Educational Studies in Mathematics

Creativity and Mathematical Problem Posing: An Analysis of High School Students' Mathematical Problem Posing in China and the United States

--Manuscript Draft--

Manuscript Number:	EDUC1235R2
Full Title:	Creativity and Mathematical Problem Posing: An Analysis of High School Students' Mathematical Problem Posing in China and the United States
Article Type:	Original Research
Keywords:	advanced high school students; cross cultural thinking; Geometry; Mathematical creativity; novelty; problem posing; problem solving; U.S and Chinese students; rural and urban Chinese students
Corresponding Author:	Xianwei Yuan van Harpen, Ph.D. Illinois State University Normal, IL UNITED STATES
Corresponding Author Secondary Information:	
Corresponding Author's Institution:	Illinois State University
Corresponding Author's Secondary Institution:	
First Author:	Xianwei Yuan van Harpen, Ph.D.
First Author Secondary Information:	
Order of Authors:	Xianwei Yuan van Harpen, Ph.D.
	Bharath Sriraman, Ph.D
Order of Authors Secondary Information:	
Manuscript Region of Origin:	
Abstract:	In the literature, problem posing abilities are reported to be an important aspect/indicator of creativity in mathematics. The importance of problem posing activities in mathematics is emphasized in educational documents in many countries, including the United States and China. This study was aimed at exploring high school students' creativity in mathematics by analyzing their abilities in posing problems in geometric scenarios. The participants in this study were from one location in the United States and two locations in China. All participants were enrolled in advanced mathematical courses in the local high school. Differences in the problems posed by the three groups are discussed in terms of quality as well as quantity. The analysis of the data indicated that even mathematically advanced high school students had trouble posing good quality and/or novel mathematical problems. We discuss our findings in terms of the culture and curricula of the respective school systems and suggest implications for directions in problem posing research within mathematics education.
Response to Reviewers:	Dear Editor, We appreciate the opportunity to improve the manuscript EDUC1235 “Creativity and Mathematical Problem Posing: An Analysis of High School Students’ Mathematical Problem Posing in China and the United States”, and hereby submit it for publication consideration in Educational Studies in Mathematics. We have taken the comments of the three reviewers very seriously and revised the manuscript according to their critiques. The specific responses to yours and the three reviewer’s queries are found below in italics. Thanks for your advices on how to make the revisions. We found them very helpful. Sincerely, Xianwei Yuan Van Harpen Bharath Sriraman Responses to Merrilyn:

1. First, all reviewers commented that the data coding, especially the categorization of student responses, needed further explanation or discussion to help the reader follow your categorization and coding decisions. One way to deal with this might be to include Appendix A in the body of the paper, as Reviewer 5 suggests. Perhaps you could also explain what it means to "combine" the categories generated from responses of the three groups to make a common rubric. Was this an additive process where all the categories were put into a single rubric? Or were some categories subsumed within others? (I suspect it was the latter.)
Response: We included Appendix A in the body of the paper. We further explained what it means to “combine” the categories. The following sentences were inserted in 4.2.
“ For example, Jiaozhou students posed dilation problems but U.S. students and Shanghai students did not. After the responses generated by each group of students were categorized, all the categories were combined to make a common rubric for all the three groups. For example, the Dilation category in the Jiaozhou rubric was subsumed into the Transformation category in the common rubric.

2. Second, there is a sense that the presentation of the findings does not quite link well enough to the creativity framework that guided the analysis. We can see that methods were used to analyse fluency and flexibility, and that originality was also analysed and reported on separately. But I wonder whether it would make the findings clearer if you presented the data analysis methods and the results in separate sections (currently they're presented together in section 4), and signaled more explicitly the findings related to fluency and flexibility. I'm thinking that a new section presenting the results could start in what is now section 4.2 with the sentence just before Table 1 starting "A comparison of the means and medians of the students' fluency and flexibility ?". This paragraph and Table 1 give the overall results. There could follow a sub-section labeled "Fluency" (currently sub-sections 4.2.1 and 4.2.2 - I'm assuming here that your analysis of triviality is relevant to
fluency?) and another labeled "Flexibility" (currently sub-section 4.2.3). A further sub-section labeled "Originality" could remind the reader that this element of creativity was analysed and reported separately. An approach like this would make it very clear that your analysis was consistent with the theoretical framework for creativity.
Response: We made the changes suggested by the editor. There are now three sub-sections under section 4.3. They are Fluency, Flexibility, and Originality.

3. Third, and related to the suggestions in the preceding paragraph, the first paragraph of the Discussion could be re-written to tie it back to the creativity framework by commenting on the findings regarding fluency and flexibility, as Reviewer 3 suggests.
Response: The first paragraph of the Discussion was rewritten to tie to the theoretical framework (Fluency, Flexibility, and Originality).

Returning to the results, you'll see that the reviewers want to know why, in section 4.1, you only statistically evaluated differences between the US and Shanghai groups while just stating, without any statistical evidence, that the Jiaozhou group performed better than the other two groups. Can this be attended to?

Response: We followed Reviewer #6’s suggestion and used participants’ scores in the mathematics content test instead of their scores on just the geometry items. Statistics tests were used to compare all the three groups’ scores as well as each pair of the three groups. [We noticed that the statistics in the last version were actually on the whole test. So you will see that, in this version, the numbers are the same as those in the last version.]

4. Also, while Figures 2, 3 and 4 are useful and interesting, their reproduction in black and white makes it difficult to distinguish between the categories - is there another way this could be represented? Table 3 is useful too, but difficult to interpret without reference to Figure 2. Could the category labels be added to Table 3 to help the reader here?
Response: Figure 2, 3, and 4 were deleted. Table 1, Table 2, and Table 3 were added with category labels included in the notes.

5. Finally, I'm going to suggest that the paper should finish on a constructive note with a discussion of its significance and suggestions for future research, rather than its

limitations. This would mean moving the current section 6 so it comes after section 5.3 and before section 5.4.
Response: The limitation section was moved as suggested.

6. As revisions tend to make a paper longer rather than shorter, you might also consider the following suggestions to keep the next version a reasonable length:

Do the sections about interviewing students (3.3, 4.3) contribute something worthwhile to the overall argument? If not, could they be omitted?
Response: Since the authors felt the need to include the interview information for Section 4.3.3 Originality, Section 3.3 is kept in this new version. The original Section 4.3 was deleted.

7. Section 5.2 doesn't present a very convincing case for a relationship between time spent at school and problem posing abilities (see Reviewer 5), and could probably be omitted.
Response: Section 5.2 was deleted.

8. The influence of curriculum is interesting, and this was something that reviewers of the original submission wanted to know more about. Rather than providing full details in Appendices B and C, it might be possible instead to add a little more of this information to the text of section 5.1.
Response: As suggested, some information on curriculum was added to section 5.1. The appendix was deleted.

As you prepare these revisions, could you also be sure to check that the manuscript confirms to APA style, especially noting requirements for abbreviations (use "i.e." and "e.g." only in parenthetical material, but "that is" and "for example" otherwise) and citing sources involving multiple authors (use "&" to join author names in parenthetical material, but "and" otherwise, except when compiling the Reference list.

Response: We have went through the manuscript and checked the style.

Responses to Reviewer #3:
I have reviewed an earlier version of this manuscript. Overall, the researchers have done the revision well. However, one of the concerns still exists and I hope the researchers can address it to make the manuscript even stronger.

The researchers framed the study in creativity. The concern is related to data coding for posed problems. The coding system does not match the creativity framework. In addition, I still hope that the researchers can revise the discussion to tie it back to creativity frame. Otherwise, there are inconsistencies among creativity frame for problem posing, data coding f posed problems, and the discussion of the findings.

Well done and I am looking forward to reading a printed version of the manuscript after addressing this concern.

Response: Thanks for the encouraging words! In section 4, data analysis and results of the mathematical problem-posing test are now separated. In the discussion of the results, three sub-sections, namely Fluency, Flexibility, and Originality, were now included to match the creativity framework. In addition, the first paragraph of the Discussion was rewritten to tie to the theoretical framework (Fluency, Flexibility, and Originality).

Responses to Reviewer #5:
1. This paper provides details of problem posing tasks to senior secondary students

enrolled in 'advanced' streams in two different countries. It is clear that the researchers have taken into consideration the comments from the previous reviews. However, responding to those comments has not completely overcome the problems inherent within the paper.

The authors are correct in describing problem posing as being an under-research area of creativity in mathematics education and it is interesting to see that the research has been conducted in two countries, one of whose students are considered to do well on international tests whilst the other's students are considered to be more creative. The fact that the results suggests that such broad generalizations are inappropriate is valuable. However, the way that the paper is currently set out this result is not highlighted, but left to one sentence at the top of page 17.

Response: Thanks for the advice. We agree with you. But that part of the paper (section 5.2) was deleted as suggested by the editor. So we did not need to expand on that statement.

2. On the other hand, the findings that are highlighted seem simplistic. Much is made of the fact that students need to have learnt mathematical knowledge in order to use it when posing problems, yet mathematics education research for many years has suggested that if children do not have knowledge then they cannot use it in a range of circumstances (not just problem posing). The paper makes much of the fact that the students who were participants in this research were enrolled in advanced mathematical programs. Consequently it is not clear how the following statement can be made "The differences in the distribution of the categories of posed problems suggest that the problems posed by students might be related to students' background mathematical knowledge. In a sense this echoes the claim that basic knowledge and basic skills in mathematics could be highly related to creativity in mathematics (Zhang) as opposed to viewing basic skills as rote or non-creative". There is no possibility for the mathematics that these students are learning to be considered 'basic'.

Response: That is a very good point. By selecting students from advanced mathematics courses in high school, the authors were trying to aim at mathematically advanced students. However, the mathematics content test, which measures basic knowledge and basic skills, showed that those participants in the Shanghai group and U.S. group did not perform very well, which means that the students did not have a strong grasp of basic knowledge and basic skills. The following paragraph was added in 5.1. to clarify this confusion.

"The results of the mathematics content test suggest that, although the participants were all taking advanced courses in their school, the U.S. participants and Shanghai participants' basic mathematical content knowledge is not as strong as the researchers expected."

3. Personally, I think that if conclusions, such as the one about the differences in curricula, require specific information then as a reader I should not be expected to look in an appendix to find that information.

Response: We agree. As suggested by the editor, some information on curriculum was added to section 5.1. The appendix was deleted.

I also feel that the examples of the problem posing test categories should be included in the paper itself. Having these as appendices does not decrease the length of the paper but just irritates the reader.

Response: We agree. The categories are now included in the paper itself.

4. I am also very hesitant about the issue of whether the amount of time that students spend at school affects their problem posing abilities. There is no research evidence presented in this paper that supports this in any way. Maybe there was information from the interviews that supports this idea but it is certainly not referred to in the paper.

Response: As suggested by the editor, the discussion was deleted.

5. Although it is mentioned that students were interviewed, it is also mentioned that the researcher did not go to China. How were the interviews conducted with the Chinese students? This is relevant when the researchers suggest that Shanghai students do not provide real-life contexts because of preference in the mathematics instruction they received but no interview data is provided to support this claim. It would seem an obvious thing to ask about in the interview.

Response: In fact, the first author, who is also the principle investigator, did go back to China to interview the students in person, although she did not go back to China to administer the tests. No interview data was collected to support the claim in this study. Therefore, the claim was deleted.

6. The issue of trivial or non-viable problems needs to be described in more detail. The decisions seem to be based on the researchers' own beliefs as a consequence I need more detail of these justifications than is currently provided.
Response: We realize the analysis of the trivial and non-trivial problems lacked rigor. The decisions were indeed made based on the researchers' own beliefs. Therefore, we cannot provide more details.

7. Statistical tests are conducted to show that there was no difference in mathematics content scores of the US and Shanghai students. However it just seems to be assumed that there is a statistical significant difference between Jiaozhau students and others.
Response: We followed Reviewer #6's suggestion and used participants' scores in the mathematics content test instead of their scores on just the geometry items. Statistics tests were used to compare all the three groups' scores as well as each pair of the three groups. [We noticed that the statistics in the last version were actually on the whole test. So you will see that, in this version, the numbers are the same as those in the last version.]

8. In the abstract, there is a statement "All participants were enrolled in advanced mathematical courses in the local high school", yet in the paper it clearly states that some of the Chinese students were boarding at the school because they lived outside the city and it was too far to travel. These students cannot be considered to have enrolled in their local high school.
Response: As suggested by the editor, this section was deleted. But to answer the reviewer's question, the city Jiaozhou was a small city with an area of 45 km2, which is less than 20 mile2. The students were boarding mainly because transportation is not convenient to commute between school and home, especially given that the students needed to get to school early and leave school late.

9. Although the language of the paper is generally good there are spots where a closer proof-read would be useful. At times "the researchers" are mentioned and at other times "the researcher" is mentioned. Occasionally, dates are missing from references. Terms such as dilation are used but without explanation.
Response: "Researcher" was changed to "researchers". An example of a dilation problem was added. (e.g., Construct a figure twice as big as the original one using a ruler and a compass.)

Responses to Reviewer #6:
First let me say that, although I did not review the previous version of this paper, I was impressed with the detail with which the authors responded to previous reviewers. I think the topic of this paper is very important simply because as the authors pointed out in their literature review problem posing whether at a professional level or at the level of school students - perhaps especially the kinds of students tested in this a study - having the facility to pose numerous and varied problems or mathematically viable possibilities in a particular situation should be a hallmark of good mathematical knowing. That said the authors have provided us with data on one task that was designed to allow these students to show such capabilities. The paper provides a careful description of how the authors carried out this study and for purposes of the dimension of international "comparison" which was part of the operational character of the study, provided us with comparisons of student performances at 2 distinct Chinese sites and one USA site. For me the most creative analysis was related to the categorizations of the responses and the particular
sub-categorizations as well which allows us an interesting picture of the kinds of mathematical actions taken by students in each of these sites. The authors provided us some nominal comparisons and provided some thinking as to why the Jiaozhou site students performed differently from students at the other sites especially the USA one in this regard. The paper is clearly written and easy to follow and does give the reader interesting insights wrt the problem posed in the title.

As will be noted below I have some questions as to why the various analyses proceeded as they did. For example, the authors chose to eliminate 2 problem posing items [see bottom of p8 and top of p9] from their analysis, both of which related to settings quite unlike the geometric one analyzed here. They claim that the other 2 settings were more sensitive to cultural differences and hence they chose to compare performance on this one "geometric' item. I guess the reader might look up the other paper referenced and get a deeper picture of these "cultural" effects, but I would have liked a little more here in this paper. For example, how was this comparison manifested in work on the other two problems? Are there other ways to handle this contrast? Does this choice make particular assumptions about the relationship between mathematics knowing of these students and their lived knowing more generally - in other words is mathematics knowing going to be subject to the lived experiences of the students? Why was this difference noted and acted upon but the differences in the schooling cultures and organizations and test pressures in the 3 sites not really taken into consideration?
Response: We included some examples to show the influence of culture in posing problems. Another reason is because we were concerned about the length of this paper.
Other comments:
1. I realize that the authors gave us a deliberately "terse" review of literature. I think this review was for the most part adequate for founding this paper but there are some elements which seem to be just hanging there: e.g. p lines 31-32 "mathematics classes in China are often described as not conducive to effective learning" which does not seem in accord with the following sentences in that paragraph nor in the data presented in this study.
Response: The following sentences were added: “For example, the teaching method in the classroom was often described as “passive transmission” and “rote drilling” (e.g., Biggs, 1991). In order to understand this “paradox of the Chinese learner” (Marton, Dall’Alba, & Lai, 1993),…”

2. Operationalizing Problem posing as creativity [section 2.5] I think the appropriate literature was used here and I liked the example problems which contrasted the three types of problems. Once again I think more could have been said about why the choice was made to use the semi-structured category. And why this contributed to the goals of the study better than the other two kinds.
Response: Similar to mentioned earliver, we tried to explain the reason we chose to focus on the geometric item. Another reason is because we were concerned about the length of this paper.

3. p10 lines16 and following; Why were only the geometry items used here? Since there was no reported statistical analysis with respect to the relationship between mathematics knowledge and the results herein what is the basis for not using the total test [as you do do in selecting students to be interviewed? Further, Why did you give us a statistical comparison of performance between US and Shanghai performances and simply assume that the Jiaozhou group performances were better. As a reader I thought all three groups performed at least at a good enough level in geometry as measured here [we don't know what this means] to be generative of possible problems related to the item given.
Response: We followed your suggestion and used participants’ scores in the mathematics content test instead of their scores on just the geometry items. Statistics tests were used to compare all the three groups’ scores as well as each pair of the three groups. [We noticed that the statistics in the last version were actually on the whole test. So you will see that, in this version, the numbers are the same as those in the last version.]

4. The discussion of viability is not very well developed for us in p 10 lines 28 and following. Similarly the categorization procedures lines 40 -47 seem rather informal and quite under-discussed here since the data from such categorizations on pp13-15 form for me such an important contribution of the paper. We really get no idea of the logic used here and in reading the categories on pp 13-15 one can imagine that they are inter-related in many ways and even possibly logically subsumed one in another. There have been logical techniques used [e.g. maximally reduced conceptual sub-universes, Taylor-Pearce in a 1971 U. of Alberta thesis] to test this and factor analytic

techniques used to better understand the variety of responses [flexibility in your study] in some statistical way. I think this paragraph [40 to 46 on p 10] needs elaboration.
Response: Unfortunately, the researchers failed to sue the methodology suggested above. The original dissertation study where the data of this paper was drawn from was an exploratory study. The researchers will keep the recommendations in mind for future use.

5. Following Guilford there are techniques to investigate and "measure" novelty using whole sample set proportions and least upper bound techniques for filtering out unusual but less significant responses. I was surprised that the authors chose not to study novelty in this paper. They gave us a defense of this position but were there other ways of handling the data on this issue that would take into account of or compensate for the differences that prompted them to leave this out here.
Response: In Section 4.3.3 Originality, we included examples of original problems posed by the three groups of students and also why they thought the problems were original or creative.

6. The comments above not withstanding I think the contrasts regarding trivial and viable problems were well handled with viability providing an interesting contrast between site performances. The discussion at the top of p 12 lines 6-8 could profitably be expanded for the reader.
Response: Looking back, we agree that we could have asked the students about how and why they pose trivial or non-trivial, viable or non-viable problems. Since we do not have data on these topics, we found it hard to expand the discussion. Sorry.

7. How was the sub-categorization say on p 14 lines 15 -20 actually done. In what way can the author understand that these categories have some stable meaning? Similarly those on p 15.
Response: The categories were determined based on the researchers' own beliefs. Therefore, it is difficult to talk about how stable they are. Again, this was just an exploratory study. Better analysis will be done in the future.

8. p17 lines12-25 Is there some reason why the relationships between knowledge and problem posing/creativity were not probed here in some way? Line 20 suggests that the Jiaozhou students scored "much more highly" on the geometry knowledge items - in what way is this true and in fact are there any ways you could better defend this judgment?
Response: The relationships between knowledge and problem posing/creativity were not meant to be discussed in this paper. That will be the focus of another paper that was submitted to the special edition of Educational Studies in Mathematics. The title of the paper is "An Investigation of Relationships between Students' Mathematical Problem-posing Abilities and their Mathematical Basics". It was accepted with revisions and is being revised currently.

9. Two small changes:
- p 16 line 19 use "similar" not "similarly"
- p 5line 20 change to "from which the data of this paper were drawn"

Response: The two changes were made.

Creativity and mathematical problem posing: an analysis of high school students' mathematical problem posing in China and the United States

Xianwei Y. Van Harpen

Illinois State University

Bharath Sriraman

The University of Montana

Abstract In the literature, problem posing abilities are reported to be an important aspect/indicator of creativity in mathematics. The importance of problem posing activities in mathematics is emphasized in educational documents in many countries, including the United States and China. This study was aimed at exploring high school students' creativity in mathematics by analyzing their abilities in posing problems in geometric scenarios. The participants in this study were from one location in the United States and two locations in China. All participants were enrolled in advanced mathematical courses in the local high school. Differences in the problems posed by the three groups are discussed in terms of quality as well as quantity. The analysis of the data indicated that even mathematically advanced high school students had trouble posing good quality and/or novel mathematical problems. We discuss our findings in terms of the culture and curricula of the respective school systems and suggest implications for directions in problem posing research within mathematics education.

Keywords Advanced high school students; cross cultural thinking; geometry; mathematical creativity; novelty; problem posing; problem solving; U.S and Chinese students; rural and urban Chinese students

1 Introduction

Creativity is a buzz word in the 21st century often invoked by policy makers, scientists, industry, funding bodies, and last but not least systems of education worldwide. In fact the vision and/ or mission statements of most school districts in the U.S., and Canada include the word "creativity" in it. Until recently, the last decade of published research includes only a handful of articles focused specifically on mathematical creativity (Leikin, Berman, Koichu, 2010). This is even more amplified within the domain of mathematics education research in their scarcity in articles that tackle giftedness and/or creativity. For instance in *Educational Studies in Mathematics (ESM)*, one of the oldest journals in mathematics education, there are 6 articles that report on studies related to giftedness (high ability) and creativity in the last 40 years starting with Presmeg (1986). In 2010 two papers focused on creativity were published in ESM. Shriki (2010) tried to move beyond creativity as process versus product dichotomy in a study involving 17-prospective mathematics teachers participating in a series of creativity awareness developing activities. This study relied on teacher reflections as a way to understand how creativity awareness can be fostered among teachers. Bolden, Harries, and Newton (2010) used questionnaires and semi structured interviews with pre-service teachers in the U.K., to resolve differences between "teaching creatively" versus "teaching for creativity", the latter of which required a deeper understanding of mathematical conceptual knowledge. Both these papers targeted prospective mathematics teachers. Other than the studies reported by Sriraman (2003, 2004, 2005, 2008, and 2009) and Sriraman and Lee (2011), there are very few attempts to understand the nature of mathematical creativity in high school students when confronted with novel mathematical tasks. The present article continues this sequence of studies but from a cross cultural viewpoint involving high school students in China and the U.S.

2 Creativity Research

2.1 A Terse Survey

Creativity research in general is somewhat divisive- In psychology some view it as effects of divergent thinking, others view it as convergent thinking. Creativity is also viewed as domain specific by some and domain general by others (Plucker & Zabelina, 2009). The research literature on mathematical creativity has historically been sparse with an over reliance on the writings of eminent mathematicians of the 19th and 20th centuries (Brinkmann & Sriraman, 2009; Sriraman, 2005). Mathematicians like Henri Poincaré (1948), Jacques Hadamard (1945), Garrett Birkhoff (1956) have attempted to demystify the mathematician's craft and explain the mystery of "mathematical" creation (Sriraman, 2005). Early accounts of mathematical creativity (Hadamard, 1945; Poincaré, 1948) influenced by Gestalt psychology describe the creative process as that of *preparation-incubation-illumination and verification* (Wallas, 1926). A large part of the creative process remains a grey area so to speak, particularly the role of the unconscious in the incubatory period before any insight (or the Aha! moment) occurs. Paradoxically, these gestalt narratives *do not explain* the Gestalt or the whole of the creative process in any field per se and are also vague because they offer no insight specifically into the mathematician's mind. We have ample accounting and understanding of the starting and ending phases of creativity, but the "middle" phases, namely incubation and illumination is still a topic of interest to psychologists, neuroscientists and educators. Other reformulations of the incubatory phase are "endocept" which is defined as non-verbalized effects of (repressed) emotional experience (Ariete, 1976). Csiksentmihalyi (1996) coined the notion of "flow" to describe a middle phase of the creative process which is generative, that is, ideas are generated freely and affective dispositions described as fun, pleasure, even enrapture are found in the literature (Ghiselin, 1952).

More recently, a number of studies have specifically examined the role of an incubation period in creative problem solving. Sio and Ormerod (2007) conducted a meta-analytic[1] review

[1] There were 117 studies included in this meta-analysis that most of them support the existence of incubation effects on problem solving.

of empirical studies that investigated incubation effects on problem solving, and found that incubation is crucial is fostering insightful thinking. Psychologists term this *the fatigue hypothesis*, that is, the mind after a period of frenzied and intense activity requires a period of rest to overcome fatigue, and the relaxation during the period of rest results in new insights. According to this report and others similar to it (Vul & Pashler, 2007), understanding the role of the incubatory period may allow us to make use of it more efficiently in task designs to foster creativity in problem solving, classroom learning, and working environments. Mathematics educators try to incorporate incubation periods in classroom activity in temporal pauses during classroom discourse (Barnes, 2000) or extended time periods for problem based learning (Sriraman, 2003), and positive incubation results in positive effects in promoting students' creativity (Sriraman, 2004, Sriraman, 2005) and this seems to be self evident for mathematicians (Kaufman & Sternberg, 2006). There are recommendations based on this line of research that students should be encouraged to engage in challenging problems and experience this aspect of problem solving (Sriraman, 2008, 2009; Sriraman & Lee, 2011; Stillman et al., 2009).

2.2 Cross Cultural Studies

During the past four decades, a large number of international evaluation studies of school mathematics have been conducted. In most of these studies, U.S. students were outperformed by students in many other countries, especially students in East Asian countries. In most cross-national studies involving Chinese and U.S. students' mathematics performance that have been reported (e.g., Husen, 1967; Robitaille & Garden, 1989), Chinese students outperformed their U.S. counterparts. However, mathematics classes in China are often described as not conducive to effective learning (Wong, 2004). For example, the teaching method in the classroom was often described as "passive transmission" and "rote drilling" (e.g., Biggs, 1991). In order to understand this "paradox of the Chinese learner" (Marton, Dall'Alba, & Lai, 1993), many comparative studies have been conducted involving U.S. and Chinese students (e.g., Cai, 1995, 1997, 1998; Ma, 1999; Stevenson, 1993; Stevenson & Stigler, 1992; Vital, Lummis, & Stevenson, 1988). But at the same time, it is widely accepted in China that U.S. students are more creative in mathematics than Chinese students (e.g., National Center for Education Development, 2000; Yang, 2007). There are studies showing that U.S. students are better than Chinese students in solving open-ended problems (e.g., Cai & Hwang, 2002) and in posing problems in mathematics (e.g., Cai, 1997, 1998). Therefore, more and more researchers have started looking at the strengths of U.S. students' mathematics learning other than merely focusing on computational skills and routine problem solving. In general there is a lack of literature addressing the differences in mathematical creativity between Chinese and U.S students, or any other large scale cross national studies.

It is difficult to compare creativity in general terms between these two general populations due to significant cultural differences- the U.S being perceived as a highly individualistic society where creativity is more or less a cultural norm whereas China is perceived as a collectivist society where conformity is the norm (Hofstede, 1980). There are some large scale empirical studies that examine temperamental differences between U.S and Chinese children ranging between the ages of 9-15 that may shed light into cultural norms (Oakland & Lu, 2006). In

Oakland and Lu’s (2006) study analyzing[2] the temperamental dispositions on a bi-polar spectrum (extroversion-introversion, thinking-feeling, practical-imaginative, organized-practical) of 3539 U.S students with 400 Chinese students of the same ages, the reported finding was that Chinese children preferred extroversion to introversion, practical to imaginative, thinking to feeling, and organized to flexible styles. They found that although Chinese and U.S. children did not differ on extroversion-introversion styles, they differed on the three other temperament styles with Chinese children more likely to prefer practical, thinking, and organized styles, which may very well be reflective values prominent in either a collectivist or individualist society (p.192).

2.3 Creativity and problem posing

In Usiskin’s (2000) eight-tiered hierarchy of mathematical talent, students who are gifted[3] and/or creative in mathematics have the potential of moving up into the professional realm with appropriate affective and instructional scaffolding as they progress beyond the K–12 realm into the university setting (Sriraman, 2005). Therefore, gifted and/or creative students in mathematics have been of special interest to many researchers in the field of mathematics education. Hadamard (1945) posited the ability to pose key research questions as an indicator of exceptional talent in the domain of mathematics. This is consistent with the paradigm in psychology that creative thinking often manifests itself in divergent thinking abilities, and we develop our study within the well defined framework of problem posing/finding or problem generating being a feature of divergent thinking and hence of creativity (Runco, 1994; Torrance, 1988). To this end, we review some of the related literature on problem posing found in mathematics education.

Krutetskii (1976) and Ellerton (1986) contrasted the problem posing of subjects with different ability levels in mathematics. In Krutetskii’s study of mathematical “giftedness”, he used a problem-posing task in which there was an unstated question (e.g., “A pupil bought $2x$ notebooks in one store, and in another bought 1.5 times as many.”), for which the student was required to pose and then answer a question on the basis of the given information. Krutetskii argued that there was a problem that “naturally followed” from the given information, and he found that high ability students were able to “see” this problem and pose it directly; whereas, students of lesser ability either required hints or were unable to pose the question. In Ellerton’s (1986) study, students were asked to pose a mathematics problem that would be difficult for a friend to solve. She found that the “more able” students posed problems of greater computational difficulty (i.e., more complex numbers and requiring more operations for solution) than did their “less able” peers.

---

[2] Cross national Studies of temperamental styles are typically based on the Myers and Briggs’ theory of temperament and the associated psychometric test called Myers-Briggs Type Indicator (MBTI). Oakland et al., (1996) adapted the MBTI to detect cross national differences in children ranging from 8 through 17 on four bipolar temperament style dimensions, namely extroversion–introversion, practical–imaginative (MBTI’s judging–perceiving), thinking–feeling and organized–flexible (MBTI’s judging–perceiving). The adapted test is called the Student Styles Questionnaire (see Oakland et al, 1996).

[3] We do not enter into a discussion of the definition of mathematical giftedness in this paper. This is a well defined term in the research literature in gifted education. In this paper, the participants by virtue of their enrollment in the advanced mathematical courses were among the high achievers in their respective schools and included students of varying mathematical abilities.

According to Jay and Perkins (1997), “the act of finding and formulating a problem is a key aspect of creative thinking and creative performance in many fields, an act that is distinct from and perhaps more important than problem solving” (p. 257). Silver (1997) claimed that inquiry-oriented mathematics instruction which includes problem-solving and problem-posing tasks and activities can assist students to develop more creative approaches to mathematics. It is claimed that through the use of such tasks and activities, teachers can increase their students’ capacity with respect to the core dimensions of creativity, namely, fluency, flexibility, and originality (e.g., Presmeg, 1986; Torrance, 1988)

The purpose of this study was to investigate mathematically advanced high school students’ abilities in posing mathematical problems. Participants were junior or senior students (16-18 year olds) in high school. As stated before, very few studies have specifically focused on high school students as opposed to pre-service teachers. By focusing on these age levels, we aim to reveal the students’ problem posing abilities at their end of K-12 school education and, therefore, shed light on the students’ creativity in mathematics after their K-12 school education.

This study reports part of a dissertation study from which the data of this paper were drawn (Yuan, 2009). Among the three tasks in the problem posing test, only one is discussed and reported in detail in this paper. The study is also different to previous studies in the sense that we focus on problem posing as an important but overlooked and least understood aspect of mathematical creativity. In the history of mathematics, there are numerous papers considered as seminal not because they proved a long standing theorem, but because they opened up entirely new areas of mathematical inquiry such as Hewitt’s (1948) paper on rings of continuous functions, in addition to Hilbert’s (1900) famous 23 problems that shaped the 20th century of mathematics.

2.4 Operationalizing Problem Posing as Creativity

The topic of problem posing has been of interest to the research community in the past decades, however, there is a lack of theory concerning problem posing. In 1982, Dillon claimed that no theory of problem finding had been constructed and that there are several different terms such as problem sensing, problem formulating, creative problem-discovering, problematizing (Allender, 1969; Bunge, 1967; Taylor, 1972). Similarly, Stoyanova and Ellerton (1996) proposed that research into the potential of problem posing as an important strategy for the development of students’ understanding of mathematics had been hindered by the absence of a framework which links problem solving, problem posing and mathematics curricula. Building on Guillford’s (1950) structure of the intellect, the framework proposed by Stoyanova and Ellerton (1996), classified a problem-posing situation as free, semi-structured or structured. According to this framework, a problem-posing situation is referred to as free when students are asked to generate a problem from a given, contrived or naturalistic situation (see Example 1 below). A problem-posing situation is referred to as semi-structured when students are given an open situation and are invited to explore the structure of that situation, and to complete it by applying knowledge, skills, concepts and relationships from their previous mathematical experiences (see Example 2 below). A problem-posing situation is referred to as structured when problem-posing activities are based on a specific problem (see Example 3 below). All three examples below are taken from Stoyanova (1998). In this study we make use of problem posing activities to study mathematical creativity in advanced high school mathematics students, and compared to existing studies that report on either students identified as gifted, or prospective mathematics teachers, our focus is on groups of students with variations in high mathematical ability.

Example 1: Make up some problems which relate to the right angled triangle. (p. 64)

Example 2: Last night there was a party and the host's doorbell rang 10 times. The first time the doorbell rang only one guest arrived. Each time the doorbell rang after that, three more guests arrived than had arrived on the previous ring. Ask as many questions as you can. Try to put them in a suitable order. (p. 66)

Example 3: Some integers are arranged in the way shown below:

1

2 3 4

5 6 7 8 9

10 11 12 13 14 15 16

17…………………………………… 25

(a) What would be the third number from the left of the 89th row of the accompanying triangular number pattern?

(b) State other meaningful questions. (p. 70)

3 Methodology

3.1 Participants

According to Peverly (2005), even within the one country, different locations in China can vary greatly in terms of culture. Thus this study selected students from two locations in China: Shanghai —an economically well developed city in the south of China and Jiaozhou —a small city that is considered as having strong historical roots in Confucian culture in the north of China. Students from the United States were from Normal[4], Illinois — a mid-western town in the United States. The U.S. students in this study were from two advanced placement Calculus classes and two Pre-Calculus classes. Those students were in the 11th or 12th grade.

In China, high school students usually are divided into two strands, namely, a science strand and an art strand. After the first semester in high school, students choose a strand and are

[4] The reader may be surprised to learn that the term "normal" schools for teachers colleges comes from the first such school in Normal, Illinois.

assigned to different classes. Science strand students take more advanced mathematics courses in high school than arts strand students. In the school in this study in Jiaozhou, in each grade, there are two art strand classes and ten science strand classes, two of which are “express” or accelerated science strand classes. Students in these two express science strand classes were admitted according to their achievement (total score of five subjects, namely, mathematics, Chinese literature, English, physics, and chemistry) in the high school entrance examination of the city, which they took after the 9th grade immediately before they entered the high school. The class in this study is one of the two 12th grade express science strand classes. Similarly to the Jiaozhou participants, the Shanghai participants in this study came from two 11th grade science strand classes and the two classes were also the top two among the ten 11th grade classes in the high school. Therefore, the Chinese participants can be considered as advanced in mathematics.

Although participants in this study were from three very different locations, by choosing students from advanced classes in high schools in each of the three locations, the researchers managed to focus on mathematically advanced high school students in each of the three locations. Initially, 68 Jiaozhou students, 73 Shanghai students, and 77 U.S. students agreed to participate in this study. However, in the dissertation study where the data of this paper was drawn from, there were four tests to take and some students had to miss one or two of the tests, therefore, not all the participants’ test papers were analyzed. In the end, 55 Jiaozhou participants, 44 Shanghai participants, and 30 U.S. participants were present for all the tests. Among the 30 U.S. students, 17 were female and 13 were male; 17 were from Advanced Placement Calculus Course students and 13 were from Pre-Calculus Course students. Among the 44 Shanghai students, 19 were female and 25 were male; all of the Shanghai students were in the 11th grade. Among the 55 Jiaozhou students, 18 were female and 37 were male; all of the Jiaozhou students were in the 12th grade.

3.2 Measures and instrumentation

The measures and instrumentation in this study include a mathematics content test and a mathematical problem-posing test. Both of the two tests were translated into Chinese for the participants in China. Several pilot tests were conducted before it was used for the study.

3.2.1 The mathematics content test. The purpose of the mathematics content test in this study is to measure the participants’ basic mathematical knowledge and skills. Instead of developing a test for this study, the researchers adapted the National Assessment of Educational Progress (NAEP) 12^{th} grade Mathematics Assessment as the mathematics content test because this assessment fits the purpose of this study very well. NAEP is the only nationally representative and continuing assessment of what America’s students know and can do in various subject areas (National Center for Education Statistics, 2009). The 2005 mathematics framework focuses on two dimensions: mathematical content and cognitive demand. By considering these two dimensions for each item in the assessment, the framework ensures that NAEP assesses an appropriate balance of content along with a variety of ways of knowing and doing mathematics. The 2005 framework describes four mathematics content areas in high school: number properties and operations, geometry, data analysis and probability, and algebra.

3.2.2 The Mathematical problem-Posing Test. Using Stoyanova and Ellerton's (1996) framework of mathematical problem posing, three situations were included in the mathematical problem-posing test, namely, free situation, semi-structured situation, and structured situation. The mathematical problem-posing test was developed based on Stoyanova's (1997) and Cai's (2000) research. There are three tasks in the mathematical problem-posing test:

Task 1. (Free problem-posing situation) There are 10 girls and 10 boys standing in a line. Make up as many problems as you can that use the information in some way.

Task 2. (Semi-Structured problem-posing situation) In the picture below, there is a triangle and its inscribed circle. Make up as many problems as you can that are in some way related to this picture. The problems could also be real-life problems. Again, do not limit yourself to the problems you have seen or heard of – try to think of as many possible and challenging mathematical problems as you can.

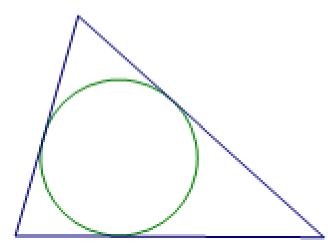

Figure 1. Semi-structured problem posing situation.

Task 3. (Structured problem-posing situation) Last night there was a party at your cousin's house and the doorbell rang 10 times. The first time the doorbell rang only one guest arrived. Each time the doorbell rang, three more guests arrived than had arrived on the previous ring.

a. How many guests will enter on the 10^{th} ring? Explain how you found your answer.

b. Ask as many questions as you can that are in some way related to this problem.

To encourage participants to try their best in posing mathematical problems, the following scenario was added in the beginning of the problem posing test.

Imagine that your school is participating in a problem posing competition in mathematics among all the high schools in town. The schools that generate the most problems or/and the best quality problems will be rewarded. In addition, the students who pose the most number of problems or/and the best quality problems will be rewarded. Last week, student Jenny from another high school created 5 really good problems for each of the three situations below. Jenny also bragged that no one else could do better than she did. Now, try to prove her wrong by making up as many problems as you can. Do not limit yourself to the problems you have seen or heard of – try to think of as many possible and challenging mathematical problems as you can.

In the larger study from which the data of this paper were drawn (Yuan, 2009), participants' responses to the problem posing test showed that the contexts of Task 1 and Task 3 had a significantly different influence on the participants' thinking processes due to the differences in the participants' culture (Van Harpen & Presmeg, 2011). For example, a Chinese student posed the following problem for Task 1. It is a common practice for a class to have a monitor, who helps the teachers to keep the students well behaved, and a class representative, who helps the subject teacher to hand out and collect student work. That is not a common scenario in the United Students.

Problem: A class of 10 students are to select a monitor. A Chinese class representative, and a mathematics class representative, how many different ways of filling in the three positions are there? One person can at most take two jobs.

A U.S. student Deanna posed a problem about parking cars for Task 3. In China, different from in the United States, it is not common for people to have their own cars or trucks.

Problem: If they parked their cars in a straight line, how long would it be? ½ of the guests drove 6 feet cars, ¼ of them drove 5 feet cars, and ¼ of them drove 9 feet trucks.

Since Task 2 only involves a geometric figure, it directed participants' attention to be more focused on the mathematics than the other two tasks. As a result, Task 2 is more culturally fair to the participants in the three groups. The length of this paper was another concern of the authors. Therefore this paper only reports data on the semi-structured problem posing situation.

3.3 Interview with the students

Eight students in the Jiaozhou group, twelve students in the Shanghai group, and twelve students in the U.S. group were interviewed. The purpose of the interviews was to find how the problems were generated so that the researcher could see the differences in the mathematical problem-posing processes between the three groups. All interviews were conducted by the first author. The interviews were audio taped and transcribed.

3.4 Data collection procedures

Since the researchers were based in the United States, both tests were administered in Chinese by the mathematics teachers of the classes in each of the two locations in China. With the U.S. students, the principle researcher (the first author) conducted the mathematical problem posing test in person. The mathematics content test was given by the mathematics teacher of the class due to a time conflict. The working time for the mathematical problem-posing test and the mathematics content test were both 50 minutes for all the students.

3.4.1. Data Collection in China. In the Shanghai high school, students have a 50 minutes self-study period between lunch and the first class period in the afternoon. The two tests were conducted two weeks apart during the self-study period. In the Jiaozhou high school, students are required to attend four 50 minutes-self-study periods every Saturday morning. The two tests were given two weeks apart during the Saturday morning self-study period. The mathematics content test required that each student have the same set of tools, including a measuring ruler, a protractor, a spinner, etc. The tools were purchased in the United States and were sent to China before the tests were conducted. The test was sent to the teachers through email and the teachers then printed the tests ready to be used. Similarly to the mathematics content test, the mathematical problem posing-test was also sent to the teachers in China via email and the teachers then printed them ready to be used with the students. No tool was needed for this test.

3.4.2. Data Collection in the United States. Because the U.S. participants in this study were at a university school, where teachers and students were more willing to participate in educational research, the teacher allowed the principle researcher to take regular class time to conduct the tests. Due to the time conflict, the principle researcher had to ask the classroom teacher to give the Mathematics Content Test to the students. The same tools were provided, including the ruler, the compass, the spinner, and the marked papers for the combination problems. The reason that this test can be given by a different person is that this test does not require any instructions other than handing out test papers, timing, and collecting test papers.

4 Data Analysis and Results

Since this paper only reports part of the data from a dissertation (Yuan, 2009), only part of the results is discussed in this section. Since the semi-structured problem posing situation task is the focus of this paper, and this task is about geometry, for the mathematics content test, only scores on geometry tasks are reported.

4.1 The data analysis and results of the mathematics content test

There are 50 items in the mathematics content test. After ranking the individual scores, a Kruskal-Wallis test was used to evaluate differences among the three groups. The outcome of the test indicated significant differences among the three groups, H=82.131 (2, N= 129), $p < .05$ two tailed. Also, Mann–Whitney U test was used to evaluate differences between each pair of the three groups. The outcome of the test indicated that there are statistically significant differences between the U.S. group and the Jiaozhou group (Mann–Whitney U = 64, $n1$ =30, $n2$ = 55, $p < .05$ two-tailed), that there are significant differences between the Shanghai group and the Jiaozhou group (Mann–Whitney U = 64, $n1$ =44, $n2$ = 55, $p < .05$ two-tailed) , and that there is no significant difference between the U.S. group and the Shanghai group (Mann–Whitney U = 653, $n1$ =30, $n2$ = 44, $p > .05$ two-tailed).

Among the 50 items in the mathematics content test, 16 are about geometry. The averages of the three groups are 14.8 for Jiaozhou group, 11.4 for Shanghai group, and 12.5 for U.S. group.

4.2 The data analysis of the mathematical problem-posing test

The problems posed by the participants in the mathematical problem-posing test were first judged as to their viability. Responses that are not viable were eliminated from further consideration. For example, responses such as “Find the area of the circle” without any other additional information were eliminated. The remaining responses that are viable were scored according to the rubrics in terms of their fluency and flexibility. The rubrics were developed by the researchers following these steps:

1. Typed all the responses into a Microsoft Word document and recorded the frequency with which each of the responses occurred. The responses generated by the three different groups of students were separated so that the researchers could see the differences among the groups.
2. Categorized the responses. As the responses were being recorded, different themes emerged, for example, some responses focused on the lengths in the figure and some response focused on the area in the figure. There are times when different concepts were involved in one problem. The researchers decided to categorize the problems according to the question of the problem. There are also problems that are difficult to fit in any of the categories. The researchers decided to put them in a category called "Others". Table 1 gives an example of each category.

Table 1
Examples of the mathematical problem-posing test categories

Categories	Examples
1. Analytical geometry	In triangle *ABC*, the coordinates of the vertex are given, *B* (0,0), *A*(2,1), and *C*(5,-1). *BD* is the height. Find the equation of *BD*. (148)
2. Lengths	If the triangle is a right triangle, the hypotenuse is 2, another angle is 60 degrees, find the radius of the circle.
3. Area	Given the radius of the circle *r*, find out the minimum area of the triangle.
4. Angles	Construct two perpendicular segments from the center of the circle to the two sides of the triangle. The angle formed by the two segments is 120 degrees. Find the angles of the triangle.
5. Transformation	What degree will the triangle have to rotate for point A to be where point B is?
6. Involving auxiliary figures	Draw a tangent line of the circle and intercept the triangle at D and E. The vertex of the triangle between D and E is M. Find the range of MD/ME.
7. Three-Dimension	The radius is 5. What is the maximum volume of a ball that can go through?
8. Probability	If you are to drop something to the circle, what is the probability of it falling into the triangle?
9. Proofs	Given triangle ABC, and D, E, F are the mid points of AB, BC, and CA. Prove that AD=AF and |AB-AC|=|BE-EC|.
10. Others	Plant 6 different flowers in the four areas and no adjacent two can be the same color. How many different ways?

After the principal researcher came up with the categories, she had her co-researchers go through them. The differences between the researchers' coding were discussed and the categories were refined over the course of three months in 2008 summer. The responses of students from the three groups to the mathematical problem-posing test were categorized. It turned out that the categories are not the same for the three samples. For example, Jiaozhou students posed dilation problems (e.g., Construct a figure twice as big as the original one using a ruler and a compass.) but U.S. students and Shanghai students did not. After the responses generated by each group of students were categorized, all the categories were combined to make a common rubric for all the three groups. For example, the Dilation category in the Jiaozhou rubric was subsumed into the Transformation category in the common rubric. The total number of viable problems generated by a student is defined as his/her fluency score. The total number of categories that a student's viable problems involve is defined as his/her flexibility score, and is not necessarily the same as the fluency score. (See Table 1 for examples of the mathematical problem-posing test categories).

3. Determined the originality of each of the responses. The originality of the responses in this test was determined by their rareness. Since students in the three groups have different textbooks and instruction, one rare response in one group might not be rare in another group. Therefore, the originality of the responses was relative to other students in the same group. For that reason, the originality was analyzed separately among the three groups and was not compared across groups. The researchers decided that if one problem was posed by 10 percent or more of the participants in the corresponding group, the problem would be considered as not original. In addition, there are problems that were posed by less than 10 percent of the total number of participants but were not considered as original. For example, the following problem is not considered as original because the mathematics involved in the problem is at a very low level to a high school student.

> *If there are four girls with brown hair and two more boys with brown hair than girls, how many people do not have brown hair?*

In scoring the responses generated by the students in this study, two researchers scored the same six copies of test papers and compared the scores.

4.3 The results of the mathematical problem-posing test

A comparison of the means and medians of the students' scores on the mathematical problem-posing test showed that Jiaozhou students and U.S. students' fluency and flexibility are similar and are both higher than those of the Shanghai students (See Table 2).

Table 2
Comparison of students' fluency scores and flexibility scores

	U.S. students	Shanghai students	Jiaozhou students
Mean of fluency scores	4.6	2.0	4.9
Median of fluency scores	4	1.5	5
Mean of flexibility scores	3.9	1.6	4.1
Median of flexibility scores	4	1	4

4.3.1 Fluency. As discussed above, in analyzing the problems generated by the students, the problems that are non-appropriate (e.g., How old are the children?) and problems that lack the information needed to determine solution (e.g., How many girls and how many boys are there at the party?) were excluded from further analysis. Those problems were considered as nonviable problems. Since the numbers of students in each of the three groups were different, the average percentage of non-viable problems generated by the students in each group was calculated with the following division:

$$\frac{\text{Number of nonviable problems}}{\text{Number of nonviable problems + Number of viable problems}}$$

It should be pointed out that the criteria classifying problems as viable or nonviable were based on the researchers' judgment. 31% of the U.S. students' problems, 42% of the Shanghai students' problems, and only 15% of the Jiaozhou students' problems were non-viable problems. Many students posed problems such as "what is the area of the circle" or "what is the area of the triangle" without giving the measures of the radius or the sides. Notice that Jiaozhou students posed the least percentage of non-viable problems, which means that Jiaozhou students tended to give necessary information for the problems to be solvable.

After the non-viable problems were eliminated, all the viable problems were analyzed for their triviality. For example, the following problem is considered as a trivial problem since the mathematics involved in the problem is at a very low level for a high school student.

If the diameter of the circle is 32, what is the circumference?

The percentages were calculated by doing the following division:

$$\frac{\text{Number of trivial problems}}{\text{Number of viable problems}}$$

9% of the U.S. students' viable problems, 8% of the Shanghai students' viable problems, and 6% of the Jiaozhou students' viable problems were trivial problems.

4.3.2 Flexibility. In counting the number of problems generated by the students in each group, the same problems generated by the same group of students were counted once. For example, the following two problems were counted as one problem and were categorized as "Given the three sides of the triangle, find the area of the inscribed circle".

Problem 1: Given that the three sides of the triangle are 3, 4, and 5, find the area of its inscribed circle.
Problem 2: Given that the three sides of the triangle are 5, 6, and 7, find the area of the circle.

Table 3 shows the distribution of the different categories posed by different groups of students. Consistently, for the three groups, the biggest two categories are Length and Area. For U.S. students and Jiaozhou students, the Area category is the largest one and the Length category is the second one. For Shanghai students, the Length category is the first and the Area category is the second.

Table 3
Distribution of the categories of the three groups' viable problems

Groups	1	2	3	4	5	6	7	8	9	10	Total
U.S.	1	39	44	3	0	3	6	3	0	7	106
(%)	(0.9)	(36.8)	(42)	(2.8)	(0)	(2.8)	(5.7)	(2.8)	(0)	(6.7)	
Shanghai	0	27	22	2	0	10	1	0	0	9	71
(%)	(0)	(38)	(31)	(2.8)	(0)	(14.1)	(1.4)	(0)	(0)	(12.7)	
Jiaozhou	11	48	61	8	1	24	14	8	10	15	200
(%)	(5.5)	(24)	(30.5)	(4)	(0.5)	(12)	(7)	(4)	(5)	(7.5)	

Note:
1. Analytical geometry
2. Lengths
3. Area
4. Angles
5. Transformation
6. Involving other figures
7.Three-D
8. Probability
9. Proofs
10. Others

However, not all the three groups posed problems for all 10 categories. The U.S. students did not pose problem involving categories 5 and 9, which are Transformation and Proof. The Shanghai students did not pose problems involving categories 1, 5, 8, and 9, which are Analytical geometry, Transformation, Probability, and Proof. The Jiaozhou students posed problems that covered all the 10 categories.
As to Category 1, Analytical geometry, only 0.9% of the U.S. students' problems were in this category and none of the Shanghai students posed problem of this category. Jiaozhou students, different from the other two groups, posed 11 problems of Analytical geometry category. See the following problem for an Analytical geometry example.

Point B and C are fixed. Point A is movable. |BC|=4 and |AC|-|AB|=2. Find the locus of A.

Another observation is that both Shanghai students and Jiaozhou students posed more problems that involve auxiliary figures (14.1% and 12%); while not many problems of that category were posed by the U.S. students (2.8%). For example,

a) Adding lines: Draw a tangent line of the circle and intercept the triangle at D and E. The vertex of the triangle between D and E is M. Find the range of MD/ME.
b) Adding triangles: If there is an inscribed triangle similar to the original one, find out the ratio of the area of the two triangles.
c) Adding circles: If the triangle is inscribed in another circle, find the ratio of the area of the two circles.

d) Adding quadrilaterals: AB, BC, and AC are given. Build a rectangle in the circle. Find the rectangle with the largest area.

Jiaozhou students posed ten problems of Category 9, Proof, while none of the U.S. students or the Shanghai students did. See the following problem for example.

Given triangle ABC, D, E, and F are the midpoints of AB, BC, and CA. Prove that AD=AF and |AB-AC| = |BE-EC|.

Distribution of sub-categories. Some of the categories are subdivided into subcategories. A closer look at those categories shows that within the subcategories, the distribution is very different, too. For example, within the Lengths category and Area category, seven subcategories appear in each (as shown in Figure 3 and Figure 4). Therefore, although Lengths and Area are the top two categories for all the three groups, the distribution of the sub-categories varies greatly.

Table 4
Distribution of subcategories of Category 1—Length

Groups	a	b	c	d	e	f	g	Total
U.S.	11	6	1	5	3	2	11	39
(%)	(28.2)	(15.4)	(2.5)	(12.8)	(7.7)	(5.1)	(28.2)	
Shanghai	7	5	0	6	3	6	0	27
(%)	(25.9)	(18.5)	(0)	(22.2)	(11.1)	(22.2)	(0)	
Jiaozhou	2	2	4	16	0	16	9	49
(%)	(4.1)	(4.1)	(8.2)	(32.7)	(0)	(32.7)	(18.4)	

Note:
a. Lengths of the sides or the perimeter of the triangle
b. Circumference of the circle
c. Ratio of the triangle's perimeter and the circle's circumference
d. Radius of the circle
e. Height of the triangle
f. Other quantities related to lengths
g. Involving real life contexts

Table 4 shows that Jiaozhou students posed much fewer problems of category a, b, and e, which involve finding the lengths of the sides of the triangle, the height and perimeter of the triangle, and the circumference of the circle. Those are more “straight forward” problems. Instead, Jiaozhou students seemed to focus more on category d, f, and g, which involve finding the radius or the circle, other quantities related to lengths, and problems that involve real life contexts. Another finding is that Shanghai students did not pose problems that involve real life contexts. That might indicate the preference in their mathematics instruction.

Table 5
Distribution of subcategories of Category 2—Area

	a.	b.	c.	d.	e.	f.	g.	total

U.S.	9	8	14	1	3	0	9	44
(%)	(20.5)	(18.2)	(31.8)	(2.3)	(6.8)	(0)	(20.5)	
Shanghai	11	2	2	1	6	0	0	22
(%)	(50.0)	(9.1)	(9.1)	(4.5)	(27.3)	(0)	(0)	
Jiaozhou	8	21	2	1	18	1	10	61
(%)	(13.1)	(34.4)	(3.3)	(1.6)	(29.5)	(1.6)	(16.4)	

Note:
a. Area of the triangle
b. Area of the circle
c. Area of the difference between the triangle and the circle
d. Sum of the areas of the triangle and the circle
e. Involving ratio of the areas
f. Other quantities related to areas
g. Involving real life context

Table 5 shows that both Shanghai students and Jiaozhou students posed more than 25% of their problems of category e—problems involving ratio of the two areas; while U.S. students posed more than 30% of their problems of category c—problems involving the difference between the two areas. Again, Shanghai students did not pose problems involving real life contexts.

4.3.3. Originality. As mentioned earlier in Section 4.2, for the U.S. group, in which there are totally 30 participants, if three or more than three participants, which is more than but including 10 percent of the 30 participants, then it is considered as not original. For the Shanghai group, in which there are totally 44 participants, the researchers decided that if one response was posed by four or more than four participants, which is about 10 percent of the 44 students, then it is considered as not original. For the Jiaozhou group, in which there are totally 55 participants, the researchers decided that if one response was posed by six or more than six participants, which is about 10 percent of the 55 students, then it is considered as not original. Below are three examples that are considered as original problems according to the criteria within the group. See Yuan and Presmeg (2010) and Yuan and Sriraman (2011) for more details on the originality of the posed problems.

A U.S. example: If AC=100 m, AB=30 m, and BC=75 m, what is the circumference of the circle? What if the triangle was inscribed inside a circle?

A Shanghai example: If the perimeter of the triangle is 20, find the maximum and minimum value of the circumference of the circle.

A Jiaozhou example: Given the sum of the three sides of the triangle ABC m, the center of the circle O, find the range of $|OA|+|OB|+|OC|$.

Some participants were interviewed to find out the thinking process in their problem posing. For example, one question asks participants which problems they posed were creative and why they thought so. U.S. student Kurt reported the following problem as creative.

What is the perimeter of the triangle if the diameter of the circle is 1?

Kurt explained that "(It's creative) just because a lot of theorems are involved to get to the right answer."

Shanghai participant Zhenyu posed the following problem.

In the right triangle ABC, A (0, 3), B (4, 0). The circle is inscribed in the triangle. If point P starts moving from point B to A, when |PC|+|PB| reached its maximum, what are the coordinates of point P?

Zhenyu likes the above problem and thinks it is creative because "it involves motion".

Jiaozhou participant Yanan posed the following problem:

If the two sides of the triangle are 3 and 6, find out the perimeter of the triangle when the area of the inscribed circle is the maximum.

Yanan explained that "I think it is creative to involve the area of the circle and the perimeter of the triangle."

5 Discussion and Concluding Points

In the problem posing test, the students were told that "Do not limit yourself to the problems you have seen or heard of - try to think of as many possible and challenging mathematical problems as you can". Despite of that information, students from the three groups were not able to pose many challenging problems. Some of the problems posed by the students lacked necessary information to find a solution for. Among the viable problems, some of them were not challenging at high school level. In other words, students' scores on Fluency were not as high as expected. The analysis of Flexibility showed that, although students posed problems of diversity as a group, most of the problems focused on two main categories, which are Area and Length. Although scores on Originality were not compared across groups, interviews with students who posed rare problems revealed a variety of reasons why the problems were considered as creative.

The findings of this study suggest that, despite the emphasis placed on this topic by the educators and governors in the United States and China (e.g., NCTM, 1989, 2000; Mathematics Curriculum Development Group of Basic Education of Education Department, 2002), problem posing is not an established element in instruction in the classrooms yet. In addition, the participants in this study were from advanced mathematics courses in high school; even those students did not perform very well on the mathematical problem-posing test. This suggests that students who are good at solving routine mathematical problems or taking routine mathematical tests might not be good at posing mathematical problems. Below, the authors attempt to explain the findings from different perspectives and also suggest future directions in research on mathematical problem posing.

5.1 Influence of Curricula

The differences in the three groups of students' performances on the problem posing test can at least partly be explained by the differences in the mathematics content they have learned. As mentioned earlier, participants from Jiaozhou, China, were in 12th grade. These students have taken topics such as introductory 3-D geometry, introductory analytical geometry, 2-D vectors, and transformation in their first year of high school. Since these students are in the science strand, they have also taken 3-D vectors and 3-D geometry in their second or third year of high school. The curriculum structure in Shanghai is very similar to that in Jiaozhou. However, since the Shanghai participants were in the 11th grade, which is the second year in high school, they have not taken as high level geometry courses as Jiaozhou participants had. In the U.S. high school, students take geometry in their first year where they are introduced to the basic postulates and theorems of geometry. For students who take pre-calculus, they get to study plane and solid analytic geometry in their third year. Since the U.S. students in this study were in pre-calculus (third year) or AP calculus course (fourth year), they should have studied high level geometry content over the years.

The results of the mathematics content test suggest that, although the participants were all taking advanced courses in their school, the U.S. participants and Shanghai participants' basic mathematical content knowledge is not as strong as the researchers expected. That might be explained by the following differences. Jiaozhou students were in their last year of high school and they would need to take the college entrance examination in six months. Therefore they needed to sustain their knowledge till the end of their high school. Shanghai students would take the college examination in 18 months and had not learn all the mathematics content yet. U.S. students did not need to take any college entrance examination. These differences suggest that Jiaozhou students' mathematics was stronger when they were tested and that also might help explain why Jiaozhou students posed problems of more diversity than the other two groups, more problems of the "Analytical Geometry" category than the other two groups. The differences in the distribution of the categories of posed problems suggest that the problems posed by students might be related to students' background mathematical knowledge. In a sense this echoes the claim that basic knowledge and basic skills in mathematics could be highly related to creativity in mathematics (Zhang, 2005), as opposed to viewing basic skills as rote or non-creative.

5.2 Implication on relationships between students' mathematical basic knowledge and mathematical problem-posing abilities

The findings from this study indicated that there are differences in the mathematical problem posing abilities among the three groups. The Jiaozhou group posed fewer nonviable problems and fewer trivial problems than the Shanghai group and the U.S. group. This result contradicts those found by Cai and Hwang' (2000), who studied sixth graders' mathematical problem posing and found out that although Chinese students did better in computation skills and solving routine problems, U.S. students performed as well as or better than those Chinese students in problem posing tasks. Again the implication is that students' problem posing abilities might be affected by their mathematical knowledge. Students from Jiaozhou in this study scored much more highly than the other two groups in the mathematics content test and the Jiaozhou students also did much better in the mathematical problem-posing test. The superior performances of Jiaozhou students in the mathematics content test and the mathematical problem-posing test suggest that there might be some correlation between the two.

In fact, in China, educators (e.g., Zhang, 2005) have reflected on the mathematics education in the past and claimed that the basic knowledge and basic skills in mathematics might or might not be highly related to creativity in mathematics, but there is definitely a kind of balance between them. Wong (2004, 2006) summarized the characteristics of the Confucian Heritage Culture (CHC) learners' phenomenon and pointed out that the Chinese students' focus on the basics might be related to the ancient Chinese tradition of learning from "entering" to "transcending the way". Wong's observation echoes that of Gardner' (1983) that imitating the master is the starting point of the path to becoming the master one day. Future research in the correlations between mathematics content knowledge and mathematical problem posing will help to validate the observations by Wong and Gardner.

5.3 Limitations of this study

In this study, participants were selected from three locations, a big city in China, Shanghai, a small city in China, Jiaozhou, and a town in the United States. Shanghai students were in the 11th grade. Jiaozhou students were in the 12th grade. Some of the U.S. students were in the 11th grade and some were in the 12th grade. The students in the three locations do not have the same mathematics curriculum. Thus the differences in the mathematical background and contexts of the three groups constituted a limitation of this research. In addition, the students were not selected randomly within the three student populations. Therefore, the findings of this study cannot be generalized to other students in the three locations.

Also, since the principal researcher of this study was based in the United States, she could not go to China to implement the tests in person. The tests given to the Chinese students in this study were all administered by the classroom teachers. Thus, it is hard to know how seriously the Chinese participants took the tests seriously. For example, Shanghai students took the tests during their self-study period between lunch and the first class period in the afternoon and it turned out that they did poorly on both the mathematical problem posing test and the mathematics content test. That indicates that some students might not have done their best on the tests due to fatigue or attitude.

5.4 Importance of problem posing research

Problem solving research has often been criticized as having reached an impasse (English & Sriraman, 2010). Polya's (1945) oft cited work provided the impetus for the ensuing research that took place in the following decades, which included focus on novice versus expert problem solving (e.g., Anderson, Boyle, & Reiser, 1985), problem solving strategies and meta-cognitive processes (e.g., Lester, Garofalo, & Kroll, 1989), and problem posing (English, 1997; Brown & Walter, 1983). However problem posing has not received the same attention as the other aforementioned areas. Problem posing has been researched to an extent with younger learners in the context of combinatorial situations (Sriraman & English, 2004) and more recently problem posing has come to the foreground in the area of mathematical modeling in the elementary and middle grades (English, 2007), but in general has received scant attention as an aspect of mathematical creativity. This study indicates the necessity for more inquiry into this line of research within mathematics education, in which learners are presented with problem posing opportunities in different areas of school mathematics, with the goal of stimulating creativity in intra-mathematical thinking as demonstrated by the Jiaozhou students, as well as diverse mathematical thinking to generate problems that are contextually different. A larger goal of bringing problem posing to the foreground in the study of mathematical creativity is to develop culturally congruent instruments that can be used to conduct larger empirical studies that compare cross-national differences. This study can be viewed as a starting point in this direction.

Acknowledgements

The first author would like to thank her dissertation committee members for their patience and expert guidance. They are Dr. Norma Presmeg (the committee chair), Dr. Nerida Ellerton, Dr. McKenzie Clements, Dr. Bharath Sriraman, and Dr. John Rugutt.

References

Allender, J. S. (1969). A study of inquiry activity in elementary school children. *American Educational Research Journal, 6*, 543-558.

Anderson, J. R., Boyle, C. B., & Reiser, B. J. (1985). Intelligent tutoring systems. *Science*, 228, 456-462.

Ariete, S. (1976). *Creativity: the magic synthesis*. New York: Basic Books.

Barnes, M. (2000). Magical moments in mathematics: Insights into the process of coming to know. *For the Learning of Mathematics*, 20(1), 33-43.

Biggs, J. B. (1991). Approaches to learning in secondary and tertiary students in Hong Kong: Some comparative studies. *Educational Research Journal*, *6*, 27-39.

Birkhoff, G.D. (1956): Mathematics of Aesthetics. In: Newman, J.R. (ed.): *The World of Mathematics,* Vol. 4, 7th edition. New York: Simon and Schuster, pp. 2185-2197.

Bolden, D.S., Harries, T.V., Newton, D.P. (2010). Pre-service primary teachers' conceptions of creativity in mathematics. *Educational Studies in Mathematics*, 73(2), 143-157.

Brinkmann, A. & Sriraman, B. (2009). Aesthetics and creativity: An exploration of the relationship between the constructs. In B.Sriraman & S. Goodchild (Eds) *Festschrift celebrating Paul Ernest's 65th Birthday* (pp. 57-80) Information Age Publishing, Charlotte, NC.

Brown, S. & Walter, M. (1983). *The Art of Problem Posing.* Philadelphia: Franklin Press

Bunge, M. (1967). *Scientific research* (Vol. 1). Berlin, NY: Springer-Verlag.

Cai, J. (1995). A cognitive analysis of U.S. and Chinese students' mathematical performance on tasks involving computation, simple problem solving, and complex problem solving. *Journal for Research in Mathematics Education monograph series.* Reston, VA: National Council of Teachers of Mathematics.

Cai, J. (1997). Beyond computation and correctness: Contributions of open-ended tasks in examining U.S. and Chinese students' mathematical performance. *Educational Measurement: Issues and Practice, 16*(1), 5-11.

Cai, J. (1998). An investigation of U.S. and Chinese students' mathematical problem posing and problem solving. *Mathematics Education Research Journal, 10*(1), 37-50.

Cai, J., & Hwang, S. (2002). Generalized and generative thinking in U.S. and Chinese students'mathematical problem solving and problem posing. *Journal of Mathematical Behavior, 21*(4), 401-421.

Csikszentmihalyi, M. (1996). *Creativity, flow and the psychology of discovery and invention*. New York: Harper Collins.

Ellerton, N. F. (1986). Children's made-up mathematics problems: A new perspective on talented mathematicians. *Educational Studies in Mathematics, 17*, 261-271.

English, L.D. (1997). The development of 5th grade students problem-posing abilities, *Educational Studies in Mathematics, 34,* 183-217.

English, L. D. (2007). Complex systems in the elementary and middle school mathematics curriculum: A focus on modeling. In B. Sriraman (Ed.), *Festschrift in Honor of Gunter Torner. The Montana Mathematics Enthusiast,* (pp. 139-156). Information Age Publishing, Charlotte, NC.

English, L.D., & Sriraman, B. (2010). Problem solving for the 21st century. In B. Sriraman & L. D. English (Eds.), *Theories of mathematics education: Seeking new frontiers* (pp. 263-285). Advances in Mathematics Education, Series: Springer Berlin/Heidelberg.

Gardner, H. (1989). *Frames of mind: The theory of multiple intelligences*. NY, Basic Books.

Ghiselin, B. (1952). *The Creative Process.* New York, Mentor.

Guilford, J.P. (1950). Creativity. *American Psychologist,* 5, 444-454.

Hadamard. J. (1945). *Mathematician's mind: the psychology of invention in the mathematical field.* Princeton, NJ: Princeton University Press.

Hewitt, E. (1948). Rings of real-valued continuous functions. *Transactions of the American Mathematical Society*, 64, 45-99.

Hilbert, D. (1900). Mathematische Probleme: Vortrag, gehalten auf dem internationalen Mathematiker-Congress zu Paris 1900. *Göttingen Nachrichten*. 253-297.

Hofstede, G. (1980). *Culture's consequences: International differences in work related values.* Beverly Hills, CA: Sage.

Husen, T. (1967). *International study of achievement in mathematics: A comparison of twelve countries* (Vol. 1-2). New York: Wiley.

Jay, E. S., & Perkins, D. N. (1997). Problem finding: The search for mechanism. In M. A., Runco, (Ed.), *The creativity research handbook* (Vol. 1, pp. 257-293). Cresskill, NJ: Hampton Press.

Kaufman, J. C., & Sternberg, R. J. (Eds). (2006). *The International Handbook of Creativity*. Cambridge: Cambridge University Press.

Krutetskii, V. A. (1976). *The psychology of mathematical abilities in school children.* Chicago: The University of Chicago Press.

Leikin, R., Berman, A., Kocihu, B., (2010). *Creativity in Mathematics and the Education of Gifted* Students, Sense Publishers, Rotterdam, The Netherlands.

Lester, F. K., Garofalo, J., & Kroll, D. L. (1989). Self-confidence, interest, beliefs, and metacognition. Key influences on problem solving behavior. In D. B. McLeod & V. M. Adams (Eds.), Affect and mathematical problem solving: A new perspective (pp. 75-88). New York: Springer-Verlag.

Ma, L. (1999). *Knowing and teaching elementary mathematics: Teachers' understanding of fundamental mathematics in China and the United States*. Mahwah, N.J.: Lawrence Erlbaum Associates.

Marton, F., Dall'Alba, G., & Lai, K. T. (1993). *The paradox of the Chinese learner (Occasional Paper 93.1).* Melbourne Institute of Technology.

National Center for Education Development. (2000). *Report on developing students' creativity and teacher training in the U.S. [关于美国创造性人才培养与教师培训的考察报告].* Retrieved March, 25, 2008, from http://www.moe.edu.cn/moe-direct/fazhanyjzx/187.htm

National Center for Educational Statistics. (2009). *The National Assessment of Educational Progress Overview.* Retrieved August 28, 2009, from http://nces.ed.gov/nationsreportcard/mathematics/

National Council of Teachers of Mathematics. (1989). *Curriculum and evaluation standards for school mathematics.* Reston, VA: National Council of Teachers of Mathematics.

National Council of Teachers of Mathematics. (2000). *Principles and Standards for School Mathematics.* Reston, VA: National Council of Teachers of Mathematics.

Oakland, T., Glutting, J., & Horton, C. (1996) *Student Styles Questionnaire.*San Antonio, TX: The Psychological Corporation.

Oakland, T. and Lu, L. (2006) . Temperament Styles of Children from the People's Republic of China and the United States, *School Psychology* International 27: 192–208.

Peverly, S. (2005). Moving past cultural homogeneity: Suggestions for comparisons of students' educational outcomes in the United States and China, *Psychology in the Schools, 42*(3), 241-249.

Poincaré, H. (1948). *Science and Method*. Dover: New York.

Plucker, J., & Zabelina, D. (2009). Creativity and interdisciplinarity: One creativity or many creativities? *ZDM: The International Journal on Mathematics Education*, 41, 5-12.

Polya, G. (1945). *How to solve it*. Princeton, NJ: Princeton University Press.

Presmeg, N.C. (1986). Visualization and mathematical giftedness. *Educational Studies in Mathematics, 17*(3), 297-311.

Robitaille, D. E., & Garden, R. A. (1989).*The IEA study of mathematics 11: Contexts and outcomes of school mathematics* .New York: Pergamon.

Runco, M.A. (1994). *Problem finding, problem solving, and creativity*. Ablex Publishing Corporation. Norwood, NJ

Shriki, A. (2010). Working like real mathematicians: developing prospective teachers' awareness of mathematical creativity through generating new concepts. *Educational Studies in Mathematics*, 73(2), 159-179.

Silver, E. A. (1997). Fostering creativity through instruction rich in mathematical problem solving and problem posing. *ZDM-The International Journal on Mathematics Education,* 97(3), 75-80.

Sio, U. N., & Ormerod, T. C. (2007). Does incubation enhance problem solving? A meta-analytic review. *Psychological Bulletin*, 135(1). 94–120.

Sriraman, B (2003) Can mathematical discovery fill the existential void? The use of conjecture, proof and refutation in a high school classroom (feature article). *Mathematics in School*, 32(2), 2-6.

Sriraman, B. (2004). Discovering a mathematical principle: The case of Matt. *Mathematics in School*, 33(2), 25-31.

Sriraman, B. (2005). Are giftedness & creativity synonyms in mathematics? An analysis of constructs within the professional and school realms. *The Journal of Secondary Gifted Education*, 17, 20–36.

Sriraman, B. (2008). *Creativity, Giftedness and Talent Development in Mathematics*. Information Age Publishing, Charlotte, NC.

Sriraman, B. (2009). The characteristics of mathematical creativity. *ZDM- The International Journal on Mathematics education,* 41(1&2), 13-27.

Sriraman, B., & English, L. (2004). Combinatorial Mathematics: Research into practice. Connecting Research into Teaching. *The Mathematics Teacher,* 98(3),182-191.

Sriraman, B., & Lee, K., (2011). *The Elements of Giftedness and Creativity in mathematics*. Sense Publishers, Rotterdam, The Netherlands.

Stevenson, H. W. (1993). Why Asian students still outdistance Americans. *Educational Leadership*. February, 63-65.

Stevenson, H. W., & Stigler, J. W. (1992). *The learning gap: Why our schools are failing and what we can learn from Japanese and Chinese education*. New York: Summit Books.

Stillman,G., Kwok-cheung, C.,Mason,R., Sheffield, L., Sriraman, B., & Ueno, K (2008). Classroom Practice: Challenging mathematics classroom practices. In E. Barbeau & P. Taylor. (Eds). *Mathematical Challenges in and Beyond the Classroom" ICMI 16 Study Volume* (pp. 243-284), Springer Science & Business.

Stoyanova, E. (1997). *Extending and Exploring Students' Problem Solving via Problem Posing: A Study of Years 8 and 9 Students involved in Mathematics Challenge and Enrichment Stages of Euler Enrichment Program for Young Australians*. Unpublished doctoral dissertation submitted to Edith Cowan University, Perth: Australia.

Stoyanova, E. (1998). Problem posing in mathematics classrooms. In N. Ellerton & A. McIntosh (Eds.) *Research in mathematics education in Australia*: *A contemporary perspective* (pp. 164-185). Perth: Edith Cowan University.

Stoyanova, E., & Ellerton, N. F. (1996). A framework for research into students' problem posing in school mathematics. In P. C. Clarkson (Ed.), *Technology in mathematics education* (pp. 518-525). Mathematics Education Research Group of Australasia. The University of Melbourne.

Taylor, I. A. (1972). *A theory of creative transactualization: a systematic approach to creativity with implications for creative leadership.* Occasional Paper, Buffalo, NY: Creative Education Foundation, No.8.

Torrance, E. P. (1988).The nature of creativity as manifest in its testing. In R. J. Sternberg (Ed.), *The nature of creativity: Contemporary psychological perspectives* (pp. 43-75). New York: Cambridge University Press.

Usiskin, Z. (2000). The development into the mathematically talented. *Journal of Secondary Gifted Education, 11*(3), 152-162.

Van Harpen, X. Y., & Presmeg, N. (2011*). Insights into students' mathematical problem posing process. Proceedings of the 35th International Conference of the International Group for the Psychology of Mathematics Education, 4*, 289-296.,

Vital, D. H., Lummis, M., & Stevenson, H. W. (1988). Low and high mathematics achievement in Japanese, Chinese, and American Elementary-school children. *Developmental Psychology, 24*(3), 335-342.

Vul, E., & Pashler, H. (2007). Incubation benefits only after people have been misdirected. *Memory and Cognition*, 35(4), 701–710.

Wallas, G. (1926). *The Art of Thought*. Harmondsworth, UK: Penguin Books Ltd.

Wong, N. Y. (2004). The CHC learner's phenomenon: Its implications on mathematics education. In L. Fan, N. Y. Wong, J. Cai & S. Li (Eds.), *How Chinese learn mathematics: Perspectives from insiders* (pp. 503–534). Singapore: World Scientific.

Wong, N. Y. (2006). From "Entering the Way" to "exiting the Way": in search of a bridge to span "basic skills" and "process abilities". In F. K. S. Leung, G.-D. Graf & F. J. Lopez-Real (Eds.), *Mathematics education in different cultural traditions: The 13th ICMI study* (pp. 111-128). New York: Springer.

Yang, G. (2007). A comparison and reflection on the school education of China and the U.S. [中美基础教育的比较与思考]. Retrieved March 25, 2007, from http://www.sm.gov.cn/bmzd/jcck/200111/Findex.htm

Yuan, X. (2009). *An exploratory study of high school students' creativity and mathematical problem posing in China and the United States.* Unpublished doctoral dissertation. Illinois State University.

Yuan, X., & Presmeg, N. (2010). An exploratory study of high school students' creativity and mathematical problem posing in China and the United States. . *Proceedings of the 34th International Conference of the International Group for the Psychology of Mathematics Education, 4*, 321-328.

Yuan, X., & Sriraman, B. (2011). An exploratory study of relationships between students' creativity and mathematical problem posing abilities—Comparing Chinese and U.S students.

In B.Sriraman, K. Lee (Eds.), *The Elements of Creativity and Giftedness in Mathematics* (pp.5-28). Rotterdam, Netherlands: Sense Publishers.

Zhang, D. (2005). *The "two basics": Mathematics teaching in Mainland China.* Shanghai, China Shanghai Educational Publishing House.