Egan J Chernoff

Bharath Sriraman

ADVANCES IN MATHEMATICS EDUCATION

Probabilistic thinking: presenting plural perspectives

Springer

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PERSPECTIVE I: MATHEMATICS AND PHILOSOPHY

Preface to Perspective I
Egan Chernoff and Bharath Sriraman

I.I. A historical and philosophical perspective on probability
Manfred Borovcnik and Ramesh Kapadia

This chapter presents a 21st century historical and philosophical perspective on probability, related to the teaching of probability. It is important to remember the historical development as it provides pointers to be taken into account in developing a modern curriculum in teaching probability at all levels. Thus we include some elements relating to continuous as well as discrete distributions. We start with initial ideas of chance two millennia ago and move on to the correspondence of Pascal and Fermat, and insurance against risk; the last century saw the axiomatic approach from Kolmogorov. Philosophical difficulties have been prevalent in probability since its inception, especially since the idea requires modelling – probability is not an inherent property of an event, but based on the underlying model chosen. Hence the arguments about the philosophical basis of probability have still not been fully resolved. The three main theories (APT, FQT, and SJT) are described, relating to the symmetric, frequentist and subjective approaches. These philosophical ideas are key to understand and develop teaching approaches. Probabilistic concepts are closer to a (consistent) way of thinking about the world rather than describing the world in a consistent manner, which seems paradoxical, and can only be resolved by a careful analysis.

I.II. From puzzles and paradoxes to concepts in probability
Manfred Borovcnik and Ramesh Kapadia

This chapter focuses on how puzzles and paradoxes in probability developed into mathematical concepts. After an introduction to background ideas, we present each

paradox, discuss why it is paradoxical, and give a normative solution as well as links to further ideas and teaching; a similar approach is taken to puzzles. After discussing the role of paradoxes, the paradoxes are grouped in topics: equal likelihood, expectation, relative frequencies, and personal probabilities. These cover the usual approaches of the a priori theory (APT), the frequentist theory (FQT), and the subjectivist theory (SJT). From our discussion it should become clear that a restriction to only one philosophical position towards probability – either objectivist or subjectivist – restricts understanding and fails to develop good applications. A section on the central mathematical ideas of probability is included to give an overview for educators to plan a coherent and consistent probability curriculum and conclusions are drawn.

I.III. Three approaches for modeling situation with randomness

Andreas Eichler and Markus Vogel

Three different approaches to the concept of probability dominate the teaching of stochastics: the classical, the frequentistic and the subjectivistic approach. Compared with each other they provide considerably different possibilities to interpret situations with randomness. With regard to teaching probability, it is useful to clarify interrelations and differences between these three approaches. Thus, students' probabilistic reasoning in specific random situations could be characterized, classified and finally, understood in more detail. In this paper we propose examples that potentially illustrate both, interrelations and differences of the three approaches to probability mentioned above. Thereby, we strictly focus on an educational perspective. At first we briefly outline a proposal for relevant teachers' content knowledge concerning the construct of probability. In this short overview we focus on three approaches to probability namely the classical, the frequentistic and the subjectivistic approach. Afterwards we briefly discuss existing research concerning teachers' knowledge and beliefs about probability approaches. Further, we outline our normative focus on teachers' potential pedagogical content knowledge concerning the construct of probability. For this, we discuss the construct of probability within a modeling perspective, with regard to a theoretical perspective on the one side and with regard to classroom activities on the other side. We further emphasize considerations about situations which are potentially meaningful with regard to different approaches to probability. Finally we focus on technological pedagogical content knowledge. Within the perspective of teaching probability, this kind of knowledge is about the question of how technology and, especially simulation, supports students understanding of probabilities.

I.IV. A modeling perspective on probability

Maxine Pfannkuch and Ilze Ziedins

We believe that [a] new and emergent direction in the teaching of probability needs to be examined. Therefore we will reflect on the practice and thinking of probabilists, from which we will propose a reference framework for curricular activities and learning. Using this framework we will analyze the modeling approaches taken by two research groups. The implications of taking a modelling

perspective on probability will be discussed in terms of the framework, school curricula and research into the development of students' probabilistic thinking.

Commentary on Perspective I: Working title
Watson, Shaughnessy, Konold or Falk

PERSPECTIVE II: PSYCHOLOGY

Preface to Perspective II
Egan Chernoff & Bharath Sriraman

II.I. Statistical thinking: no child left behind
Björn Meder and Gerd Gigerenzer

Is man an "intuitive statistician"? Or is the human mind biased and error-prone when it comes to probabilistic thinking? While researchers in the 1950s and 1960s suggested that people reason approximately in accordance with the laws of probability theory, research conducted in the heuristics-and-biases program during the 1970s and 1980s concluded the opposite. To overcome this conceptual impasse, more recently psychologists began to identify and characterize the circumstances under which people—both children and adults—are capable of sound probabilistic thinking. One important insight from this line of research is the power of representation formats. For instance, information presented by means of natural frequencies or icon arrays fosters the understanding of statistical information and improves probabilistic reasoning, whereas conveying information through conditional probabilities tends to impede understanding We review this research and show how its findings have been used to design effective tools and teaching methods for helping people—be it children or adults, laypeople or experts—to reason appropriately with statistical information. For example, using natural frequencies to convey statistical information helps people to perform better in Bayesian reasoning tasks, such as understanding the implications of diagnostic test results or assessing the benefits and risks of medical treatments. Teaching how to understand statistical information and how to use it for sound probabilistic inferences should be an integral part of comprehensive education, to provide children and adults with the risk literacy needed to make better decisions in a complex and uncertain world.

II.II. The A-B-C of probabilistic literacy
Laura Martignon

The A-B-C of probabilistic literacy for reckoning with risk and good decision-making should be conveyed to children at an early stage, more precisely, before they reach their eleventh year of age. This conviction is based on the view of experts, who sustain that the mathematical competencies of adults in general, not especially trained in mathematical subjects, are those they developed when they were 9, 10 and 11 years old. This paper will show how children can be provided with a box of elementary tools for reckoning with risk and for decision making

under uncertainty. Children, as has been demonstrated empirically, can acquire this tool box through a mosaic of simple, play-based activities, by means of magnet-plates, play-cards, tinker-cubes and systematic use of the software Tinker-Plots. We present a sequence of such activities that build upon each other, beginning with comparing proportions and risks, performing variations of the Wason selection task, moving on to Bayesian reasoning tasks and finally tasks devoted to the assessment and communication of risks. These results, as will be reported, were guided and inspired by empirical studies on human decision making obtained by the Centre of Adaptive Behaviour and Cognition,
directed by Gerd Gigerenzer.

II.III. Intuitive conceptions of probability and the development of basic math skills

Gary Brase, Sherri Martinie and Carlos Castillo-Garsow

The idea of probabilities has been described as a "Janus-faced" concept, which can be thought of either in terms of frequencies or in terms of subjective confidence. This dualism contributes to debates about the nature of human rationality, and therefore the pedagogical assumptions and goals of education. For this reason, the present chapter explores the evidence regarding how quantitative information is intuitively understood in the human mind over the course of elementary school education. Are particular interpretations of probability equally weighted or does one interpretation predominate as mathematical concepts are being acquired? We find multiple, converging lines of evidence that indicate a frequency interpretation of probabilistic information is developmentally primary and privileged. This has implications for mathematics education, even before the introduction of actual probabilities, in areas such as learning fractions and decimals. Educational practices should work to bootstrap from these privileged representations (rather than fight them) and built towards a more inclusive and comprehensive model of probability knowledge. We conclude that a fundamental issue is not just whether students think about probabilities as a frequentist or as a subjectivist, but rather how they recognize when to be one versus the other.

II.IV. Testing a model on probabilistic reasoning

Francesca Chiesi and Caterina Primi

Stanovich and colleagues (2008) outlined how people can reach a correct solution when a task besides the normative solution elicits competing response options that are intuitively compelling. First of all people have to possess the relevant rules, procedures, and strategies derived from past learning experiences, called mindware (Perkins, 1995). Then they have to recognise the need to use and to inhibit competing responses. Starting from this assumption, Stanovich and colleagues developed a taxonomy of thinking errors that builds on the dual-process theories of cognition. The present chapter presents a set of experiments designed to test the Stanovich and colleagues' taxonomy inside probabilistic reasoning. Since rules concerned with probabilistic reasoning (i.e., the mindware in Stanovich and colleagues' terms) are learned and consolidated through education, we carried on the researches with students of different grade levels. In particular, we assessed the role of the mindware gap (i.e., missing knowledge), taking into account individual

differences in cognitive ability and thinking dispositions, taking into account superstitious thinking as contaminated mindware (Study 1). Then, we conducted a set of experiments (Study 2) in order to investigate the override failure (i.e., the failure in inhibiting intuitive competing responses) in which participants were instructed to reason on the basis of logic or provided with example of logical vs. intuitive solutions of the same task. In this way, we aimed at stressing the need to use the rules. Our results provide support for the claim that the mindware plays an important role in probabilistic reasoning independent of age. Moreover, we found that cognitive capacity would only lead to increase reasoning performance if individuals possess the necessary knowledge about normative rules. Finally, superstitious beliefs might have a detrimental effect on reasoning. The overall findings offer some cues to cross the bridge from a psychological approach to an educational approach.

II.V. Revisiting the medical diagnosis problem: reconciling intuitive and analytical thinking

Lisser Rye Ejersbo and Uri Leron

A recurrent concern in mathematics education – both theory and practice – is a family of mathematical tasks which elicit from most people strong immediate ("intuitive") responses, which on further reflection turn out to clash with the normative analytical solution. We call such tasks cognitive challenges, because they challenge cognitive psychologists to postulate mechanisms of the mind which could account for these phenomena. For the educational community, these cognitive challenges raise a corresponding educational challenge: What can we as mathematics educators do in the face of such cognitive challenges? In our view, pointing out the clash is not enough; we'd like to help students build bridges between the intuitive and analytical ways of seeing the problem, thus hopefully creating a peaceful co-existence between these two modes of thought. In this article, we investigate this question in the context of probability, with special focus on one case study – the Medical Diagnosis Problem – which figures prominently in the cognitive psychology research literature and in the so-called rationality debate. Our case study involves a combination of theory, design and experiment: Using the extensive psychological research as a theoretical base, we design a new "bridging" task, which is on the one hand formally equivalent to the given "difficult" task, but on the other hand is much more accessible to students' intuitions. Furthermore, this new task would serve as "stepping stone", enabling students to solve the original difficult task without any further explicit instruction. These design requirements are operationalized and put to empirical test.

II.VI. Rethinking probability education: perceptual judgment as epistemic resource

Dor Abrahamson

In this chapter I propose a rethinking of how students should be introduced to the fundamental principle of classicist probability theory, the rule of ratio, as it applies beyond the simple cases of single events. I propose that students' perceptual judgment of the stochastic propensities inherent to random generators should constitute an epistemic resource for making sense of the classicist approach to

probability, particularly in the case of compound events. I argue that under auspicious conditions, students' perceptual judgment of the stochastic propensities of a random generator can play a similar epistemic role as do actual experiments with the device in terms of evoking sensations of relative likelihood that, in turn, can be linked to the distribution of possible events in the event space. In order to create these auspicious pedagogical conditions, I submit, educational designers should create materials and activities geared to accommodate humans' evolved perceptual inclinations, such as sensitivity to proportional relations in the visual field. The chapter attempts to promote this rethinking by furnishing intuitive, empirical, and theoretical evidence as support for its validity.

II.VII. Sticking to your guns: a flawed heuristic for probabilistic decision-making

Deborah Bennett

Should we switch and find our initial choice to be a winner, we anticipate "kicking ourselves." If only we hadn't switched doors—a kind of counterfactual thinking according to psychologists. Refusal to switch doors, even in the light of increased odds of winning, is a type of status quo bias. Individuals are hesitant to deviate from the status quo. Linked to this bias is another bias, the preference for inaction over action. Individuals hold themselves more responsible for negative outcomes due to their actions than similar or identical outcomes due to their inactions. The sins of commission (acting) seem to be greater than the sins of omission (not acting). All of these cognitive biases can lead to a sort of memory distortion, and researchers have documented that counterfactual thoughts and anticipatory regret affect people's decision strategies. Our mental sample space is distorted by an overrepresentation of negative results attached to losses and regret. Their vividness makes them seem more frequent than they really are. There is some evidence that believing in good luck imbues individuals with a sense of control and confidence. If so, then perhaps tempting fate by switching doors makes us feel a loss of control; we have invited disaster and angered the gods. Finally, Risen and Gilovich (2007, 2008) conclude that individuals are of "two minds" in making everyday judgments about likelihoods—what their guts tell them and what their rational minds tell them. This study attempts to examine our conjecture by testing subjects with a simplified version of the Monty Hall Problem: after making an initial choice, new information is revealed that doesn't require an omniscient host.

II.VIII. Developing probabilistic thinking: what about peoples' conceptions

Annie Savard

Since the important work on reasoning under uncertainty by Kahneman and Tversky in 1970's, the description of how people think about probability by using intuitions, conceptions and misconceptions have been studied in psychology and mathematics education. Over the years, the body of the literature have identified and studied many of them. Some conceptions, such as representativeness and availability, are well known. But not all of the conceptions have been studied many times and the conceptions presented in the literature usually don't rely them to each other. Therefore, it is now difficult to have a broader perspective on people conceptions of probability. In addition to that, some epistemological differences

exist between the conceptions. Not all of them use the same kind of reasoning for addressing different aspects of probability. Thus, a broader perspective of people conceptions of probability involve not only knowing about conceptions and link them together, it is also knowing about the mathematical aspect involved. This chapter will define what is a conception and present a classification of some of them presented in the literature, based on their epistemological differences.

Commentary on Perspective II: Working title
Watson, Shaughnessy, Konold or Falk

PERSPECTIVE III: STOCHASTICS

Preface to Perspective III
Egan Chernoff and Bharath Sriraman

III.I. Prospective primary school teachers' perception of randomness
Carmen Batanero, Pedro Arteaga, Luis Serrano and Blanca Ruiz

Subjective perception of randomness has been researched by psychologists and mathematics educators, using a variety of tasks, resulting in a number of different descriptions for the biases that characterize people's performances. Analyzing prospective teachers' possible biases concerning randomness is highly relevant as new mathematics curricula for compulsory teaching levels are being proposed that incorporate increased study of random phenomena. In this chapter we present results of assessing perception of randomness in a sample of 208 prospective primary school teachers in Spain. We first compare three pairs of random variables deduced from a classical task in perception of randomness and deduce the mathematical properties these prospective teachers assign to sequences of random experiments. Then, the written reports, where prospective teachers analyse the same variables and explicitly conclude about their own intuitions are also studied. Results show a good perception of the expected value and poor conception of both independence and variation as well as some views of randomness that parallel some naïve conceptions on randomness held at different historic periods.

III.II. Challenges of developing coherent probabilistic reasoning: rethinking randomness and probability from a stochastic perspective
Luis Saldanha and Yan Liu

The concept of probability plays a vital role in mathematics and scientific research, as well as in our everyday lives. It has also become one of the fastest growing segments of the high school and college curriculum, yet learning probability within school contexts has proved more difficult than many in education realize.

This chapter is in two broad parts. The first part synthesizes a discussion of randomness and probability that is situated at the nexus of bodies of literature concerned with the ontology of stochastic events and epistemology of probabilistic ideas held by people. Our synthesis foregrounds philosophical, mathematical, and psychological debates about the meaning of randomness and probability that

highlight their deeply problematic nature, and therefore raises the equally problematic question of how instruction might support students' understanding of them. We propose an approach to the design of probability instruction that focuses on the development of coherent meanings of randomness and probability—that is, schemes composed of imagery and conceptual operations that stand to support students' coherent thinking and reasoning about situations that we see as entailing randomness and probability. The second part of the chapter reports on aspects of a sequence of classroom teaching experiments in high school that employed such an instructional approach. We draw on evidence from the experiments to highlight challenges in learning and teaching stochastic conceptions of probability. Our students' challenges centered on re-construing given situations as idealized random experiments involving the conceptualization of an unambiguous and essentially repeatable trial, as a basis for conceiving of the probability of an event as its anticipated long-run relative frequency.

III.III. "It is very, very random because it doesn't happen very often": examining learners' discourse on randomness

Simin Jolafee, Rina Zazkis and Nathalie Sinclair

The notion of randomness is central to the study of probability and statistics and it presents a challenge to students of all ages. However, it is usually not defined in textbooks and curriculum documents, as if the meaning of randomness should be captured intuitively. In fact, the word "random" does get used in everyday language, but not always in the same way that it is used in mathematics. Even in mathematics, the notion of randomness has been a challenging one—not only did it emerge relatively recently in the history of mathematics, it has also undergone various attempts to be adequately defined. Given the importance of randomness in the study of probability and its complexity as a concept, our goal in this chapter is to better understand the ways learners use and talk about it. In this chapter, we first provide an overview of some of the ways in which randomness is defined in mathematics—these aspects of randomness will help structure our analysis of learners' uses and descriptions of it. We then provide a brief overview of the research in mathematics education and highlight the main resources that learners use to explain randomness. Following this, we present two empirical studies involving prospective teachers and undergraduate students, each aimed at further probing understandings of randomness using different methodological approaches.

III.IV. Developing a modelling approach to probability using computer-based simulations

Theodosia Prodromou

The introduction of digital technology into secondary schools is ideally suited for supporting students as they manipulate data and portray it in a range of different representations to draw inferences from it without resource to a classical understanding of probability theory. As a result, probability is overlooked from school curricula and is gradually becoming almost a non-existent topic. The aim of recent curricula (e.g. ACARA, 2010) to support the parallel development of statistics and probability and then progressively build the links between them seems

utopic since statistics prevails over probability in mathematics curricula. In this chapter, it is argued that it is worthwhile to consider an alternative approach for teaching probability− presenting probability as a modelling tool, which reflects the mindset of an expert when using probability to model random behaviour in real-world contexts.

III.V. Promoting statistical literacy through data modelling in the early school years

Lyn D. English

This chapter addresses data modelling as a means of promoting statistical literacy in the early grades. Consideration is first given to the importance of increasing young children's exposure to statistical reasoning experiences and how data modelling can be a rich means of doing so. Selected components of data modelling are then reviewed, followed by a report on some findings from the third-year of a three-year longitudinal study across grades one through three.

III.VI. Learning Bayesian statistics in adulthood

Wolff-Michael Roth

Drawing on first-person methods for the study of cognition (Roth 2012), this chapter is designed to articulate invariants of learning an advanced statistical topic: Bayesian statistics. The first case study focuses on the learning of some fundamentals (such as those that one may find as content of a Wikipedia page); the second case study presents and analyzes a learning episode in the case of quantitative social science research that takes into account prior studies for establishing prior probabilities required for calculating posterior probabilities given the information collected in the study of interest. The analyses show – consistent with pragmatic theories of language (Wittgenstein 1953/1997) – that the essential dimension of learning is what equations, terms, and formulae require to be done rather than their (elusive) "meanings." Because learning to think within a Bayesian framework is like learning an entirely new language, even scientists with statistical background may find it difficult to learn using the Bayesian approach. Even less should we expect just plain folks to use these forms of thinking or fault them for not using these.

Commentary on Perspective III: Working title

Watson, Shaughnessy, Konold or Falk

PERSPECTIVE IV: MATHEMATICS EDUCATION

Preface to Perspective IV

Egan Chernoff and Bharath Sriraman

IV.I. Experimenation in probability teaching and learning

Per Nilsson

This chapter provides a discussion on teaching and learning probability in the relationship between theoretical and empirical probability. We examine a small-

scaled teaching experiment, which involves acts of experimentation. The purpose of the experiment is particularly on exploring and illustrating critical aspect of probability teaching, which involves students' concrete production and experimentation with data. Developing a background for the teaching experiment we proceed with reviewing previous research, which are highlighting issues on the relationship between empirical and theoretical probability in situations of experimentation.

IV.II. Investigating the dynamics of stochastic learning processes

Susanne Prediger and Susanne Schnell

Our didactical research perspective focuses on stochastic teaching-learning processes in a systematically designed teaching-learning arrangement. Embedded in the methodological framework of Didactical Design Research, this perspective necessitates the iterative interplay between theoretically guided design of the teaching-learning arrangement and empirical studies for investigating the initiated learning processes in more and more depth. For investigating the micro-level of students' processes, we provide a theoretical framework and some exemplary results from a case study on students (in grade 6) approaching the distinction between short term and long term in the teaching-learning arrangement "Betting King".

IV.III. Counting as a foundation for learning to reason about probability

Carolyn A. Maher and Anoop Ahluwalia

Based on findings from long-term and cross-sectional studies in a variety of contexts and across a variety of ages, we have found that in the activity of problem solving on strands of counting and probability tasks, students exhibit unique and rich representations of counting heuristics as they work to make sense of the requirements of the tasks. Through the process of sense making and providing justifications for their solutions to the problems, students' representations of the counting schemes become increasingly more sophisticated and show understanding of basic combinatorial and probabilistic reasoning.

IV.IV. Levels of probabilistic reasoning of high school students about binomial problems

Ernesto Sánchez and Pedro Rubén Landín

In this chapter some aspects of the process in which students come to know and use the binomial formula are described. In the context of a common high school probability and statistics course, a test of 8 problems was designed to explore the performance of students in binomial situations. For investigating the influence of instruction to overcome some common cognitive bias or its persistency, the first three problems are wording in such way that may induce bias and each one is structurally equivalent to another problem in the test whose wording avoids any bias. Also, the second and third problems were administered before and after the course to assess the changes produced by instruction. A hierarchy of reasoning designed in a previous study was adapted and used to classify in different levels of reasoning the responses of the students. The classification of students' responses

points out that the components of knowledge: classical definition of probability, the rule of product of probabilities, combinations and binomial formulae are indicators of transitions between levels. The influence of the wording of the problems is strong before instruction, but weak after it.

IV.V. Children's construction of sample space with respect to the law of large numbers

Efi Paparistodemou

The chapter describes how children use an expressive microworld to articulate ideas about how to make a game seem fair or not with the use of randomness. An open computer game was designed for children to express understandings of randomness as formal conjectures, so that they were able to examine the consequences of their understandings. The study investigates how twenty-three children, aged between 5½ and 8 years, engaged in constructing a crucial part of a mechanism for a fair or not spatial lottery machine (microworld). In particular, the children tried to construct a fair game given a situation in which the key elements happened randomly. The children could select objects, determine their properties, and arrange their spatial layout in the machine. The study is based on task-based interviewing of children who were interacting with the computer game. The findings identify children's initial meanings for expressing stochastic phenomena and describe how the computer tool-based game helped to shift children's attempts to understand randomness from looking for ways to control random behaviour, towards looking for ways to control events. Evidence is presented that the children constructed a set of 'situated abstractions' for ideas such as the 'law of large numbers'. The computer game offered children the opportunity to make their own constructions of sample space and distribution. The children used spontaneously five distinct strategies to express the idea that their construction could only be judged with respect to a large number of trials. It is apparent that the game provided children the opportunity to express the idea that stability can come from increasing outcomes with different strategies.

IV.VI. Researching conditional probability problem solving

Pedro Huerta

In this chapter, in particular, we talk about conditional probability problems, offering a systematic method to study them. We show a theoretical study into those problems; identifying a particular family of problems we call ternary problems of conditional probability. We define the notions of Level, Category and Type of a ternary problem of conditional probability in order to classify them into families and sub-families to study them better. We also offer a tool we call trinomial graph that functions as a generative model (in the sense of Fischbein, 1976) for this family of problems and that we consider would be useful for teachers and researchers. We show the syntax of the model that allows the translation of a problem in terms of trinomial graphs, and the consequences of this translation. Among others, one of their uses is shown in this part of the chapter: how these problems can be analysed before they become a task for students, depending on the task variables (in the sense of Kulm, 1979) researchers and teachers want to take

into account in their works. At the end of the chapter, there are two main related protagonists: ternary problems of conditional probability and students solving them. As a result, the students' probabilistic thinking is observed in a broader problem solving context, as suggested by Borovcnik (2005), in relation to the task variables: structure, context and data format. We report some of the results of our investigation into students' behaviours, showing how these depend in any manner on those task variables.

IV.VII. Real life experiences as hindrance in probabilistic situations

Ami Mamolo and Rina Zazkis

In this chapter we consider the specific mathematics embedded in a particular task and analyze the responses of practicing and prospective secondary school mathematics teachers as they addressed the scenario. Unlike conventional probability tasks, such as tossing a coin or throwing a die, a special feature of the presented task is that the embedded experiment – picking "any real number" – cannot be carried out. We begin by examining different aspects of probability tasks, the context in which they are presented and the associated interpretations.

IV.VIII. Influence of culture on high school students' probabilistic thinking

Sashi Sharma

This chapter has five sections. The first section outlines the importance of probability in both out-of-school and formal (school) situations and makes a case for teaching probability. The next section considers the issues in probability education. Specifically, different interpretations of probability are considered: theoretical, frequentist and subjective. Although we use informal probabilistic notions daily in making decisions, research on probability has mostly focused on the classical and frequentist approaches, research on the subjective approach is scare. Further, the common culture may influence the informal ideas of probability. Yet, there appears to be minimal literature that deals with the educational implications of the role of culture in probability education. The third section draws on mathematics education research to discuss the interaction between mathematical cognition and culture. It will be argued that probability is no different and early notions as well as misconceptions need to be addressed via the cultural lens. This would help clarify the aims, purpose and limitations of probability education. The next section will report on research on the effects of culture on students' probabilistic thinking. I will draw on examples from my work and few others who have studied cultural influences on probabilistic thinking to explain how probability is related to human culture and tied to cultural practices. In particular, cultural beliefs (religion, superstations, language and experiences) strongly affected student thinking. The final section will consider the issues arising out of the literature and offer suggestions for meeting these challenges. Specifically, suggestions for teaching and further research will be outlined.

IV. IX. Primary school students' attitudes to and beliefs about probability

Steven Nisbet and Anne Williams

This chapter relates to the role of attitudes and beliefs in the teaching and learning of probability in schools. A study was conducted in which two Year 7 teachers in an Australian primary school and the students in their combined class participated in a teaching experiment. The study involved implementing a program of probability games and activities which aligned with both the Probabilistic Reasoning framework of Jones, Thornton, Langrall, and Tarr (1999), and the formal Year 7 curriculum. The program was designed to improve attitudes to probability, challenge beliefs about luck, and support the learning of probability concepts. Data were collected from students and teachers with respect to attitudes, beliefs, and understanding before and after the program. It was concluded that an activity approach to the teaching of probability improved students' attitudes to and beliefs about probability, at least in the short term. Students had a greater appreciation of the relevance of probability in the world around them and their superstitions about luck lessened. There was evidence of positive links between attitudes and understanding. It was noted also that a lack of prerequisite number skills impacted on students' motivation to remain involved. At the end of the study, teachers were more confident and enthusiastic about teaching probability in the future.

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DISCUSSION
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