# The didactical nature of some lesser known historical examples in mathematics

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## 1. Introduction

In the field of history of mathematics the work of famous mathematicians, such as Cauchy, Newton and Leibniz, have been carefully studied which certainly have provided valuable knowledge regarding the development of mathematics. The results are well-documented in the literature. However, it may also be important to take into account the efforts of the less known (or sometimes unknown) mathematicians, contemporary to the famous mathematicians. In a didactical perspective, the study of the work of the less known mathematicians can provide valuable insights regarding mathematical thinking and conceptual development that often cannot be derived from the work of the well-known mathematicians. For instance, by studying the work of the less known mathematicians we get more information of the struggle behind famous mathematical results, which often include valuable didactical knowledge. Moreover, the general view of certain mathematical concepts at particular time periods can be better understood on the basis of not only the work of the leading mathematicians, but also on the basis of contemporary mathematicians working in the shadow of the famous mathematicians. Furthermore, in the work of the less known mathematicians one can sometimes find alternative solutions to a difficult mathematical problem, even before it was posed as a problem. However, these alternative solutions may not be as straightforward as the solutions documented in the modern textbooks. But they can provide valuable knowledge regarding mathematical thinking as well as the development in mathematics.

In this paper we try to highlight the didactical nature of some historical mathematical examples. We use examples from some less-known mathematicians in order to show how didactics of mathematics is naturally interweaved in the methods devised in the history of mathematics to solve difficult problems. In Sections 2 and 3 of this paper we consider some less known work from Islamic and Indian mathematics between the 11th and 16th centuries. Apparently, these mathematicians seem to have displayed an understanding of ideas such as limits, derivatives and integrals long before Newton and Leibniz developed the modern idea of calculus. For instance, we consider the 11th century Islamic mathematician ibn al-Haytham's (965-1040) work of calculating the volume of a parabola revolved around a line perpendicular to its axis. In fact, al-Haytham used a method similar to integration to correctly solve the problem. Moreover, within his calculations he actually derived the formulae of a sum of squares and a sum of the fourth powers. He did not have the machinery of Bernoulli numbers or polynomial computations but he used a similar algorithmic and recursive method. Al-Haytham accomplished this about 650 years before Newton an Leibniz began developing modern calculus. Moreover, we also consider the Islamic mathematician Sharaf al-Din al-Tusi who appeared to use methods similar to derivatives to find maximum values of functions already during the 12th century.

The so called "Kerala school", which consisted of Indian mathematicians between the 14th and 16th centuries, was developing methods similar to those in modern calculus. It appears that these mathematicians developed power series expansions for trigonometric functions without having the binomial expansion machinery that for instance Newton had at his disposal. In Section 3 of this paper we consider the Kerala shool's work with infinite series as well as the Indian mathematician Bhaskara II who addressed the idea of infinitesimals already in the 12th century. Moreover, it turns out that Bhaskara II used the idea of what is now known as Rolle's Theorem, that is 500 years before Rolle did.

In Section 4 we consider different views of some fundamental mathematical concepts based on examples from the 17th and 19th centuries. From the 17th century we consider a debate between the philospher Thomas Hobbes and the mathematician John Wallis. One of their issues dealt with whether there existed an angle between a circle and its tangent, the so called "angle of contact". Wallis claimed that "the angle of contact" was nothing, meanwhile, Hobbes argued that it was impossible that something that could be perceived in a picture could be nothing. It seems that one problem was that Hobbes and Wallis sometimes did not base their arguments on mathematical definitions. Instead their arguments were often based on intuitive and visual thinking. Another problem was that at least Hobbes in some cases did not clearly distinguish between mathematical objects and other objects.

A closely related issue to the debate between Hobbes and Wallis is the Swedish 19th century mathematician E.G. Björling's view of fundamental concepts in mathematical analysis. Björling lived during a time period when mathematical analysis underwent a significant shift from being considered as an empirical science based on time and space to being considered as a purely conceptual science (Jahnke, 1993, p. 267). It seems that Björling had a tendency to sometimes consider mathematical definitions as descriptions of entities rather than conventions. A typical example is that Björling considered particular examples of functions almost as if they were "existing independently of any definition". For instance, he argued that the function $f(x) = \frac{x}{|x|}$ (written with modern terminology) had two values at $x = 0$.

Meanwhile, the derivative did not exist, since the function representing the first derivtive