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EXTRAPOLATION OR EXPANSION?: CHARACTERISTICS OF IMPACT EXPOSED IN A LONGITUDINAL STUDY OF ONE SCHOOL'S PARTICIPATION IN SUCCESSIVE MATHEMATICS TEACHING DEVELOPMENT PROJECTS.

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Abstract:

Communities of practice theory models development as extrapolation, whereas cultural historical activity theory models development as expansion. This paper explores the differences between these models and their underlying principles. A longitudinal case study of one school team that worked within a series of mathematics teaching developmental research projects over a period of six years is analysed to expose evidence of development, which is examined for indications of extrapolation and expansion. The projects were designed on principles of communities of inquiry, which it is claimed radically transform community of practice theory, entailing a shift into the critical paradigm. The paper engages with the analysis and synthesis of a large volume of qualitative data that accrues in teaching development projects.

Extrapolation or Expansion?: Characteristics of Impact Exposed in a Longitudinal Study of One School's Participation in Successive Mathematics Teaching Development Projects.

Extrapolation and *expansion* are terms used to describe development in communities of practice theory (CPT)[1] and cultural historical activity theory (CHAT) respectively. The purpose of this paper is to illuminate these terms and demonstrate how they are of value in the analysis and interpretation of events within mathematics teaching developmental research. Although both are sociocultural theories CPT and CHAT have significant differences, especially regarding the notion of goal directedness and personal agency. In the first part of this paper these differences are used to argue that development as extrapolation is characterised by roots in prior experience, whereas expansion is characterised by creative innovation that is essentially oriented towards the future. The second part of the paper describes mathematics teaching developmental research projects led by a team based in southern of Norway. The projects are founded on principles of community and inquiry; that is CPT extended to accommodate inquiry, in the learning and teaching of mathematics, together with inquiry as a tool in the development of teaching practice. It is asserted that this extension and the development of a community of inquiry is not a mere modification of CPT but constitutes a paradigm shift.

A longitudinal case study of one school teacher team that has worked within the projects over a period of six years is used as an example, to explore and expose signs of development and examine these signs for evidence of extrapolation and expansion. The paper concludes with reflection on what can be learnt from these experiences.

Extrapolation and Expansion as Models of Development

The notion of 'community of inquiry' (COI) as a frame for the teaching and learning mathematics, and for the development of teaching mathematics, is well established, theoretically and empirically (e.g. Berg, 2009; Cochrane-Smith & Lytle, 1999; Elbers, 2003;

Elbers & Streefland, 2000; Goos, 2004; Graven, 2004; Jaworski, 2006a, 2006b; Schoenfeld, 1996; Wells, 1999). On the surface it might not appear surprising that many mathematics teaching developmental research projects that set out to create a COI use CPT, (Lave, 1988; Lave & Wenger, 1991; Wenger, 1998) as a framework to conceptualise community, practice, participation and other ideas implicit in COI. Wenger's (1998) analysis of community, practice, and identity has been demonstrated to be relevant and useful. However, some of those who have used CPT in this manner have found it necessary to 'extend' the theory. Graven (2004), for example, adds a dimension of 'confidence' in the panoply of teachers' professional knowledge, experience and practice. Also, indeed crucial to the development of the argument in this paper, Jaworski (2006a) extends CPT by explaining that the introduction of inquiry transforms 'alignment', one of Wenger's (1998) 'modes of belonging', into *critical alignment*. Jaworski asserts that inquiry is 'a tool for developing practice' (Jaworski, 2005, p. 103), and that a community of inquiry is not a 'type' of community of practice; it is not meant that 'inquiry' is the practice. As will be argued below COI theory is different; CPT is not merely 'extended' but radically transformed by the introduction of inquiry. Moreover, it is possible to produce a mapping from Jaworski's (2006a) explanation of critical alignment to Friere's (1972) account of conscientization and thus argue that this transformation constitutes a paradigm shift (Goodchild, 2008).

Critical inspection of CPT has drawn attention to several 'weaknesses' when the theory is applied to mathematics teaching and development. However, before going any further it must be noted that these are not weaknesses of CPT per se, they concern what adaptations of CPT are necessary before it can be used it as an adequate theoretical perspective for teaching developmental research. Three key issues have attracted attention, mediation, goals and agency, which will be explained below. However, first we note that Kanes and Lerman (2008) draw attention to characteristic differences in the way fundamental

concepts are explained within different articulations of CPT, in particular Wenger's (1998) 'Communities of Practice', and Lave and Wenger's (1991) 'Situated Learning'. Consequently, a discussion that is based on the notion of a single, unified CPT could be fundamentally flawed – especially if it also includes other original contributions such as Lave (1988, 1996), and Scribner and Cole (1981). The assumption here is that there is sufficient common ground between these theories to establish a basis for the present discussion.

CPT lacks a theory of mediation and teaching (Graven, 2004; Jaworski, 2007; Kanes & Lerman, 2008). Graven claims that Wenger (1998) 'undermines the role of teaching' (Graven, 2004, p. 185), and Jaworski (2007), uses Lave's (1996) argument of 'teaching as learning in practice' to assert that CPT 'is unhelpful in characterizing or analyzing mathematics teaching' (Jaworski, 2007, p. 1691). Nevertheless Jaworski still acknowledges that CPT is useful as a framework for 'characterizing and analyzing learning: for example teachers' learning of mathematics teaching' (2007, p. 1691). More fundamentally, Kanes and Lerman (2008) assert that CPT lacks a theory of mediation and a clear theorisation of tools and artefacts, as a result, they argue, the theory is difficult to apply to developmental and change processes. Berg (2009), however, associates Wenger's 'negotiation of meaning' with the notion of 'mediation of meaning' (Kozulin, 2003).

The theorisation of goals in CPT is inimical to problematising purposeful development. Scribner and Cole (1981) offer a definition of practice as "a recurrent, goal-directed sequence of activities" (p. 236), that is, it is the practice which is goal directed rather than the participants within the practice. Lave (1988) asserts that a study of goals is not appropriate because goals can be changed in the course of action and only known in retrospect. This can be understood in the context of teaching mathematics, a teacher might share with students that the goal of the lesson is that the students will develop an understanding of *derivative*, say, but for the students who do not yet possess any knowledge

of the derivative concept the learning goal cannot make sense. Alternatively, in the science classroom, students might engage in some experimental activity explained by the teacher, who has a clear learning goal that the students will reach as they later reflect on the outcomes of the experiment. It is only after the event and due reflection that students can be aware of the goal of their actions. However, it may be reasonable for students to have as a learning goal the resolution of a mathematical problem embedded in an adidactical situation created by the teacher, with the expectation that the milieu, also part of the teacher's creation, will provide sufficient feedback to confirm that the problem has indeed been solved (Brousseau, 1997). That is, the goal is the resolution of tension between knowing and not-knowing represented by the problem and new insight that enables the resolution, rather than an image of what the resolution might be. Nardi (1996) also draws attention to the treatment of goals in CPT as one of the defining characteristics that separates the theory from other sociocultural theories. From the perspective of teaching development, the possibility of goal directed action is crucial; developmental activity is not a process of thrashing around in the dark in the hope of hitting on something worthwhile, it is rather the purposeful integration of theory, thought experiment and practical engagement with the intention of achieving a desired outcome.

Closely connected to the position of goals in CPT is that of personal agency. Wenger (1998) explains that CPT, in his development, aims at a central position between the extremes of social theory that focus on social structures and individual agency, so perhaps it is not surprising that he does not linger on this theme. However, inquiry and critical reflection are tools intended to enhance and direct an individual's capacity to act on their situation. Moreover, Greene (1988), taking Dewey's writings as a starting point, argues that inquiry is a means of the individual breaking free from the constraints of socially reproductive practices. Agency is not only about the choices that are entailed in aligning (or not) to practice, from a

critical perspective it is about exposing and challenging those aspects of practice that interfere with development.

Notwithstanding the foregoing, CPT is useful in explaining a form of development in participation in practice as will be demonstrated in the analysis of the longitudinal case study that forms the empirical basis of this paper. It is possible to infer a theory of development from Wenger's (1998) discussion of learning, identity and modes of belonging. Wenger lists twelve principles of learning within a social perspective (1998, pp. 226-228), here the focus is directed on the principle of learning as a transformation of identity and 'ability to participate in the world by changing all at once who we are, our practices, and our communities' (1998, p. 227). Identity is about the relationship between the individual and the practice and focuses attention on the individual (Wenger, 1998), Wenger explains that identity formation and learning are a matter of three modes of belonging: *engagement*, *imagination* and *alignment*. Engagement relates to participation in a practice as a member of the community of practice and alignment is about 'the coordination of … energies, actions and practices. … we do what it takes to play our part' (1998, p. 179). Thus engagement and alignment are about the individual adapting her/himself to the practice. Imagination, however, is a mode of belonging that enables an individual to perceive her/himself in an extended historical, spatial and social context of practice, engagement and alignment than actually experienced. 'My use of the concept of imagination refers to a process of expanding our self by transcending our time and space and creating new images of the world and ourselves' (Wenger, 1998, p. 176). The word 'expansion' will be used in a different context below and it will be necessary to mark a distinction between the way the word is used in each context, hence attention is drawn to the object of expansion, 'self', and the outcome, 'new images of the world and ourselves'. Wenger also uses the word 'extrapolation', because imagination, as a mode of belonging, is

about the creation of new images based on what is known and experienced, either directly or vicariously through other participants.

In the general sense, 'extrapolation' is based on an assumption that there is an underlying rule determining the behaviour of an object and that the rule can be determined from using all known data, hence it is possible to predict beyond experiences. An example from the realm of mathematics reveals the risk if extrapolation is taken as a means of prediction is the well known 'circle and spots' problem often used in school mathematics classes to draw attention to the *fallibility of intuition*. Spots are placed successively on the circumference of a circle; as each spot is marked it is joined to all existing spots so that no more than two chords intersect at any given point; the number of regions within the circle are counted as each new spot is added and chords drawn. Initially counting the regions for 1, 2, 3, 4, and 5 spots, leads to the sequence 1, 2, 4, 8, 16, … By extrapolation one might predict that the next members of the sequence are 32, 64, 128, etc. because a rule ($n \rightarrow 2^{n-1}$) has been be assumed. However, when the sixth spot is added the region count is 31. The assumed 'rule' was not correct. In this mathematical example the assumed rule and the prediction can be tested against a concrete situation. In the context of social practice however, engagement and alignment lead to the assumption of rules which extrapolation is likely to confirm because the extension of self, practice or community inevitably 'fits' with existing conceptions. Extrapolation is, by its very nature reproductive rather than transformative. Thus, the modes of belonging: engagement, imagination and alignment, lead to changes within practice that reproduce underlying assumptions of participation. Development that seeks fundamental transformation of practice needs a critical dimension that challenges the assumed rules. This critical dimension is provided through the addition of *inquiry*, which transforms 'alignment' into *critical alignment* (Jaworski, 2006a).

The argument presented here is not that CPT is wrong, indeed quite the contrary, CPT offers a theoretical model of practice, and development within practice that explains some types of change in teaching. CPT is useful where teaching assimilates new ideas, materials or tools into existing practice rather than being transformed as the practice accommodates and adapts to new possibilities; examples can be found in (Cohen, 1990; Cuban, 2001; Hennessy, Ruthven, & Brindley, 2005). However, mathematics teaching developmental research seeks to go beyond the reproduction of practice through the development of critical alignment; a theory that can be applied to such transformation is required; CHAT fulfils this purpose.

CHAT brings to the foreground mediation, goal directed action and agency (Roth & Lee, 2007); thus it should not be surprising that it is used as a theoretical framework for developmental activity in teaching (e.g. Engeström, 1994; Gordon & Fittler, 2004; Roth & Tobin, 2005). Very briefly, CHAT theorises goal directed actions (of individuals or groups) as the substance of historically enduring object oriented human activity. Actions are mediated, that is actions are both enabled and gain meaning through, cultural tools and artefacts – the most basic of these being language. Other sources of mediation are the community, rules, and division of labour, which are the social and cultural context of the activity. Engeström (1987) presents an extended version of Vygotsky's (1978) notion of mediated action with the person or group in goal directed action on some object in dialectical relationship with mediating artefacts, rules, community and division of labour, thus representing an 'extended activity system'. The immediate purpose here is to outline a model of learning and development as 'expansion' based on Engeström's writing (1987, 1999, 2001) , rather than provide a thorough introduction to CHAT, which can be found elsewhere, for example Roth and Lee (2007).

Expansion refers to transformation of the entire activity system (Engeström, 2001), of one or more elements within the system and/or the dialectical relations between the elements. (Note, 'elements' is used here to refer to characteristic dimensions of activity, not component

parts that are assembled into an activity. Leont'ev asserts "activity is the nonadditive molar unit of life … activity is not a reaction or aggregate of reactions, but a system with its own structure, its own internal transformations, and its own development" (1979, p. 46). Expansion is about transformation rather than reproduction, as Engeström asserts "Expansive learning activity produces culturally new patterns of activity. Expansive learning at work produces new forms of work activity" (2001, p. 139). Expansion is explained by Engeström as taking place in cycles of activity which are successively dominated by internalisation and externalisation. Consider the trajectory of a teacher's career. Initially the novice mathematics teacher is in a phase of internalization as s/he learns the basic craft of the practice, how to interpret the curriculum, how to manage classes, how to respond to students' questions, how to organise learning experiences, how to prepare students for examinations and so on. In the first instance these may be learned in the form of responsive behaviours, but alongside the teacher also begins to internalize the underlying principles and structures of teaching mathematics in school that give meaning to the regular actions of the craft, and incidentally provide a foundation for personal development in the form of extrapolation. However, the internalisation of the basic principles and structures expose long standing contradictions and tensions, or double binds within the activity (Engeström, 2001). Following the example of mathematics teaching a double bind might be experienced from the apparent contradiction of a demanding curriculum, high stakes examinations, classes including the full attainment range, pressure from teachers' peers to conform to school norms, public and political pressure expressed through the media to improve student performance, and advice from professional groups to teach for understanding. Extrapolation of participation within the practice will not resolve this double bind; the resolution requires culturally new patterns of work activity, that is, expansion.

Externalisation begins in the form of discrete, creative individual innovations, one might conceive of these as experimental actions, external to the activity system; Cole and Engeström (1993) refer to these as 'violations'. As experimental activity produces desirable outcomes so the innovations become internalised in the activity and a new phase of internalisation begins. Whereas internalisation might be accompanied or followed by extrapolation as a rather natural feature of participation, expansion as Engeström presents it, requires mediation. In a COI the mediation is provided by inquiry. Cole and Engeström assert:

> The new activity structure does not emerge out of the blue. It requires reflective analysis of the existing activity structure – participants must learn to know and understand what they want to transcend. And the creation of a new activity system requires the reflective appropriation of advanced models and tools that offer ways out of the internal contradictions. (1993, p. 40).

Expansion as a model of development has been worked out through CHAT, which is based upon a notion of goal directed action. However, whereas individual innovations may be goal directed towards resolving the internal contradictions of the activity, the eventual outcome of an expansive cycle cannot be predicted in advance (Cole & Engeström, 1993). Ironically the converse may be true of extrapolation, given its tendency to confirm and reproduce assumptions of structures and practices. The differences between extrapolation and expansion as forms of development are summarised in table 1.

To demonstrate how extrapolation and expansion are both of value in explaining outcomes of mathematics teaching development activity we use two instantiations of development of a school mathematics teacher team that participated in a series of projects. We first set out the underlying methodologies, and then outline the data upon which the instantiations of development are constructed.

Extrapolation	**Expansion**
Contingent	Goal directed
Reactive	Experimental
Progressive	Innovative
Incremental	Creative
Conforming to and confirming rules and structures of practice	Critically-challenging rules and structures of practice
Reproductive	Transformative
Safe	Risky
Characterised by assimilation and adoption	Characterised by accommodation and adaptation
Example: Cohen, (1990)	Example: Liljedahl, (2010)

Table 1. Differences between extrapolation and expansion as forms of development

Methodology

We report episodes from a school team that participated in three mathematics teaching developmental research projects led by a team of didacticians[2] at the University of Agder over the period 2004-2010. Not all the teachers took part in all three projects; however the continuous involvement of the school opens the possibility of a longitudinal case study that can be explored for evidence of development. Two projects ran in parallel over the period 2004-2007: Learning Communities in Mathematics (LCM) and ICT and Mathematics Learning (ICTML). Following on from these a 'binary' project ran for the period 2007-2010, Teaching Better Mathematics and Learning Better Mathematics (LBM/TBM). TBM focuses principally on researching teaching development and is mainly the concern of didacticians. LBM focuses on the development of teaching and learning mathematics in schools and kindergartens and is led collaboratively by representatives of school authorities, teachers from

participating schools and didacticians.[3] The common theoretical principles and developmental goals of the projects are outlined below and afterwards, the essential differences between the projects will be explained.

The projects have shared three basic associated developmental goals: the improvement of students' learning opportunities and performance in mathematics; the development of mathematics teaching; and the development of a didactical environment in which teachers and didacticians collaborate in the development of teaching and learning mathematics in school (Jaworski, 2006b). Each of these developmental goals has an associated research question that relates to how the desired developmental goal may be achieved. In this paper a more sharply focused question is addressed:

> *What can be learned about the characteristics of development of mathematics teaching from a school team that has participated in the projects over a period of six years?*

Developmental research entails interconnected cycles of research and development (Goodchild, 2008, Gravemeijer, 1994); this is illustrated in figure 1.

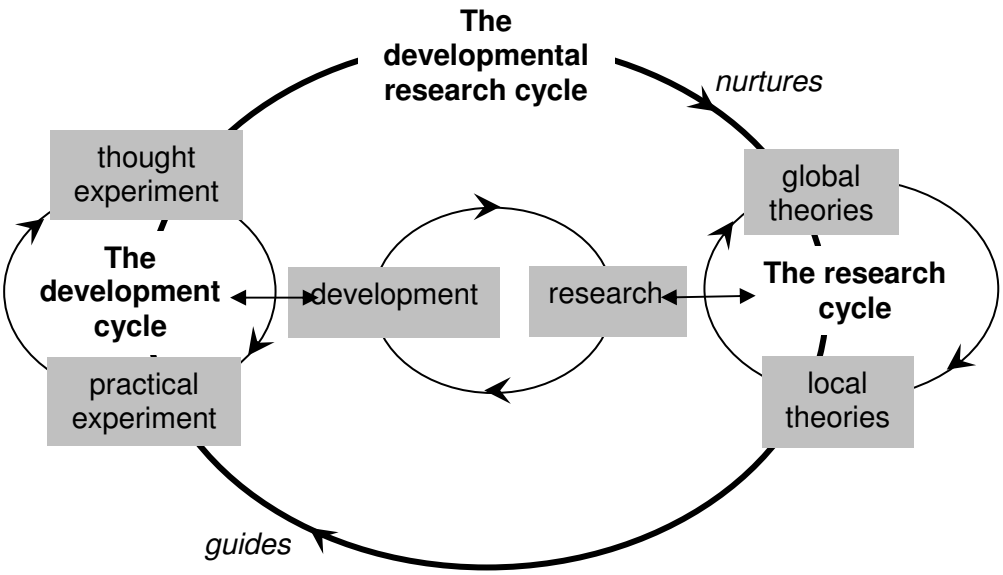

Figure 1. The developmental research cycle (Goodchild, 2008) based on Gravemeijer (1994)

Research seeks knowledge and principles of general applicability, whereas the developmental activity is localised in particular settings. Research activity is concerned with taking global theories of learning and development and interpreting these at a local level to inform developmental activity. The developmental activity is concerned with the interpretation of local theory in terms of thought experiment (i.e. planning) and implementation. Systematic observation and analysis of data arising from developmental activity is then used to examine and inform theory; it is within this phase that this paper is located. CPT (Wenger, 1998) and reflective inquiry (Dewey, 1933) form the 'global' framework and these are interpreted to provide a local theoretical underpinning of COI (Cochrane-Smith & Lytle, 1999) and inquiry in mathematics (e.g. Mason, Burton & Stacey, 1982; Polya, 1957/1945). Implementation within the projects has included, principally: teams of at least three mathematics teachers in each school and the active support of the school principal; developmental workshops for which didacticians take responsibility for detailed planning, and teachers advise about needs and focus; and school meetings, observation in classrooms, followed by joint reflection on classroom episodes. The workshops have comprised teachers reporting activities from their own classrooms, didacticians making theoretically informed presentations about, for example, inquiry approaches to mathematics teaching and issues regarding learning different topics within mathematics (such as mental calculation, algebra, proof, and so on). In addition there have been opportunities for teachers to meet in small groups, across schools and grade levels to work on mathematical problems, discuss pedagogical issues and begin planning for their own classes. The projects have also included events aimed at disseminating experiences, including three national conferences – two arranged by the project and one arranged by the Research Council of Norway to 'showcase' the projects. Other local events have included workshops with an 'open invitation' for all teachers in the participating authorities' schools.

The team of didacticians has remained fairly stable over the six years, varying in number between 10 and 13.

An important consideration in the projects has been the relationship between teachers and didacticians and the roles they take. This has taken account of responsibilities, as well as practical, developmental and ethical principles. Teachers are responsible for their classes, they have experience of teaching and learning that must be acknowledged and respected. Didacticians have to take responsibility for activities which teachers cannot undertake due to their regular workload, such as the detailed planning of workshops. Any development of teaching and learning will occur because teachers make it happen. Thus, the projects are concerned with the development of teachers' knowledge of teaching and learning mathematics, students' knowledge of mathematics, and didacticians knowledge of the developmental process. The research is about exploring the developmental processes rather than finding out what teachers or learners of mathematics 'do'. Consequently, all participants are inquirers into their own practice, all are thus 'researchers', and the research is based upon a principle of co-learning agreement (Wagner, 1997).

LCM included teams of teachers from eight participating schools, two primary (grades 1-8), two lower secondary (grades 9-10), two combined primary/secondary (grades 1-10) and two upper secondary (grades 11-13). ICTML took a specific focus on the use of ICT as a tool for teaching and learning mathematics and comprised the two lower secondary and two combined schools participating in LCM. The binary project LBM/TBM included one each of the primary, lower secondary and combined schools that continued from LCM; and one new school from each of these classifications. Also the project included four kindergartens and two new upper secondary schools in addition to the continuation of the upper secondary teachers from the LCM project. Thus LBM/TBM comprised four kindergartens, two primary schools, two lower secondary schools, two combined schools and three upper secondary

schools.[4] Apart from the inclusion of kindergartens and some changes in school participation there were other significant differences between the LCM, ICTML and LBM/TBM projects that reflected the knowledge gained from the accumulating experience. Whereas in the former projects, it was, initially, the didacticians who set the goals and steered the activity, in LBM leaders from the school authorities and school teams met regularly to discuss progress and suggestions for future activities. The involvement of the school authorities was welcomed as it introduced a level of recognition and encouragement of the teachers' activity within the projects. An unanticipated consequence was that the introduction of additional levels of management and steering combined with the contractual agreement being made with the school authorities rather than the individual schools appeared to weaken the relationship between didacticians and schools. The inclusion of kindergartens and additional schools also stretched didacticians' capacity to make the desired regular visits to observe in classrooms and contribute to school team meetings.

The research element is based on the systematic collection of naturally occurring data; that is video and audio recordings made whenever a project event takes place. Few actions were taken with the sole purpose of generating data, of these focus group interviews were held in each project school in the spring 2006, and again spring 2010. The data corpus also includes all documentary material related to the projects. Careful organisation and cataloguing of the huge data corpus facilitates the following of school teams and quick access to documents and recordings. In this paper a phenomenological approach to data analysis is taken (Miles & Huberman, 1994); this comprises a retrospective engagement with all data that pertains to one school team with the aim of producing an authentic account of events that might be used as evidence. The purpose is to expose instantiations of development which is then examined for signs of *extrapolation* and/or *expansion*. The longitudinal case study upon which this paper is based focuses on the team of teachers within a lower-secondary school

that participated throughout six years in all the projects described above. Table 2 lists pseudonyms of all the people who form part of the case study, an abbreviated time line of events relating to the school (Austpark) upon which this paper focuses is provided in figure 2, brief details of the events noted on the time line are provided in table 3.

We continue by setting out two instantiations of development that have emerged over the six years of the project. These developments have occurred slowly and the history of each is rooted in events that precede the projects' inceptions. Emergence of the development is exposed by examination of data arising from the sequence of events that have occurred over the whole six year period. Reporting these instantiations of development entails a narrative of key events. The first narrative describes the development of a community of inquiry within the school, which, using the criteria listed above bears the hallmark of expansion. The second narrative describes the development of mathematics teaching at the school. This appears to be largely of the character of extrapolation, although there is some indication of creative innovation that might eventually result in expansion. The second narrative concludes with a consideration of why extrapolation might be more characteristic of teaching development than expansion. A list of pseudonyms of all the key participants in the story of Austpark school is provided in table 2.

Instantiation of Development 1.

Episode 1 (numbers #1, #2, etc. refer to events listed in table 3)

Teachers at Austpark had set themselves on a developmental trajectory before the LCM and ICTML projects existed. In November 2002 Gunnar had attended a national conference and been impressed by the presentation of the keynote speaker, John Mason (#1). In 2003 the school had engaged in a project run by the local authority, "Regn med Kristiansand" (Count with Kristiansand), and late in the same year teachers at Austpark had written a proposal for a school-based mathematics teaching development project. It was while attending a "Regn med

Kristiansand" meeting that Gunnar heard about seminars at the university, which had been planned to introduce *didacticians* to LCM and ICTML (#2). At the end of January 2004 Gunnar attended the second of these seminars and was convinced that the LCM and ICTML projects had much in common with the proposal he and his colleagues had formulated some weeks earlier. Gunnar contacted the LCM and ICTML project leaders to request that Austpark be invited to participate (#3). An invitation was extended and following several meetings to negotiate the terms of participation, Austpark decided to participate in both projects.

Austpark teachers	**Project participation**
Harald (promoted to school principal in 2007)	LCM, ICTML and LBM
Gunnar,	LCM, ICTML and LBM
Frode, Elise	LCM and LBM
Jakob (2 years temporary appointment from 2005)	LCM and ICTML
Helga, Gunn, Runar, Eivind, Sigurd, Ingunn, Arild	LBM
Didacticians involved in the Austpark story	
Eli (leader of LCM and didactician initiator of LBM/TBM)	
Aud (leader of ICTML and didactician leader of LBM/TBM)	
Otto (a teacher who had a didactician role in ICTML	
Kai (a didactician PhD fellow whose research focused on teachers' implementation and orchestration of dynamic geometry software)	

Table 2. Key 'players' in the Austpark story. (All names are pseudonyms).

During the process of negotiation the teachers' and school leaders' enthusiasm was apparent to Eli, the LCM project leader, who recalled in a didacticians' meeting some months later:

> I had high hopes for Austpark because of the early involvement of *Gunnar* and that really insightful reflection (#4) that we had from him after our first workshop planning meeting. (#9)

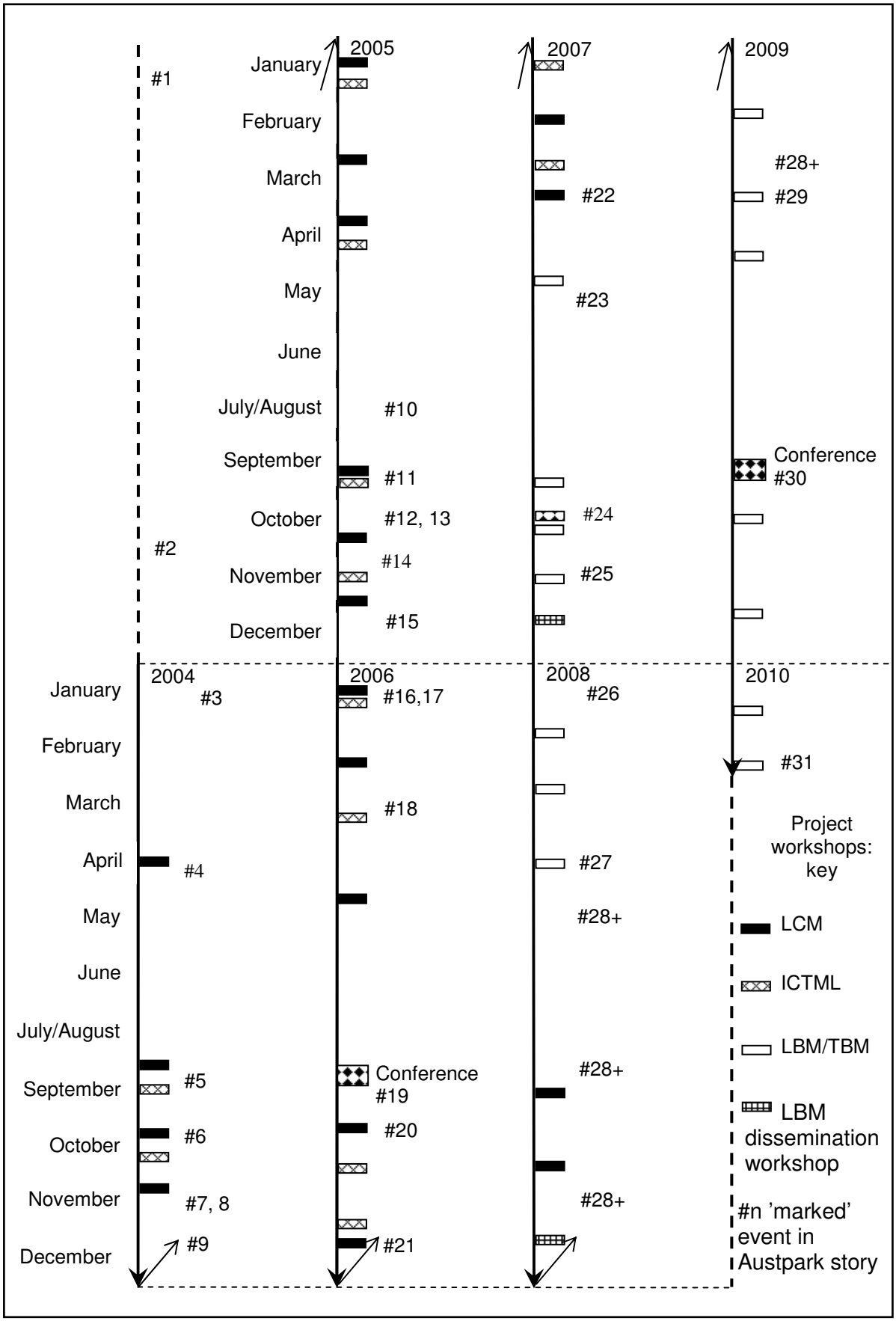

Figure 2. Time line: Workshops, conferences and other events in the Austpark story.

Event	Description
#1	November 2002, Gunnar attends national teachers' conference, John Mason is keynote speaker.
#2	Autumn 2003 The Research Council of Norway announces support of the proposed LCM & ICTML projects. Didacticians decide to hold a series of introductory seminars to inform and discuss core concepts of the projects.
#3	Gunnar attends the second introductory seminar. Subsequently he contacts didacticians to signal wish for Austpark school to participate in the projects.
#4	Pilot workshop at the university, most participants are didacticians who will be active in the projects. Gunnar and two teachers from other schools participate. Gunnar sends an "insightful" reflection on the pilot workshop. He expresses enthusiasm towards the project describing it as "wonderful", "a gift from above".
#5	First project workshops. LCM: September 01; ICTML: September 15.
#6	Second LCM workshop.In plenary Gunnar talks about the impact of hearing John Mason speak at the conference in November 2002 and reading "Researching your own practice:The discipline of noticing" (Mason, 2002).
#7	School meeting: Gunnar, Harald, Frode, Elise, Principal, Aud, Eli.
#8	Incident resulting in major structural damage at Austpark school.
#9	Didacticians meeting: review of progress in project schools.
#10	Jakob, recently qualified with Masters in Mathematics Education joins Austpark.
#11	Concern about the lack of development in use of ICT at Austpark, Aud contacts the school to request a meeting.
#12	Eli and Aud combine to contact Austpark principal to expressing anxieties about school's commitment to the projects.
#13	Aud and Otto meet Austpark teachers and leaders to offer support.
#14	Eli and Aud visit Austpark school to discuss progress and commitment.
#15	Gunnar introduces class to dynamic geometry software (DGS).
#16	In group discussion in a workshop Gunnar tells of his experience using DGS.
#17	Otto provides school-based training sessions in use of DGS.
#18	Focus group: Eli, Aud, Kai, Jakob, Elise, Harald, Frode, and Vice Principal.
#19	LCM/ICTML Conference at the university. All teacher teams present their work in the projects. Conference draws participants from throughout Norway.
#20	In workshop groups teachers discuss goals for continued activity in the projects.
#21	Jakob presents in workshop plenary: a specially planned class activity was video recorded and used to stimulate reflection in a school team meeting.
#22	Workshop plenary, teachers talk about the impact of the projects and plans.
#23	New project Learning Better Mathematics (LBM). Project leaders from each school meet to report activity and discuss plans.
#24	National conference arranged by Research Council (RCN)
#25	Presentation of school goals for LBM.
#26	Schools' project leaders meeting.
#27	Austpark teachers tell about their approach to teaching linear functions in workshop plenary.
#28	Schools' project leaders meetings (07.05, 03.09, 19.11.2008, 11.03.2009)
#29	Focus group: Aud, Helga, Ingunn, Gunnar, Sigurd, Eivind, Harald, Elise.
#30	LBM/TBM Conference at the university. All teacher teams present their work in the project. Conference draws participants from throughout Norway.
#31	Final LBM workshop. Teachers report impact of project in their schools.

Table 3. Summary of 'marked' events relating to Austpark School

The enthusiasm and commitment of the school team was evident in other ways. Time had been set aside within the teaching schedule each week to facilitate the project teachers meeting and they were planning observation of each other's lessons. Further, in the second LCM workshop (#6) Gunnar made a lively presentation of John Mason's book "Researching your own practice" (Mason, 2002).

Comment.

It appears that the developmental process was initiated by one inspired teacher who managed to convince some of his colleagues to collaborate with him. Adey (2004) draws attention to the importance of teacher's ownership of the developmental process and their acceptance of the theoretical foundation of the innovation proposed. In this respect Austpark made a promising start in the projects. It is also evident that there was support from the school leadership, which was a requirement for participation in the projects, and identified as a crucial factor in successful professional development in schools (Adey, 2004, Hargreaves & Fullan, 1992).

Episode 2

However, by mid November, 2004 didacticians began to perceive that Austpark teachers' intentions were not being fully realised in practice, and Eli had the impression of a small group of individuals each pursuing their own plans independently. Eli followed the remark reproduced above by reflecting on her visit to the school (#7) some days prior to the meeting:

> I got the impression of three teachers who were all very interesting and positive but they were doing quite disparate things and they didn't seem to be talking [together] about what they were doing. (#9)

Matters were exacerbated when at about the same time the school was subject to a burglary resulting in the theft of computers, serious structural damage and the loss of

academic records (#8). By the end of October 2005 didacticians were expressing serious concerns about the commitment of teachers at the school and it was decided that the project leaders needed to take action if the projects were to survive at Austpark (#11, #12). Two meetings were arranged at the school. In the first (#13) it was agreed that Otto would provide some in-school workshops on the use of dynamic geometry software (DGS), and in the second (#14) the teachers and school leaders were challenged to renew their commitment to the projects.

Austpark remained in the projects. Otto provided two workshops in the school to support teachers' competence development in the use of DGS in their teaching. The teachers' use of DGS with their classes was followed closely by didactician Kai who describes and analyses the development in his doctoral dissertation, (Erfjord, 2008). Later the Austpark teachers reported their use of DGS at the projects' conference that was held in September 2006 (#19).

In the focus group held in March 2006 (#18) Austpark teachers reflected on the values of meeting as a team, and the challenges they faced in making opportunities for this to happen. The value of sharing and reflecting on experiences, as well as discussing plans for lessons was recognised. Harald and Gunnar had also managed to organise observation in each other's lessons, and they remarked on how they could learn from each other's strengths. The teachers recognised the benefit of working as part of a larger team and noted the 'vulnerability' of the isolated teacher. However, finding opportunities to meet proved challenging. Ideally, teachers would meet in teams that could focus on the same grade but the project team taught different grades and so the scheduled meeting for the project team did not turn out to be so practicable. There was also an issue when a project 'partnership' planned activity for their same grade classes, it was necessary to draw in other teachers who were not participating in the project so that students shared similar experience within the grade. The

appointment of Jakob in August 2005 (#10) contributed to raising awareness of the challenges of meeting. Austpark was Jakob's first teaching post, he was well-qualified having specialised in mathematics and mathematics education. Jakob was encouraged to join the project team and did so with enthusiasm, and he made a significant contribution in the development of DGS use at Austpark. However, the teaching schedule had been made without consideration that Jakob would be a member of the project team and he experienced some difficulties in meeting with the other team members. The Austpark leaders' awareness of these issues, and their commitment to the projects' goals resulted in significant changes at the school, as indicated below.

Comment.

At this point it appears that a 'culture of collaboration' (Hargreave's, 1992) is understood by the teachers, even if their understanding emerges from their awareness of the difficulties of its realisation. Nurturing this culture was crucial for the success of the projects at Austpark. The school's structural constraints that prevented the teachers meeting was an issue that required decisive action from the school leadership. In addition the introduction of DGS required the teaching team to have better access to the schools computer room than they believed possible, and here, following Aud's intervention (#13), the school leadership sanctioned the team's booking of the room for substantial periods taking priority over previously agreed school curriculum intentions (to introduce all pupils to basic word-processing and spreadsheet applications). Another source of leadership is also evident here in the active intervention of the projects' leaders. Similar action was taken by the project leaders in another school at about the same time and the teachers' reflected with some gratitude for the intervention describing it as 'a kick in the back end!' (Erfjord, 2008, p. 139). The above episodes support Hargreaves and Fullan's (1992) observation when they draw attention to the important role of leadership in development, to support teachers 'to get beyond the

uncertainties and disappointments of early implementation difficulties', and to 'provide a supportive context' (p.14).

Episode 3

In the first two years of LCM common developmental goals had been set. In the first year the team of didacticians took the leading role in deciding these, their decisions being informed by careful reflection on the feedback being received from teachers. In the second year goals were set through discussion within the whole project community. At the beginning of the third and final year of the LCM project each school team was asked to set out the development plans for mathematics teaching in their own school for the coming year (#20). The Austpark team's goals included the achievement of better dialogue between teachers and between students, and the observation, video recording and subsequent group reflection and discussion of lessons. Evidence of their commitment to these goals followed in a workshop in December 2006 (#21) when Jakob presented activity of the Austpark team. Jakob described the joint preparation of a lesson, its subsequent implementation, which was video recorded, and the ensuing reflection and discussion within the teacher team based on the video recordings. The discussions led to suggestions for modifications to the lessons in subsequent implementations. In many ways the sequence of events described were exemplary of the outcomes the projects sought to achieve.

The LCM and ICTML projects terminated in 2007 and there was a smooth transition into the follow-on project LBM/TBM. Austpark wanted to be part of the new project. LBM/TBM set out to build on the knowledge and experience gained by both didacticians and teachers in the earlier projects. As a result school teams were asked, at the outset, to provide a project plan for their activity (#25). Austpark's plan included the involvement of *all the mathematics teachers at the school*, a schedule for regular team meetings and a statement of goals:

- We want to have an even better understanding about how students learn mathematics and what we can do to adapt this knowledge to our teaching.
- We want to exploit teachers' resources such that they can be shared within a learning community.
- We want to benefit from the competence the university can offer. (#25, authors' translation).

In the focus group that took place 16 months later (#29) the teacher team was invited to reflect upon their participation within the LBM/TBM project and how this has been experienced within their school. They were not prompted to reflect on the goals they had set earlier, rather they were encouraged to express how they felt about their participation. As the Austpark teachers reflected on what the project had contributed to the school it became evident that, at least in their perceptions their achievements were consistent with the goals they had set themselves. There was also much said about collaboration between teachers, observation of each other's lessons, joint planning, shared reflection and learning from others' experiences. There were also claims made about how the teaching of mathematics had been transformed; this will be considered in the next section. In the final LBM/TBM workshop (#31) participants were invited to tell about the impact the project had in their school. Harald, now Austpark's principal spoke about how the mathematics milieu had been transformed, that there was a new 'culture' of mathematics teaching which had infected other subject groups – humanities, sciences and languages. In later correspondence Harald explained further

> We see that it takes time to incorporate this culture (of sharing) … I see that it is important that we set aside time to share with each other in continuation, otherwise the culture may quickly disappear. But it is good to see that even if we have lost several mathematics teachers during the projects, the culture continues to "live" with us. (e-mail April 27, 2010, authors' translation).

Comment.

The above is necessarily a highly abbreviated account of the development of a new culture of sharing at Austpark, however we believe it is sufficient to assert that this instantiation of development exposes evidence of an expansive cycle. Mathematics teachers at Austpark were motivated to develop their teaching and recognised the value of inquiry and collaboration before joining the projects. Participating in the projects, possibly, forced the school leaders' attention to a fundamental contradiction, teachers were too busy to meet and discuss and the isolated teacher can achieve little if the 'rule' is to ensure students within a grade have common experiences. Perhaps if the project leaders had not taken the initiative to challenge the school leaders (#14) the development would not have occurred; it is impossible to say. However, the situation did ensure that all participants were made aware of the obstacles to progress and steps were taken to remove these. These steps included Otto's in-school sessions that included all the mathematics teachers and more effort made to facilitate teachers meeting. At the time of transition to the new project the school's decision was that all mathematics teachers should be included (#25). Harald describes the result as a change in culture, it might also be considered as an expansion of the teachers' activity system. A schedule that included teachers meetings constituted a change in rules. The inclusion of all teachers in the project activity meant a change in the community and their joint enterprise. Shared planning, mutual observation, and joint reflection required changes in role definitions and the division of labour. It is also possible to consider changes in the tools or artefacts of teaching and learning of which more will be said in the next section. Harald's words are reproduced above where he reflects that the development of the culture of sharing took time; this is entirely consistent with Engeström's (2001) assertion that an expansive cycle is a long term event. Harald's further observation that the culture has been preserved despite changes in

mathematics teachers suggests that the change is substantial and resilient, and does represent expansion of the activity system and not merely changes in individual teacher's actions.

Instantiation of Development 2.

Episode 1

The most detailed analysis of an episode of teaching development at Austpark is provided by Erfjord (2008) who examines the manner in which DGS was introduced following the intervention of Otto (#17). Erfjord's study is supported by classroom observation, participation in teacher planning meetings and an interview in which he invites the teachers to reflect on their experiences; it thus provides the best and most direct view of teaching development at Austpark. Initially it was proposed by the teachers that they would make the introduction of DGS at all grade levels so that all the project teachers were involved in collaborative effort. Consistent with their rationale for joining the projects they wanted to collaborate on something, they wanted to try something 'new' that their students would find motivating and enjoyable. The inclusion of DGS in the ICTML project appeared to provide an ideal opportunity to address these goals. However, it was finally agreed to introduce DGS only at grade eight where it was believed to fit better with the curriculum. This involved Frode and Jakob, and Eivind who at this stage was not participating in the projects. The manner of introduction of DGS is revealing. Jakob had experience of using DGS during his teacher education programme and one of his fellow students had developed a pack of worksheets suitable for introducing DGS at grade eight. Jakob shared these worksheets with his colleagues and it was decided that they would use them with their classes. The grade eight geometry syllabus included elementary construction of angles. Another significant influence discussed by the teachers was the requirement of the national tests that students sit at grade ten, in these students are required to use the traditional ruler and compasses methods. It was thus decided to introduce and use DGS and the traditional methods alongside each other.

Comment.

Erfjord's description of the classes suggests that the approach taken was not radically different from that used regularly; he describes it as 'supportive' and 'step by step' (2008, p. 243). This suggests that the teachers were in control of the subject content, which they released to the students in chunks, which the teachers assumed the students would find manageable. The approach appears to be consistent with the description of German lessons recorded in the TIMSS video study (Stigler & Hiebert 1999), 'the teacher owns the mathematics and parcels it out to students' (p. 25). The teachers claimed that they did not perceive the worksheets, or their implementation, as consistent with 'inquiry' as promoted in the projects and they explained that shortage of time prevented an approach that was more inquiry oriented. Nevertheless, the teachers were able to claim success from their effort claiming the students 'loved it' and noted the 'good results by the unruly boys'. (Erfjord, 2008, p. 244).

Episode 2

Other, but indirect, views of teaching development at Austpark are gained from the teachers' presentations at workshops and conferences (#19, 21, 22 24, 27, 30). These combine to create a picture that is largely consistent with Erfjord's analysis, but with additional evidence of teachers exploring new possibilities in their work.

Austpark teachers were invited to give a presentation of their approach to teaching linear functions in a workshop that was held in April 2008 (#27). In this, Runar and Gunnar additionally presented results they had collected from their own classes' work on related tasks from the PISA studies. They noted how the vocabulary used in the PISA tasks differed from that used in their regular teaching, such as a PISA task using the expression 'growth rate' whereas in mathematics lessons they used 'gradient' and 'slope' when referring to graphical representations. They also told how their students had interpreted questions differently from

that expected in the task. For example, in a task that required the interpretation of a graph to comment on differences in the rate of growth between males and females at different stages of childhood and adolescence. Many of the students had given a (pseudo)scientific reason for the differences rather than a mathematical interpretation of the graphs that modelled heights. This research undertaken by the teachers followed a previous workshop in which PISA tasks were taken as a starting point for mathematical inquiry.

Comment.

Liljedahl (2010) reports a number of case studies in which teachers he has worked with experience 'profound' change in their practice and the events that trigger these changes. One such trigger is when a teacher is challenged to try out a novel task with their classes and observe the results. In the Austpark teachers' report of their use of the PISA test items with their classes it appears that they began to see issues of mathematical language and students' situated responses in a new light. Our data does not include evidence of how this might have had an impact on their classroom practice. However, this use of the PISA tasks occurred through the teachers' own initiative and their subsequent interpretation of students' responses to the PISA tasks reveals that Gunnar and Runar are interested to learn from and about their students' engagement in mathematics.

Episode 3

In addition to the foregoing Runar presented his approach to teaching linear functions at grade 10. He outlined the content of the intended curriculum and explained that his approach was based, first on a review of work from previous grades, and then how this was extended at grade 10. The developments were based upon relaxing the requirement for students to draw up tables of values before drawing the graph of a linear equation, and the inclusion of inverse problems where questions are posed about the function modelled by a given straight line graph. Runar explained how 'inquiry' is introduced by the removal of

information from tasks. This appears to be based upon one of the suggestions made by Prestage and Perks (2001) in their book 'Adapting and Extending Secondary Mathematics Activities', which had been introduced in a previous workshop.

Comment.

Runar's presentation reinforces the picture of the teacher being in control of the mathematics which is dispensed in 'manageable' chunks for the students' consumption. His explanation of how 'inquiry' is incorporated into the teaching appears to be based on an approach to changing tasks, which he has adopted from the project. This may indicate a development in his practice that is consistent with the 'progressive' and 'safe' characteristics of 'extrapolation'. However, other reports reveal the teachers being prepared to be 'experimental', 'innovative'' risk' takers as the final two episodes indicate.

Episode 4

In September 2009 the LBM/TBM project organised a conference with a national invitation to mathematics teachers and teacher educators (#30). All participating schools in the project presented their work, with each school level collaborating in a joint session. The lower secondary schools, including Austpark, decided that they would use a large part of their session to give participants the opportunity to experience the type of 'inquiry' approach they used. Participants were split into three groups and they rotated around short presentations provided by each of the three schools. The Austpark teachers presented two tasks which they explained were used with their classes. The first concerned designing house, it was explained how the task could be opened up by providing less information – A highly structured version of the task was presented with details and dimensions for different rooms and an open version which was set *simply* as: "design a house with at least five walls". The teachers spoke about the circumstances in which they would use open and closed versions and their experiences from work with students. They spoke about the advantages of using open tasks and the

possibilities they opened to challenge pupils at different levels of attainment, they also spoke about the challenges they faced as teachers, in particular they emphasised the most difficult of all was to 'guard their mouths so that the students would experience mastery through wondering, reflection, cooperation and discussion'[5]. The second task was based on a travel graph of two groups making a journey to a holiday cabin, and the teachers spoke about how these graphs could be the context for asking a wide range of questions. The opened-up design task provides opportunities for the students to take control of their mathematical activity, whereas the travel graph task suggests the teacher carefully controlling the students' mathematical experience through purposeful questioning.

Episode 5

Further evidence of the teachers exploring new ideas comes from their presentations within project workshops. In the final LCM workshop plenary (#22) Harald presented an episode from one of his lessons. The task was based upon students making a selection of four cards from a pack of playing cards and taking the numerical value of each and using these with the arithmetical operations to produce the value of a fifth card also selected from the pack. The task was intended as a lesson starter but Harald had used it to stimulate and engage students throughout the whole of a double lesson (90 minutes). Harald explained that he had been inspired to use the task following a visit to an upper secondary school in London where he had seen the task being used. He continued his presentation by expressing his positive feelings about being part of the project community, how it provided confidence to try out new ideas and the pleasure of being able to discuss experiences with other interested teachers. In the previous section attention was also given to Jakob's workshop presentation (#21), this was based on students' creation of dominoes to make a game in which the requirement was to match fraction calculations (sums, products, etc.) with their result. This also offers an example of teachers exploring tasks beyond the standard text book interpretation of the curriculum.

Comment.

It is possible to characterise two extreme approaches to fulfilling the demands of the curriculum and simultaneously being innovative in teaching. One extreme is to accept the textbook as a fair interpretation of the curriculum and adapt this to the requirements of the innovation. Such an approach is demonstrated in episode three and the travel graph task in episode four. At the other extreme it is possible to introduce special 'novel' tasks and relate these to the curriculum; this is illustrated by the 'design a house' task in episode four, and the tasks in episode five, *albeit no attempt was made to relate these to the curriculum*. Given the strong influence of the curriculum 'rule' in teachers' activity this second extreme appears to be at risk if the tasks are not related to the curriculum and a model of students' learning progression. The special tasks are, nevertheless, important because, as indicated in the above, it is with these that there appears to be some expansive development of teaching, and as such they provide experiences that may inspire the imagination for the progressive extrapolation that characterises approaches based upon the use of the textbook.

The overriding impression is that to a large extent teaching has assimilated 'inquiry' in the form of adapting tasks and, to a limited extent, adapting approaches taken to novel tasks. It does not appear that teaching has generally adapted to the extent that 'inquiry' as an approach to teaching and learning mathematics has introduced new or transformed relationships between the mathematics, teacher and students. Nevertheless, we note that the teachers themselves claim that their teaching has been transformed. In the focus group interview (#29, March 2009), after five years of the school's engagement with the projects Sigurd remarks:

> We have certainly succeeded at Austpark, mathematics teaching today is approached in a different way than three or four years ago, I think in fact I dare say, at least in the very, very many classes. Eh, very many of us have opened up very (...) and dare to

> challenge more of the students more than we did before. I do not think the 'board' is gone, but its (use) is very reduced (Sigurd, focus group, March 2009. Author's translation)

The modest claims made in this paper for the extent of impact on teaching are based on the juxtaposition of all the data available and other studies of the impact of teaching development that point to the possibility of teachers' overestimation of the depth of changes in their practice (Cohen, 1990).

The account has drawn attention to the very powerful external structures (rules) that constrain teachers' activity – the curriculum, national tests, resources and time available. There are also less visible constraints such as students' expectations and the teachers' beliefs about the 'normal desirable state' of student activity (Brown & McIntyre, 1993) which they seek to achieve with their classes. The introduction of a new classroom approach or activity entails risk, and any misadventure occurs in full view of the students, whose continued trust in the teacher's competence is sought. Erfjord's analysis, especially, reveals that teachers experience this as a double bind, however the moral, professional and social obligations to the constraints restrict teachers to a form of incremental development that is based on the extrapolation of experience – personal, shared or what might be imagined in practice. Erfjord also draws attention to the demands of 'double innovation' that occurs when teachers are required to introduce new tools and new approaches simultaneously. The point is that the teachers do not feel empowered to take control, there appears to be a lack of any possibility of agency in their situation. In such circumstances development is slow, Jaworski (1998) describes it as 'evolutionary' but here we see it as consistent with Wenger's (1998) description of 'extrapolation'.[6]

Conclusion

As we review the Austpark story and the participation of Austpark teachers through six years of mathematics teaching development projects it is possible to see signs of development. We do not claim that the projects have been the cause of the development, rather that they have served the teachers' attempts to work on their practice. The development of collaboration of the teacher team has occurred through the transformation in the working practices, the rules, division of labour and community. The account provided above, it is argued, is consistent with Engeström's description of expansion. Changes in teaching, however, appear modest and better characterised, for the most part, as extrapolation. It has been suggested that, perhaps, the nature of the constraints restricts teaching development. This is not meant to be interpreted as being hopeless for radical change. Harald described the changes in the working practices at Austpark as a 'change in culture', and the transformation provides the opportunity, motivation and favourable conditions for continued extrapolation of teaching practice. We reflect on Stigler and Hiebert's (1999) suggestion that the superiority of Japanese students in international comparisons of mathematics performance might be, in part, the consequence of decades of teachers' commitment to lesson study. The expansion of the mathematics teachers' activity system at Austpark has led to an approximation to lesson study and it is hoped that it will contribute to the continual improvement of students' experience of, and performance in mathematics.

The analysis of the two instantiations of development at Austpark demonstrates the value of adopting both community of practice theory and cultural historical activity theory as complementary stances to explore mathematics teaching development. The use of the CHAT draws attention to development that appears to represent a radical change to practices because it is accompanied by changes to the rules, community and division of labour in the activity system; it represents a change in the culture. This is expansive development and appears

consistent with Liljedahl's (2010) accounts of 'profound' change, and it is believe the 'self-sustaining generative change' described by Franke, Carpenter, Fennema, Ansell and Behrend (1998). The development of mathematics teaching at Austpark appears less marked. The teachers, for a variety of reasons as outlined above do not have the same degree of agency as with the organisational structures of their working practices. Development as extrapolation does not involve the same level of risk as expansion, and given high stakes examinations that students face the costs of failure in a developmental activity could be very high indeed.

In the methodology section we claimed that this paper addressed the phase in the developmental research cycle in which development informs theory. To be clear, the engagement with the data collected from the developmental activity has required not only systematic analysis but also careful reflection on the theoretical frameworks within which the analysis has been conducted. The need for an adequate theorisation of development has resulted in the juxtaposition of CPT and CHAT. Examination of the evidence has then resulted in our recognition of complementary models of development – extrapolation and expansion. We have suggested above that in mathematics teaching development extrapolation is more likely because of the constraints of curriculum and high stakes examination. Nevertheless, this observation leads us to inquire into fresh or modified approaches that might facilitate teachers' creative innovation in their classroom practice.

Finally, to compile this research report we have been confronted with the challenges common to many large teams working in teaching development projects. Teaching development is a slow process; substantial quantities of qualitative data accumulate over the period of many years. The data must be analysed, synthesised and presented in a manageable form. The temptation is to 'cherry pick' the data and write research reports on events or brief episodes, each interesting and informative but nevertheless do not address a fundamental requirement and wish to report on how the global developmental aims and research questions

have been met. Our engagement with the data to produce the case history of the Austpark teachers has opened a way for us to meet this challenge.

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Appendix: Glossary of Abbreviations Used

CHAT: Cultural historical activity theory

CPT: Community of practice theory

COI: Community of inquiry

ICTML: ICT and mathematics learning (project 2004-2007)

LBM/TBM: Learning better mathematics/teaching better mathematics (project 2007-2010)

LCM: Learning communities in mathematics (project 2004-2007)

DGS: Dynamic geometry software

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[1] The authors are conscious of the density of abbreviations used in this paper and have decided to include a glossary in appendix 1.

[2] Participants in the projects based at the university are referred to as didacticians rather than the more usual researchers because within the projects teachers are also regarded as having an active role in researching teaching and learning.

[3] LCM, ICTML and TBM were supported by The Research Council of Norway. Additional support for TBM and support for LBM came from the Competence Development Fund of Southern Norway.

[4] The two upper secondary schools that had participated in the LCM project were combined into one school sharing the same campus in 2007, thus explaining why two and two only results in three schools!

[5] Quoted from material handed out prepared by the Austpark teachers and given to session participants.

[6] Erfjord's analysis is carried out within a CHAT framework and he refers to the development of DGS use at Austpark as expansion. We do not dispute that development that occurred but in the more critical interpretation being used in this paper we suggest that extrapolation is a better characterisation of what occurred.