest that the problems posed by students might be related to students' background mathematical knowledge. In a sense this echoes the claim that basic knowledge and basic skills in mathematics could be highly related to creativity in mathematics (Zhang, 2005), as opposed to viewing basic skills as rote or non-creative. In this study, the two Chinese groups' performances in the mathematics test and the mathematical problem-posing test were very different. This is attributable to differences in the school systems, which makes students' experiences with mathematics very different. Therefore, cross national studies should avoid over generalizing their findings to a certain culture or nation.

The findings from this study indicated that there are differences in the mathematical problem posing abilities among the three groups. The Jiaozhou group posed fewer nonviable problems and fewer trivial problems than the Shanghai group and the U.S. group. Also, Jiaozhou students tended to focus on the specific mathematics involved in the problem posing tasks rather than the contexts and had a clear idea about how they generated the problems. Shanghai students and U.S. students, however, tended to make the contexts fun and rare rather than focusing on the mathematics[4]. This result contradicts those found by Cai and Hwang' (2000), who studied sixth graders' mathematical problem posing and found out that although Chinese students did better in computation skills and solving routine problems, U.S. students performed as well as or better

[4] Qualitative data not reported in the paper were collected and this statement came from them.

than those Chinese students in problem posing tasks. Again the implication is that students' problem posing abilities might be affected by their mathematical knowledge. Students from Jiaozhou in this study scored much more highly than the other two groups in the mathematics content test and the Jiaozhou students also did much better in the mathematical problem-posing test. The superior performances of Jiaozhou students in the mathematics content test and the mathematical problem-posing test suggest that there might be some correlation between the two.

In fact, in China, educators (e.g., Zhang, 2005) have reflected on the mathematics education in the past and claimed that the basic knowledge and basic skills in mathematics might or might not be highly related to creativity in mathematics, but there is definitely a kind of balance between them. Wong (2004, 2006) summarized the characteristics of the Confucian Heritage Culture (CHC) learners' phenomenon and pointed out that the Chinese students' focus on the basics might be related to the ancient Chinese tradition of learning from "entering" to "transcending the way". Wong's observation echoes that of Gardner' (1989) that imitating the master is the starting point of the path to becoming the master one day. Future research in the correlations between mathematics content knowledge and mathematical problem posing will help to validate the observations by Wong and Gardner.

5.2 Future directions for problem solving research: Problem solving research has often been criticized as having reached an impasse (English & Sriraman, 2010). Polya's (1945) oft cited work provided the impetus for the ensuing research that took place in the following decades, which included focus on novice versus expert problem solving (e.g., Anderson, Boyle, & Reiser, 1985), problem solving strategies and meta-cognitive processes (e.g., Lester, Garofalo, & Kroll, 1989), and problem posing (English, 1997; Walter & Brown, 1983). However problem posing has not received the same attention as the other aforementioned areas. Problem posing has been researched to an extent with younger learners in the context of combinatorial situations (Sriraman & English, 2004) and more recently problem posing has come to the foreground in the area of mathematical modeling in the elementary and middle grades (English, 2007), but in general has received scant attention as an aspect of mathematical creativity. This study indicates the necessity for more inquiry into this line of research within mathematics education, in which learners are presented with problem posing opportunities in different areas of school mathematics, with the goal of stimulating creativity in intra-mathematical thinking as demonstrated by the Jiaozhou students, as well as diverse mathematical thinking to generate problems that are contextually different.

Acknowledgements

The first author would like to thank her dissertation committee members for their patience and expert guidance. They are Dr. Norma Presmeg, Dr. Nerida Ellerton, Dr. McKenzie Clements, Dr. Bharath Sriraman, and Dr. John Rugutt.

References

Allender, J. S. (1969). A study of inquiry activity in elementary school children. *American Educational Research Journal, 6,* 543-558.

Anderson, J. R., Boyle, C. B., & Reiser, B. J. (1985). Intelligent tutoring systems. *Science*, 228, 456-462.

Barnes, M. (2000). Magical moments in mathematics: Insights into the process of coming to know. *For the Learning of Mathematics*, 20(1), 33-43.

Birkhoff, G.D. (1956): Mathematics of Aesthetics. In: Newman, J.R. (ed.): *The World of Mathematics,* Vol. 4, 7th edition. New York: Simon and Schuster, pp. 2185-2197.

Birkhoff, G. D. (1969). Mathematics and psychology. *SIAM Review, 11*, 429-469.

Bolden, D.S., Harries, T.V., Newton, D.P. (2010). Pre-service primary teachers' conceptions of creativity in mathematics. *Educational Studies in Mathematics*, 73(2), 143-157.

Brinkmann, A. & Sriraman, B. (2009). Aesthetics and creativity: An exploration of the relationship between the constructs. In B.Sriraman & S. Goodchild (Eds) *Festschrift celebrating Paul Ernest's 65th Birthday* (pp. 57-80) Information Age Publishing, Charlotte, NC.

Bunge, M. (1967). *Scientific research* (Vol. 1). Berlin, NY: Springer-Verlag.

Cai, J. (1995). A cognitive analysis of U.S. and Chinese students' mathematical performance on tasks involving computation, simple problem solving, and complex problem solving. *Journal for Research in Mathematics Education monograph series.* Reston, VA: National Council of Teachers of Mathematics.

Cai, J. (1997). Beyond computation and correctness: Contributions of open-ended tasks in examining U.S. and Chinese students' mathematical performance. *Educational Measurement: Issues and Practice, 16*(1), 5-11.

Cai, J. (1998). An investigation of U.S. and Chinese students' mathematical problem posing and problem solving. *Mathematics Education Research Journal, 10*(1), 37-50.

Cai, J., & Hwang, S. (2002). Generalized and generative thinking in U.S. and Chinese students'mathematical problem solving and problem posing. *Journal of Mathematical Behavior, 21*(4), 401-421.

Ellerton, N. F. (1986). Children's made-up mathematics problems: A new perspective on talented mathematicians. *Educational Studies in Mathematics, 17*, 261-271.

English, L.D. (1997). The development of 5th grade students problem-posing abilities, *Educational Studies in Mathematics, 34*, 183-217.

English, L. D. (2007). Complex systems in the elementary and middle school mathematics curriculum: A focus on modeling. In B. Sriraman (Ed.), *Festschrift in Honor of Gunter Torner*. *The Montana Mathematics Enthusiast*, (pp. 139-156). Information Age Publishing, Charlotte, NC.

English, L.D., & Sriraman, B. (2010). Problem solving for the 21st century. In B. Sriraman & L. D. English (Eds.), *Theories of mathematics education: Seeking new frontiers* (pp. 263-285). Advances in Mathematics Education, Series: Springer Berlin/Heidelberg.

Gardner, H. (1983). *Frames of mind: The theory of multiple intelligences*. NY, Basic Books.

Guilford, J.P. (1950). Creativity. *American Psychologist*, 5, 444-454

Hewitt, E. (1948). Rings of real-valued continuous functions. *Transactions of the American Mathematical Society*, 64, 45-99.

Hilbert, D. (1900). Mathematische Probleme: Vortrag, gehalten auf dem internationalen Mathematiker-Congress zu Paris 1900. *Göttingen Nachrichten*. 253-297.

Husen, T. (1967). *International study of achievement in mathematics: A comparison of twelve countries* (Vol. 1-2). New York: Wiley.

Jay, E. S., & Perkins, D. N. (1997). Problem finding: The search for mechanism. In M. A., Runco, (Ed.), *The creativity research handbook* (Vol. 1, pp. 257-293). Cresskill, NJ: Hampton Press.

Kaufman, J. C., & Sternberg, R. J. (Eds). (2006). *The International Handbook of Creativity*. Cambridge: Cambridge University Press.

Krutetskii, V. A. (1976). *The psychology of mathematical abilities in school children.* Chicago: The University of Chicago Press.

Leikin, R., Berman, A., Kocihu, B., (2010). *Creativity in Mathematics and the Education of Gifted* Students, Sense Publishers, Rotterdam, The Netherlands.

Lester, F. K., Garofalo, J., & Kroll, D. L. (1989). Self-confidence, interest, beliefs, and metacognition. Key influences on problem solving behavior. In D. B. McLeod & V. M. Adams (Eds.), Affect and mathematical problem solving: A new perspective (pp. 75-88). New York: Springer-Verlag.

Lester, F. K. & Kehle, P. E, (2003). From problem solving to modeling: The evolution of thinking about research on complex mathematical activity. In R. A. Lesh & H. M. Doerr (Eds.), Beyond constructivism: Models and modeling perspectives on mathematics problem solving, learning, and teaching (pp. 501-518). Mahwah, NJ: Lawrence Erlbaum Associates.

Ma, L. (1999). *Knowing and teaching elementary mathematics: Teachers' understanding of fundamental mathematics in China and the United States*. Mahwah, N.J.: Lawrence Erlbaum Associates.

National Center for Education Development. (2000). *Report on developing students' creativity and teacher training in the U.S. [关于美国创造性人才培养与教师培训的考察报告]*. Retrieved March, 25, 2008, from http://www.moe.edu.cn/moe-direct/fazhanyjzx/187.htm

National Center for Educational Statistics. (2009). *The National Assessment of Educational Progress Overview*. Retrieved August 28, 2009, from http://nces.ed.gov/nationsreportcard/mathematics/

Peverly, S. (2005). Moving past cultural homogeneity: Suggestions for comparisons of students' educational outcomes in the United States and China, *Psychology in the Schools, 42*(3), 241-249.

Poincaré, H. (1948). *Science and Method*. Dover: New York.

Polya, G. (1945). *How to solve it*. Princeton, NJ: Princeton University Press.

Presmeg, N.C. (1986). Visualization and mathematical giftedness. *Educational Studies in Mathematics*, 17(3), *297-311*.

Robitaille, D. E., & Garden, R. A. (1989).*The IEA study of mathematics 11: Contexts and outcomes of school mathematics* .New York: Pergamon.

Shriki, A. (2010). Working like real mathematicians: developing prospective teachers' awareness of mathematical creativity through generating new concepts. *Educational Studies in Mathematics*, 73(2), 159-179.

Silver, E. A. (1997). Fostering creativity through instruction rich in mathematical problem solving and problem posing. *ZDM-The International Journal on Mathematics Education*, 97(3), 75-80.

Sio, U. N., & Ormerod, T. C. (2007). Does incubation enhance problem solving? A meta-analytic review. *Psychological Bulletin*, 135(1). 94–120.

Sriraman, B (2003) Can mathematical discovery fill the existential void? The use of conjecture, proof and refutation in a high school classroom (feature article). *Mathematics in School*, 32(2), 2-6.

Sriraman, B. (2004). Discovering a mathematical principle: The case of Matt. *Mathematics in School*, 33(2), 25-31.

Sriraman, B. (2005). Are giftedness & creativity synonyms in mathematics? An analysis of constructs within the professional and school realms. *The Journal of Secondary Gifted Education*, 17, 20–36.

Sriraman, B. (2008). *Creativity, Giftedness and Talent Development in Mathematics*. Information Age Publishing, Charlotte, NC.

Sriraman, B. (2009). The characteristics of mathematical creativity. *ZDM- The International Journal on Mathematics education,* 41(1&2), 13-27.

Sriraman, B., & English, L. (2004). Combinatorial Mathematics: Research into practice. Connecting Research into Teaching. *The Mathematics Teacher,* 98(3),182-191.

Sriraman, B., & Lee, K., (2011). *The Elements of Giftedness and Creativity in mathematics*. Sense Publishers, Rotterdam, The Netherlands.

Stevenson, H. W. (1993). Why Asian students still outdistance Americans. *Educational Leadership*. February, 63-65.

Stevenson, H. W., & Stigler, J. W. (1992). *The learning gap: Why our schools are failing and what we can learn from Japanese and Chinese education*. New York: Summit Books.

Stillman,G., Kwok-cheung, C.,Mason,R., Sheffield, L., Sriraman, B., & Ueno, K (2009). Classroom Practice: Challenging mathematics classroom practices. In E. Barbeau & P. Taylor. (Eds). *ICMI Study 16 Volume on Mathematical Challenges* (pp. 243-284), Springer Science & Business.

Stoyanova, E. (1998). Problem posing in mathematics classrooms. In N. Ellerton & A. McIntosh (Eds.) *Research in mathematics education in Australia*: *A contemporary perspective* (pp. 164-185). Perth: Edith Cowan University.

Stoyanova, E., & Ellerton, N. F. (1996). A framework for research into students' problem posng in school mathematics. In P. C. Clarkson (Ed.), *Technology in mathematics education* (pp. 518-525). Mathematics Education Research Group of Australasia. The University of Melbourne.

Taylor, I. A. (1972). *A theory of creative transactualization: a systematic approach to creativity with implications for creative leadership.* Occasional Paper, Buffalo, NY: Creative Education Foundation, No.8.

Torrance, E. P. (1988).The nature of creativity as manifest in its testing. In R. J. Sternberg (Ed.), *The nature of creativity: Contemporary psychological perspectives* (pp. 43-75). New York: Cambridge University Press.

U.S. Department of Education. (1998). *Pursuing excellence: A study of U.S. twelfth-grade mathematics and science achievement in international context, NCES 98-049.* Washington, DC: U.S. Government Printing Office. National Center for Education Statistics.

Usiskin, Z. (2000). The development into the mathematically talented. *Journal of Secondary Gifted Education, 11*(3), 152-162.

Vital, D. H., Lummis, M., & Stevenson, H. W. (1988). Low and high mathematics achievement in Japanese, Chinese, and American Elementary-school children. *Developmental Psychology, 24*(3), 335-342.

Vul, E., & Pashler, H. (2009). Incubation benefits only after people have been misdirected. *Memory and Cognition*, 35(4), 701–710.

Brown, S. & Walter, M. (1983). *The Art of Problem Posing.* Philadelphia: Franklin Press

Wong, N. Y. (2004). The CHC learner's phenomenon: Its implications on mathematics education. In L. Fan, N. Y. Wong, J. Cai & S. Li (Eds.), *How Chinese learn mathematics: Perspectives from insiders* (pp. 503–534). Singapore: World Scientific.

Wong, N. Y. (2006). From "Entering the Way" to "exiting the Way": in search of a bridge to span "basic skills" and "process abilities". In F. K. S. Leung, G.-D. Graf & F. J. Lopez-Real (Eds.), *Mathematics education in different cultural traditions: The 13th ICMI study* (pp. 111-128). New York: Springer.

Yang, G. (2007). A comparison and reflection on the school education of China and the U.S. [中美基础教育的比较与思考]. Retrieved March 25, 2007, from http://www.sm.gov.cn/bmzd/jcck/200111/Findex.htm

Yuan, X. (2009). *An exploratory study of high school students' creativity and mathematical problem posing in China and the United States.* Unpublished doctoral dissertation. Illinois State University.

Yuan, X., & Sriraman, B. (2011). An exploratory study of relationships between students' creativity and mathematical problem posing abilities—Comparing Chinese and U.S students. In B.Sriraman, K. Lee (Eds.), *The Elements of Creativity and Giftedness in Mathematics* (pp.5-28). Rotterdam, Netherlands: Sense Publishers.

Zhang, D. (2005). *The "two basics": Mathematics teaching in Mainland China.* Shanghai, China Shanghai Educational Publishing House.