Nyaya Methodology and Western Mathematical Logic: Origins and Implications

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Abstract: In this chapter we compare the Nyaya school of logic, namely one of the six major schools of Hindu philosophy to Aristotelian logic. In particular we compare the intent, nature and structure of syllogisms in Nyaya to that in Western mathematical logic to highlight differences in premises and conclusions. In doing so, we draw on the foundational writings and commentaries on Nyaya methodology from both Hindu and Buddhist sources, as well as the major sources of Aristotelian logic. We explore the possibility of Nyaya methodology being influenced by Aristotelian logic before arguing that the Nyaya system developed on its own trajectory. Some modern implications of Nyaya methodology are given.

Keywords: Indian philosophy; Hindu philosophy; Mathematical Logic; Nyaya logic; Aristotelian logic; Syllogisms; Milinda; Law of Contrapositive; Buddhist logic

I. Debate Over Importance of Nyaya Philosophy

Nyaya syllogism and system of logic has its roots in the interpretation of the ancient Vedic texts of India in the 1st to 2nd century (Dasgupta 1922, 276). Indian logic is one of the three major traditions of logic dating back to 6th century BCE (Fathi, 2013). The field of logic may be conceptualized as branching into two traditions- the Aristotelian tradition which modern logic traces its origins to, and a dialectic tradition which Buddhist logic and Hegelian dialectics trace their origins to. As Fathi (2013) puts it there is a false schism that has been interposed by historians of logic by viewing Aristotelian logic as a product of Western culture, whereas non-Aristotelian logic is viewed as being a product of the East and not being able to confer an objective true/false status to propositions. One explanation is found in the fact that much of the Vedic texts are shrouded in symbolism and verse. However Nyaya logic aims for clear analytic reasoning. It is one of the six major schools of Hindu philosophy, also known as “astika” schools.

Since the middle of the 20th century, the Nyaya logical system has drawn a lot of praise and attention from scholars. However, the acclamations were not always so forthcoming. Many 19th century European logicians came to criticize the Nyaya syllogism. The Scottish metaphysician Sir William Hamilton describes the Nyaya syllogism as “merely a clumsy agglutination of...counterforms” in his book *Discussions on Philosophy* (Ganeri 1996, 5). Even the prominent J.S. Mill, son of the author of *History of British India*, who worked at the Colonial Indian Office of Britain for 35 years, considered Indian thought undeveloped (Ganeri 1995, 9).

On the other hand, such eminent thinkers such as George Boole, Charles Babbage,

Agustus De Morgan claim influence from some ancient Indian philosophy. Mary Boole, wife of George Boole, goes as far as claiming that part of the greatness of three men came from “the effect of intense Hinduizing” (Ganeri 1995, 5). De Morgan criticises the British on their ignorance of ancient Indian thought in a preface of a 19th century algebra text by Ram Chundra, saying that Christian influence of the British prevents them from seeing a “body of literature and science [in India] which might well be the nucleus of a new civilisation” (Ganeri 1995, 6).

Much of the debate in the late 19th century can be understood in light of colonialism. There was a devaluation of rational elements from both European and Indian scholars. It is Jonardon Ganeri's claim that many Indian scholars were in search of an Indian identity. Since identity can only be affirmed in light of differences of your oppressor, India looked to their non-rational, mystical past and highlighted these non-logical areas of their culture since Europe was thought to be hyper-rational (2). From the Western perspective, European thinkers of the time tended to see themselves as the continuation of the Greek tradition in the vein of Plato and Aristotle. So it was natural that any attempt to critique the Nyaya philosophy from an Aristotelian viewpoint would lead one to think that the methods of Nyaya are “manifestly superfluous” and “clumsy,” words used by prominent German logicians of the time (7).

Since India's independence, there has been a resurgence of interest in Nyaya logic from both sides of globe. It is generally accepted that Nyaya is different from Aristotle's system, though there are still debates as to how much influence Aristotle had on certain aspects of their logic. In any case, we should not take the Nyaya tradition lightly or see it merely as a derivative of the Aristotle syllogism. In fact, even J.S. Mill's method of Agreement and Difference, which he believed himself to have discovered was already present in the Nyaya text *Muktavali* (Davies 127). Had Mill been not so ignorant of ancient Indian ideas, perhaps he could have progressed further and expanded his influence in modern logic.

Our interest in Nyaya logic will focus specifically on its relation to mathematics. Although Nyaya texts do not discuss operations on numbers, they display a prescience of set theory and the foundations of mathematics. This paper will compare the two different logical systems of the Nyaya and Aristotle. We specifically look at the syllogisms defined by Nyaya philosophers in the *Nyaya Sutras* and by Aristotle in *Prior Analytics* and show how a basic assumption on the nature of logical argument and theory of knowledge influenced their respective cultures' academia for many centuries. We'll show how both systems are not without flaw, but while the West clung to unproductive assumptions about deduction in Aristotle's system, the Nyaya's irresolution on the inductive step challenged them to produce new understandings on what is a valid logical claim. Remarkably, some of these findings led them to conclusions about the nature of number and a notion of set which correspond to findings by logicians and mathematicians in West had in the 19th and early 20th.

II. Comparisons Between the Aristotelian Syllogism and Nyaya Syllogism

Webster's dictionary defines syllogism as a deductive scheme of a formal argument consisting of a major and a minor premise and a conclusion. This is the definition of the Aristotelian syllogism and is indicative of the influence Aristotle's logic has had on Western culture. While we accept this definition, we will also accept any number of propositions that lead to a conclusion as a valid representation of a syllogism.

Mathematics rarely uses a syllogistic form explicitly. Consider, a simple algebra problem, $3x-2=0$, where you are asked to solve for a real-valued x. Although you *deduce* $x=2/3$, you didn't place your arguments in any syllogistic form. You could have, but it would just add complication

to what you are trying to show. However, for many centuries, many philosophers adhered to the Aristotelian syllogism as the major means of proof and were confused as to the true nature of reasoning in mathematics (Russell 199). In order to see how Aristotle's syllogism has drawbacks when used in a mathematical setting, we first need to understand the subtleties of it.

Aristotle (384BC-322BC), considered to be the founder of logic, wrote six texts devoted to the laws of thought. These texts were later bound together and given the name *The Oreganon*, or *Instrument of Knowledge*. We will look at Aristotle's treatise *Prior Analytics* since this is where he defines his syllogism in detail.

A Western student of philosophy and logic will most likely come across this particular syllogism somewhere in their studies,

(*1) All men are mortal (Major premise)
Socrates is a man, (Minor premise)
Therefore, Socrates is mortal. (Conclusion)

We learn that in the premise “All men are mortal”, “men” is the subject and “mortal” is the predicate. Furthermore, we learn that every premise must have a subject and a predicate. Although syllogism *1 is often attributed to Aristotle, it is not an Aristotelian syllogism. Let's discuss why *1 is not an Aristotelian syllogism in the true sense.

First, since logic needs to be exact we should be as precise in our translation of the actual syllogisms in Aristotle's *Prior Analytics* and *Prosterior Analytics*. Jan Lukasiewicz makes note that Aristotle never used the form “All B is A”. Instead, he used the expression “A belongs to B” (Lukasiewicz 3). Also, Aristotle presented his major and minor premise together and used the Greek word that is more closely translated as an implication (ie “then”) rather than inference (ie “therefore”) (2). So, we should change *1 to,

(*2) Mortality belongs to all of humanity
and humanity belongs to Socrates,
Then, mortality belongs to Socrates.

This might seem trivial and makes it seem a bit awkward, but it gets the point across.

Second, although *2 is an example of a logical form, it is about something, namely about men and mortality. But logic is not about something, it is only concerned with valid forms. In *Prior Analytics*, there is no syllogism with particular terms. Instead of using concrete subjects and predicates, Aristotle introduced variables to stand for them (which might be Aristotle's greatest achievement on the subject especially when considering how the use of variables by Diophantus changed the subject of algebra). So now we have,

(*3) A belongs to all B
and B belongs to all C,
then A belongs to all C.

This is the Aristotelian syllogism known as “Modus Barbara” (Lukasiewicz 88). We may now talk meaningfully about its characteristics.

Bertrand Russell sites *1 word for word in attempt to criticize some aspects of it (Russell 196-198). We have shown that *1 is not a true Aristotelian syllogism, but let's briefly examine Bertrand Russell's argument against the validity of *1. He says that in this syllogism “the distinction between particulars and universals, is blurred” (Russell 198). Although he agrees that “human” is a predicate of “Socrates”, he finds fault with “human” as a predicate of “Greek”, which is a predicate of a predicate. If we think of this in terms of set theory, he claims Aristotle does not distinguish between an element and a set that contains only that element. This “made it impossible to have a correct theory of the number *one*, and let to endless bad metaphysics about

unity” (198). Although Russell may have a case against Aristotle's conception of logic on the grounds of different levels of predication (an area we'll come back to when discussing the nature of number), he used the wrong example to prove his point. The error of assigning *1 to Aristotle must be attributed to subsequent logicians who perpetuated it.

Aristotle states that premises come in “either universal or particular or indefinite” (Aristotle, Prior Analytic 1). While Russell found no fault in the premise “Socrates is a man”, which deals with a particular subject, Aristotle actually refuses to take notice of any singular terms such as “Socrates” in his discussions on logic. There have been a few expositions explaining why Aristotle refused singular terms in explanation of his syllogism (Ghose, Lukasiewics, Bochenski). Lukasiewics believes that this is a major deficiency in his system (Lukasiewics 4). Although Aristotle claimed only universal terms are needed in science since science is about the essential (Aristotle, Posterior Analytics), the exclusion of particulars made it impossible to analyze any type of scientific proposition with singular terms.

The common explanation for why Aristotle excluded singular terms from his logic was that he was a Platonic realist, meaning that he believed that the objects in reality, which had some essential universal quality, were distinct from intelligible objects, which would have some particular form. Aristotle was interested in the essential for his logical system. However, we need not necessarily resort to Platonic realism to explain Aristotle's exclusion of particular terms.

In Amitabha Ghose's article “Singluar Propositions and Aristotle's Conception of Logic”, he shows that by rejecting singular terms from his logic, Aristotle avoids problems with infinite collections. If Aristotle allowed singular terms such as “Socrates” in a premise, then if the collection of “humans” were infinite, the proposition “Socrates is a human” would mean every one of the infinite number of humans would need to be given an unique name. This brings to mind countability. Ghose entertains the possibility of an algorithm to assign the natural numbers as names to each human, but claims that the difficulty of assigning names would be enough to give Aristotle pause. In his treatise *Physics,* Aristotle considers the idea of infinity when describing motion, but he always did so in describing infinitesimal segments and not in magnitude (Arisotle *Physics,* Book 6). Also, if this is the reason why he rejected singular terms, any discussion of it is absent in existent texts. However, while Ghose's interpretation may be flawed, it does suggest how one can approach the idea of infinity in Aristotle's system. We will come back to this idea of infinite sets and how it lead to problems in the attempt to formalize mathematics in the early 20^{th} century.

We now leave our discussion on Aristotle and turn to the Nyaya syllogism. The origins of the Nyaya philosophy can be traced back to the debates of the correct interpretation of the Vedic scriptures regarding certain rituals such as sacrifice (Dasgupta 1922, 276). As with other Hindu and Buddhists sects, Nyaya considered liberation from duhkha, or suffering the highest ideal. What set Nyaya apart from the other sects was that Nyaya concerned itself with the rational as a means to liberation. The word Nyaya means “that by which one is led to a conclusion” or “correct reasoning” (King 130).

The earliest parts of Nyaya philosophy was developed by Gautama in the Nyaya sutras. There is some debate as to when Gautama lived. Dr. S. C. Vidyabhusana dates him at around 550 BC in Mithila (North Behar) (Dasgupta 1922, 279). However, most scholars place him in the 2^{nd} or 3^{rd} century AD. This may be due to the hypothesis that much of the syllogism was borrowed from Aristotle and placing Gautama anywhere earlier would go against this claim.

The most convincing account of Aristotelian influence on the Nyaya syllogism is a 1922 article entitled “Influence of Aristotle and the Development of Inference in India” by M. M. Satis

Chandra Vidyabhusana. He argues that while Gautama may have existed in 550 BC and contributed to the Nyaya sutra, he wasn't the sole author (Vidyabhusana 470). He proposes that Aksapada (1^{st} -2^{nd} century AD), who some have claimed to be Gautama himself, was the contributing author of the subject of syllogism in the sutras (471). He admits that Aksapada was probably not the first to use the syllogism in India, but entertained the possibility that Aristotle's syllogism had reached India during this time by performing side by side comparisons of the two systems and showing that cultural transmission was physically possible. Vidyabhusana's claim is convincing. However, many scholars find many of the similarities fabricated. For example, P.T. Raju refutes this claim in one line in a footnote in his 1971 book *The Philosophical Traditions of India*: "There is no evidence of borrowing" (134).

In either case, we come across words denoting logic in earlier literature than the Nyaya sutras (Dasgupta 1922, 232). So we know that Nyaya did emerge independently as a logical system. It is also well documented that India had originated the ideas of tarka, pramana, and anya-mata-pariksa (Vidybhusana 474). These ideas should be thought of as a part of the syllogism, thus setting it apart from the Aristotelian tradition.

The Nyaya sutras comprises of four distinct subjects: (1) Tarka (the art of debate), (2) Pramana (the means of valid knowledge), (3) Avayava (the doctrine of syllogism), and (4) Anya-mata-pariksa (the examination of contemporaneous philosophical doctrine) (Vidyabhusana 469). The syllogism is just one part, but is important to understand the logical nature of a Nyaya argument. The Nyaya syllogism consists of five parts: (1) Pratijna (The Premise), (2) Hetu (The Cause), (3) Udaharana (The Example), (4) Upanaya (The Application of that example), (5) Nigamana (The Conclusion) (King 131). Let's look at a specific example of a syllogism found in the Nyaya sutras.

(*4) The hill is fiery (The Premise)
Because it has smoke (The Cause)
Whatever has smoke has fire, for example, the oven (The Example)
This hill has smoke, which is associated with fire (The Application)
Therefore, this hill has fire (The Conclusion) (Phillips 55)

Now let's "translate" this into an Aristotelian syllogism.

(*5) Sign of fire belongs to all smoke (The Major Premise)
and smoke belongs to this hill, (The Minor Premise)
then fire belongs to this hill (The Conclusion) (Phillips 55)

We should note that this is not a Aristotelian syllogism in the true sense since "this hill" is a singular term. If we allow singular terms in Aristotle's logic, *5 may seem as a reduction of *4 while proving the same thing. This is why some Western scholars in the 19^{th} century considered the Nyaya syllogism redundant and clumsy. The criticism is unwarranted because while the Aristotelian syllogism is purely deductive, the Nyaya syllogism is always grounded in reality by necessitating an example. The Nyaya enterprise is concerned with true knowledge based on induction while Aristotle was interested in what constitutes valid logical form regardless of the meaning of the terms. This is why Aristotle was able to use symbols to stand for the terms, while Nyaya refrained from symbolization.

Puligandla explains the difference between the two systems: "The Indian view is based on the conviction that logic is an instrument for the discovery and understanding of reality," while "the Western tradition, having divided the formal from the empirical, is faced with the serious problem of accounting for the fact of the application of logic in the study of the world" (King 132). The Western tradition has been plagued with the idea of dualism in various forms,

such as the mind-body problem and the free will/determinism debate. The interface between our cognition and the "external world" (wherever that boundary is is beyond me) seems to be a serious problem when one stays within the system of cognitive forms, such as Aristotle had. In *Prior Analytics,* Aristotle was not concerned about proving the premises and conclusion as true, but wanted to show when an argument is true assuming those premises are true. He has missed the induction step in his syllogism. This doesn't make his syllogism useless. He only separated the subjects, which is analogous to how mind and body were separated in early Western philosophy. Philosophers call Aristotle's distinction between the logical subjects "analysis" (for deduction) and "synthesis" (for induction). This idea that deduction could be severed completely away from induction influenced many philosophers and mathematicians for hundreds of years.

The culmination of this separation can be seen in Immanual Kant's *Critique of Pure Reason* where he defines apriori propositions (true by only the definition of its terms), which in turn influenced the logical positivists, who had many subscribers who were interested in the nature of mathematicians. For the positivists, "mathematics is the science of the forms of our expression, as for Kant it was the science of the forms of our intuition" (Kattsoff 245). However, this "intuition" that Kant uses is not the intuition that is known in India. As P.T. Raju explains in his article "Intuition as a Philosophical Method in India," intuition is something that requires training (Raju 1952, 187). In other words, there is correct way to intuition through experience. Intuition isn't completely divorced from our reality, something mystical as has been portrayed of much Indian thought. Therefore, to the Nyaya philosopher, it is a valid form of knowledge that can be used as an induction step.

III. Valid Knowledge and Logical Methods in the Nyaya System

Peter M. Scharf succinctly summarizes the methodology of Nyaya logic: "The Nyaya system of India philosophy analyzes the means of gaining knowledge (pramana) and the objects of knowledge (prameya) in order to arrive at correct knowledge (tattvajnana)" (151). In other words, Nyaya is primarily an empirical system that uses inference as its main method of determining the truth of a proposition. The Nyaya believed "from correct knowledge one attains the highest good (nihsreyasa). The elimination of false knowledge leads to absolute liberation" (Scharf 152).

Since the Nyaya focus on the empirical for their induction step in their logic, this begs the question as to what constitutes valid knowledge. M. Hiriyanna describes how Nyaya divides its theories of knowledge into two classes. One is called svatah-pramanya, or self-validity, and the other is paratah-pramanya, or extraneous-validity. In the self-validity case, unless there is doubt, there is no need for proof of its validity. In the extraneous-validity case, there must be an appropriate test, called samvadi-pravrtti, or practical verication (Hiriyanna 98). According to Nyaya philosopher Vatsyayana Paksilasvamin, a "critical proof is the proof of things desired, supported by observation and authority" (Generi 2001, 10).

Nyaya not only allowed observation as a valid means of knowledge, but also testimony from and trustworthy authority. By allowing testimony, they were able to use Vedic scripture and testimony from yogis in their logic. The experience of the yogis is still an inductive step, since it is a special kind of experience brought about through the training of the intuition. There is no deductive realm to draw from in the Nyaya system, which goes against much of the Western's conception of the nature of mathematics. As Bertrand Russell claims, only logic, pure mathematics, law, and theology are deductive inferences, meaning that they are derived from first principals (Russell 199). Nyaya would argue that there is no deductive logic, not even in

theology. Induction is part and parcel to the world. We are not the skin-encapsulated beings that act as the privileged vessels of apriori truths.

The question of whether there truly exists a deductive argument is taken seriously by Hilary Putman, W.V. Quine, Garrett Birkoff, and John von Neumann in the 20th century . However, their skepticism of deductive statements didn't come from theological standpoint, but partly from empirical findings in quantum mechanics(Katsoff 55-76). Again quantum mechanics overturns our most basic common sense. Yet, we needed induction to show us that.

IV. Flaws in the Law of Contrapositive

While the logic of Aristotle, due to its unprecedented rigor, wasn't challenged and therefore didn't really progress much in the centuries up until the time 18th and 19th century. On the other hand, due to the religious debates with the Buddhist of the time Nyaya logic continued to evolve. It is in the 6th and 7th century, largely due to the works of Nyaya philosopher Uddyotakara (580 AD), that Nyaya logic began to progress. It was during this time that the famous proofs on the existence of Ishvara and atoms were formulated (Keith 61). More importantly, Uddyotakara noticed a mistake in the Nyaya logic that is common to Aristotle's logic as well. Both systems allowed contraposition as a valid argument (Chakrabarti 1989, 121). Contraposition is the law which allows the simultaneous inversion and negation of the subject and predicate. For example, if “A belongs to all B”, then “non-B belongs to non-A” is a valid argument. However, if there is a universal proposition when negated doesn't contain any objects with that property, then the law of contraposition fails. Neither the Nyaya or Aristotle took note of this. An example of an invalid contraposition is

(*6) Mortality belongs to all men,
then non-men belongs to all non-mortals (Chakrabarti 1989, 124).

This is clearly invalid due to the probable nonexistence of non-mortals. While the Western world didn't address this mistake until George Boole excluded existential import from universal propositions, Uddyotakara thought about the implications of the universal and empty classes when looking at propositions about the properties “knowability” and “nameability” (Matilal 155). Since Nyaya logic asserts that everything knowable is nameable, it should follow that everything that is not nameable should be unknowable. Uddyotakara had a problem with this since the term unknowable has no rightful place in an inductive system of logic since it has no referent in reality. He disagreed that statements of universal inclusion and universal exclusion should be allowed as premises in a proposition (Chakrabarti 1989, 125). He divided the types of classes into three: (1) universal type (anvayin), which can be thought of as the universal set, (2) empty type (vyatirekin), which is similar to the empty set, and (3) normal (anvayavyatirekin), which is similar to a nonempty subset of the universal set. This was the first time in recorded history of discussion of the universal and null classes, which as we know were important in the formulation of mathematics in the early 20th century (126).

The Buddhist continued debating the Nyaya. They adhered to an atheistic world view where suffering is caused by the attachment to impermanent things. To the Buddhist, everything is flux and therefore nothing can be predicated. If it is predicated, it is an illusion (maya) caused by attachment which perpetuates suffering (Nicholson). The Buddhists often challenged Nyaya philosophers such as Uddyotakara about their notion of a generic property which causes the same cognition in those who perceive it (Scharf 161). The Buddhist claim that though the same cognition may exist in every observer, it may not be due to a single specific generic property. However, if the Nyaya could not show generic properties exist, their method predication could

be challenged. For example, the Buddhists argued that there does not exist a set of properties 'x', such that 'x' causes the recurrent cognition that “x is a cow” (161). The Nyaya argued that perception of a cow consists of two parts. First, the unqualified (nirvikalpaka) perception, which is the awareness of the cow as an object. Then, the qualified (savikalpaka) perception, which is the recognition of “cowness” by its generic property *and* other properties. The Nyaya point to the fact that the generic property exists since we know what a cow is and what a cow is not. The Buddhist consider the generic property as a set of temporary constituents that are constantly changing. Therefore, there is no set of properties that can be reduced to a generic property “cowness” (163). This debate is similar to what is an allowable predicated set in the theory of sets.

It is through these debates with the Buddhists that the Nyaya focused on the relation of properties to the objects of knowledge. Still, while the Aristotelian logic remained relatively stagnant through the centuries, the Nyaya evolved. By the 13th century, a new form of logical thought began to take form. The leader of this new Nyaya logic, now called Navya-Nyaya (or Neo-Nyaya), was Gangesa Upadhyaya. In the Navya-nyaya school, the universal property first examined by Uddyotakara was taken as a serious concept. Gangesa showed that if A is a univeral property, then in the Nyaya logical system, one can prove that A is not a universal property (Matilal 154). (A comparison to the infamous Russell paradox comes to mind). This paradox confirmed what Uddyotakara suspected when he claimed that universal premises whose inverse does not contain any objects cannot be used in valid arguments.

V. Navya-Nyaya Theory of Number

In the philosophy of mathematics, there is a debate of the nature of what constitutes a mathematical “object”. Is it something that comes from our experience, such as the idea of “two” from experiencing “twoness”? Or is “two” just a formal convention that has no real existence outside its definition? “Number is treated either as a real independent existent (Plato) or as a pure symbol (Hilbert) or as a construct from logical concepts (Russell) or even *as if* it were real (Vahinger)” (Katsoff 20). In *Metaphysics*, Aristotle claims number (arithmos) is “a measure, evidently” (32). So number must have been thought to be empirically known by Aristotle, which would exclude from any of the philosophers mentioned above in the quote. However, Aristotle entertained the notion of “essence,” or the things true nature, in *The Posterior Analytics*, which would place him in the same camp as Plato. For Aristotle, would a pure mathematical statement about “2” represent the essence of say “2” or would it represent the measure of all things with the property of “twoness”? In other words, is “2” a deductive term or an inductive term? Unfortunately, it was Aristotle's idea of “essence” in the purely concocted arena of deduction that influence subsequent philosophers up to the 20th century. As Bertrand Russell puts it, “essence” was “a hopelessly muddle-headed notion” from the start (Russell 200).

The Navya-Nyaya philosophers understood that “2” was a different sort of property than say “red”. Although an object may have the property of “redness”, no single object has the property “twoness”. The Navya-Nyaya also recognized that there were two different types of “2”. There was the “universal (jati) property” where each object that belonged to a pair was related to “twoness”. But along with that there was also the “abstract property” where the pairs themselves are the objects and not individual elements which comprise them. The individual objects that comprise the pairs are related to the object “pairs” by what is called paryapti (Shaw 284). From this, we can develop the natural numbers as distinct objects. This idea of paryapti is similar to the Frege-Russell definition of number. As Frege describes, "number itself as the

extension of a concept, and extensions of concepts are, according to my definitions, ranges" (Shaw 289). In J.L. Shaw's paper, “Number: From the Nyaya to Frege,” he shows that the extension of a concept Frege talks about is the same idea as the abstract property and the “ranges” correlate to the universal property (283).

This abstract quality of the Navya-Nyaya theory of numbers is consistent with the axioms of Peano Arithmetic without needing to invoke a new Axiom of Infinity to deal with the numbers larger than the largest known class of objects (Chakrabarti 561). Let's explain. The third Peano postulate states that “no two numbers have the same successor.” Now take the universe, which has n distinct objects. Then the class of things with n+1 objects is the empty class. Similarly, the class of things with n+2 objects is also the empty class. Therefore, n and n+1 have the same successor, which contradicts the third postulate making Peano's set of axioms incomplete. This is why there is an Axiom of Infinity. However, if we want to reduce arithmetic to logic, we shouldn't need to resort to existence claims about the nature of the universe. (Refer back to Ghose's argument as to why Aristotle might have refused to use particulars in his syllogism).

By considering numbers as properties rather than extensions of classes, as the Navya-Nyaya have done, eliminates the need for an Axiom of Infinity. So even though n+1 might not exist as a class, the property n+1 exists. So while n and n+1 would be identical to the null class, the properties remain distinct. This is not a difficult concept to grasp. For example, the idea of a class consisting of 5000 pound men is within our imaginative faculties, giving it an abstract quality and existence, even though the class of 5000 pound men is the null class (I hope). Similarly, if n is within our capacity of understanding, n+1 is a property that can be understood. This is different than the property of a color that has never been seen, since there is no experience and it is beyond our intuitions.

VI. Aristotle v. Nyaya: Final Word

Nyaya logic was shown to be known and studied by many of leading logicians of the 19th and 20th century. Much of Aristotle's work in science, such as in astronomy and physics, had such a strong influence on subsequent scientists that many didn't observe the universe around them to see that he was in fact wrong. We should be highly skeptical of those who are persuasive with word. Aristotle had a way with argument. It shouldn't be surprising then that his logic could be flawed as well. It took the Galileo of logic, Gottlob Frege, to set in motion a new calculus of predicated logic.

Nyaya logic, while superficially showing similarities to the Aristolean system, differed in a fundamental disagreement about the nature of the validity of arguments. While Aristotle believed that deduction is the seat of argument, he perpetuated a dualistic philosophy that has continued in the West. In the 20th century there have been Western philosophers and logicians who have questioned whether true first principle deduction exists. If quantum laws fail to obey classical logic, then all logic must begin with an inductive step. Nyaya never questioned this assumption. For scientific inquiry, this is obvious. As Feynman said, “It doesn't matter how beautiful your theory is, it doesn't matter how smart you are. If it doesn't agree with experiment, it's wrong.” The concept of number may be different in that there might not exist in reality any number greater than class of all objects in the universe. (The task of counting objects in the universe presents us with a conundrum of how to delineate objects in a physical sense. The quantum foam may prevent us from doing this. This is beyond the scope of this paper).

Physics tells us that friction is the cause of motion. In the West, it was a frictionless universe where Aristotle's logic was for the most part accepted for centuries. In the East, Nyaya

had a formidable intellectual adversary in the Buddhists, which challenged their views. The Devil's Advocate is sometimes necessary. Nyaya logic progressed over the thousand years of debate. Discussions were undoubtedly lively and in the course of time Nyaya had a theory that was able to distinguish between sense and reference, property and class. In the next part of the paper, we will look at these debates and how they influenced the Nyaya.

The Nyaya Syllogism's Conceptual Origins and Implications

In the first part of this paper, we discussed elements of the Nyaya syllogism and how certain assumptions as to the nature of knowledge led the logic of the Nyaya along a different trajectory than that of Aristotle. We examined specifically how the two systems drew ontological distinctions between induction and deduction in relation to their place in a syllogism. The Nyaya believed experience as something not to be detached from the logical form. The system initiated by Aristotle saw logical form as apriori. As Quine and others discovered, in the quantum age, we should not be so certain as to logic apart from a universe structured as to accommodate these logical forms.

In this paper we will look closer at the origins of Nyaya logic as it developed from debates. We'll see how these methods of debate may have shaped their attitude toward proof and argument for hundreds of years to follow. As before, since the Buddhist play an instrumental role in the Nyaya logic system, we'll examine the major philosophers as they emerge in contrast to the Nyaya.

We begin our study with a discussion as to what constitutes a logic and how a culture such as that of ancient India may have developed these elements.

VII. Origins of Logic

We take logic for granted. It seems to be part and parcel of being human. We see logic in our language, so we may be tempted to believe the foundations of logic came about very early in the development of language. Let's go into the history of logic and see how this will relate with the origins of Nyaya logic.

As social creatures the need for effective communication necessarily arises. The ability of ascertaining truth from falsity could be a deciding factor in survival. With the onset of deception, it became increasingly important to determine the validity of a claim from reason. Logical arguments can be found in the very earliest of literature such as in Homer's *Iliad* (Moravcsic 1).

According to Moravcsic, early peoples exposed to regularities in nature must have drew inferences from certain “signs”. We showed in the first paper that the Nyaya were concerned with interpretation of signs in sacred Vedic texts. Moravcsik explains how these “signs” show early people what the gods wanted (4). Although this is still far from showing logical clarity, it does mark a prototype for a premise where signs precede some event. We can imagine that debate over what a sign signifies could have been the start of debate and argument.

However, we not only need the ability to form an argument, but also need to have a culture that can recognize and appreciate logical form. In *The Discovery of the Mind*, Bruno Snell marks the move from a proposition being true solely on religious grounds to deductive steps by rules of inference (Moravcsic 2). This he believes is the origins of logical thought in culture which is often conducted as a debate.

The earliest instance of logical debate in India, as in the form of question and answer occurs in the 7th century BCE in the text *Brhadaranyaka Upanisad*. Here, a king named Janaka

and his entourage questions the sage Yajnavalkya to test his wisdom and mettle.

Janaka: Here sire, I'll give you a thousand cows! But you'll have to tell me more than that to get yourself released!
Yajnvalkya: (thinking) The king is really sharp! He has flushed me out of every cover. (4.3.33-4)

We see in Janaka and Yajnvalkya's debate that there is more than just argument. There is competition to test the opponents ability to pose a sound argument. There are elements of cross-examination and attempts to find flaws in the logic of the other.

The shift that Bruno Snell mentions from belief that something is true based on what god or a king has said to a logical examination is evident in *Milinda-panha* (around 150 BCE).

Nagasena: When a king, your Magesty, discusses a matter, and he advances a point, if any one differ from him on that point, he is apt to fine him...
Milinda: Very well. It is as a scholar, not as a king, that I will discuss. (Ganeri 2004, MP 2.1.3)

There is acknowledgment from Milinda that reasoning conducted by a scholar is different than the usual engagement with a king. This is the necessary separation that Bruno Snell explained.

VIII. The Original Debate: Milinda-panha

In *Milinda-panha*, there is the first instance of formal debate in Indian literature. Each opponent defends a position. There is an "unravelling", called *nibbethanam*, which consists of an explanation of the stance. Then there is a "winding up", called *nigraha,* where one party questions the reasons. When there is agreement, it is called "reaction", or *pitikamman* (Ganeri 2004, 310). The debate resolves when somebody retracts doubt or is convinced of the others argument.

We will see if any of the elements of the Nyaya syllogism can be found in these proto-debates. To recall, the Nyaya syllogism consists of five steps.

The hill is fiery (The Premise)
Because it has smoke (The Cause)
Whatever has smoke has fire, for example, the oven (The Example)
This hill has smoke, which is associated with fire (The Application)
Therefore, this hill has fire (The Conclusion) (Phillips 55)

Now compare this to a section of debate in *Milinda-panha*.

Milinda: What is the distinguishing characteristic, Nagasena, of investigation (*vikara*)?
Nagasena: Threshing out again and again.
Milinda: Give me an illustration.
Nagasena: It is like the case of the copper vessel, which when it is beaten into shape, makes a sound again and again as it gradually gathers shape. The beating

> into shape is to be regarded as reflection and the sounding again and again as investigation. Thus it is, great king, that threshing out again and again is the mark of investigation.
> Milinda: Very good, Nagasena. (Ganeri 2004, MP 2.3.13-14)

The Premise in the syllogism is reflected by the first question and answer in the debate. We could combine these into one statement and call it The Premise, i.e. "The distinguishing characteristic of investigation is threshing out again and again." We also see something similar to The Example in the syllogism with the analogy of the copper vessel. We may even grant the debate to have The Conclusion. But this analogous reasoning does not appear to have the same logical rigor as the syllogism. It is missing the means of knowledge, the sign, that is exemplified in The Cause and The Application of the syllogism. Analogies, though appealing due to the reduction of something difficult to apprehend to something we all understand, is not yet logic. However, it is through these debates that the rigors found in the later development of Nyaya logic have their foundation. We will revisit the evolution of debate, but now we move to other essential aspects of logic that are found in *Milinda-panha*.

IX. Logical Objects

For an argument to be persuasive, it needs to explain the semantic characteristics of its objects. We see that even in *Milinda-panha,* their main concern was with the "distinguishing characteristic" of certain terms. If we want to define a logical object, we must set its boundaries. For an intangible concept such as "investigation", this seems like a necessary thing to do. But what about tangible objects? Most of us take it for granted that an object, such as a car or even a person is distinguished by their visual outline. For most of us, the sun is a ball of light and in the background a sky of blue or, if we take the image from space, of black. Seldom do we consider what "sun" is absent an observer. We have evoked the meaning, "sun," through our visual cortex. Why couldn't have we extended the boundaries? Let's inflate it to the radius of a certain temperature threshold. Or why not set its boundaries to the extent of its light? This may be difficult to consider since we are within in this sphere. This is the first problem with defining a logic. How do we deal with semantics? Where do we draw the lines between objects? This cutting up of the world into separate objects is something that seems very natural, but should we be skeptical of how we do this? This was one of the preoccupations of the Buddhists.

The type of debate that Milinda engaged in was known as *vitanda*, or "refutation-only" (Ganeri 2004, 312). In this type of debate, there is only one thesis, and the opponent only refutes what is said until there is agreement. However, the questions that the opponent raises could be conceived as counter-stances. A popular example of this is, "When did you stop beating your wife?" Questions that require the opponent to accept the embedded premise led the predecessors of the Nyaya system to examine the nature of objects in the premisses. Certain Buddhist philosophers had already given up on trying to pin down logical objects. Their idea eventually became known as "The Principle of Four-Cornered Negation" (Raju 694).

X. Four-Cornered Negation

In the 6th Century BCE in India, Sanjaya, a Buddhist philosopher who predated Siddhartha Gottama, gave peculiar answers to yes/no questions. When asked, "Is it good?", he replied "No." When asked "Is it not good?, he again replied "No." For both "Is it both good and not good?" and "Is it neither good nor not good?", he again answered "No" (694).

Sanjaya's teachings may have influenced *Milinda-panha*, since we have the same kind of experimenting with responses to difficult questions in order to avoid entrapment. For instance:

> Milinda: He who is born, Nagasena, does he remain the same or become another?
> Nagasena: Neither the same nor another.
> Milinda: Give me an illustration.
> Nagasena: Now what do you think, O king? You were once a baby, a tender thing, and small in size, lying flat on your back. Was that the same as you who are now grown up?
> Milinda: No. That child was one, I am another.
> Nagasena: If you are not that child, it will follow that you have had neither mother nor father, no! Nor teacher. You cannot have been taught either learning, or behaviour, or wisdom.
>
> (Ganeri 2004, MP 2.2.1)

The argument is that the king is neither identical to the baby nor completely distinct from it. The argument is based on understanding the meaning of "I" or self. Something which Wittgenstein would suggest that this is merely a game of words.

Gautama, the founder of Nyaya logic, refused to enter into a *vitanda* debate with anyone who took this stance. Nyaya philosophers eventually abandoned *vitanda* all together because there was no constructive effort. The Nyaya believed this standpoint was taken up to gain an easy victory over their opponent. However, the foundation of the Buddhist's philosophy was understanding the impermanence and mutual relation of everything in the universe (Stcherbatsky 1-20). By affirming or denying, the philosopher has placed a concept in a box, restricting its position. This is why Sanjaya was also known to remain silent when given a question. Buddhists of this type were often difficult to engage in a debate with because of lack of consistency. Even other Buddhists called the followers of Sanjaya, "eel-wrigglers."

As a side note, Pyrrho, one of the most famous Greek sceptics, held a similar belief. He maintained, "I am not only not certain of the knowledge of any object, but also not certain that I am not certain of such knowledge" (699). Might we suggest that Pyrrho who accompanied Alexander the Great on his invasion of India was influenced by the followers of Sanjaya?

Another example of ambivalence in setting a definition from *Milinda-panha* illustrates another aspect of Buddhist philosophy and its relation to the Nyaya's induction step:

> Milinda: Does memory, Nagasena, always arise subjectively, or is it stirred up by suggestion from outside?
> Nagasena: Both the one and the other.
> Milinda: But does not that amount to all memory being subjective in origin, and never artificial?
> Nagasena: If, O king, there were no artificial memory, then artisans would have no need of practice, or art, or schooling, and teachers would be useless. But the contrary is the case.
> Milinda: Very good, Nagasena. (Ganeri 2004, MP 3.6.11)

Nagasena suggests that no memory, or even knowledge, is entirely within or without the

thinker. Memory, like knowledge, is a process, an interaction, between the environment and the observer. So if there is a thought, we cannot say it is cut off from our experience. It is an experience and a valid source of knowledge, thus fitting into the inductive step.

XI. Kathavatthu and the Vadayutti

In the 3rd century BCE, *Kathavatthu* (Points of Controversy) describes the method of dialogue called *vadayutti*. The dialogue is a strictly prescribed format for correct argumentation. A dialogue is separated into eight openings, or *atthamukha* (Ganeri 2004, 315).

(1) Is A B?
(2) Is A not B?
(3) Is A B everywhere?
(4) Is A B always?
(5) Is A B in everything?
(6) Is A not B everywhere?
(7) Is A not B always?
(8) Is A not B in everything?

After a proponent has selected an opening the dialogue is then divided into five stages:

(1) The way forward (*anuloma*)
(2) The way back (*patikamma*)
(3) The refutation (*niggaha*)
(4) The application (*upanayana*)
(5) The conclusion (*niggamana*)

In the *anuloma* the opponent questions the respondent, makes them take a position and tries to refute that position. Then the opponent comes up with a counter-thesis so that in the *patikamma* the roles are reversed and the opponent's counter-thesis is questioned. Then in the *niggaha* the proponent argues against the counter-thesis. Finally, in the *upanayana* and *niggamana* the proponent's argument is repeated and reaffirmed.

We will give a famous example of the way forward process and omit the way back.

Theravadin: Is the soul known as a real and ultimate fact? (1st opening)
Puggalavadin: Yes.
Theravadin: Is the soul known in the same way as a real and ultimate fact is known?
Puggalavadin: No, that cannot be truly said.
Theravadin: Acknowledge your refutation.
*Puggalavadin: If the soul be known as a real and ultimate fact, then indeed, good sir, you should also say, the soul is known in the same way as any other real and ultimate is known.
**Theravadin: That which you say here is false, namely, that we should say, “the soul is known as a real and ultimate fact”, but we should not say, “the soul is known in the same way as any other real and ultimate fact is known.”
***Puggalavadin: If the later statement cannot be admitted, then indeed the former statement should not be admitted either.
Theravadin: In affirming the former, while denying the latter, you are wrong.
(Ganeri 2004, 316)

Puggalavadin put forward a thesis of the existence of personal souls. Theravadin then challenges that claim. In this we can see the origins of the Nyaya syllogism taking shape. There is the setting of a premise and giving an explanation. The acknowledgment of a refutation mimics that of asking for an example. In the *upanayana* there is the application and in the *niggamana*, the conclusion. We still are missing the cause which so vital to the syllogism. Without it there is no connection between valid knowledge and the thing which is to be proven. However, there is implication being used. From * to ** we see material implication being implemented. In symbols, we have,

(A is B → (C is D)) = ~((A is B)&~(C is D))

There is also a derivation of the contraposed version of the conditional from ** to ***.

(A is B) → (C is D) iff ~(C is D) → ~(A is B) (Ganeri 2004, 317)

One could also see the application of *modus ponens* in this dialogue. In all these cases, we can say with certainty that logic is being employed in these dialogues. We can also assume that this was a major development in Nyaya logic. As to the doubts that the Nyaya syllogism was not of Indian origin and was merely an adoption of the Aristotelian syllogism, the predecessors leading up to the Nyaya syllogism suggest strongly that Aristotle was not needed in its development.

XII. The Nyayasutra

Henry Colebrooke introduced Indian logic in the form of Nyaya syllogism to the Western philosophical community in 1824 (Keith 132). He was unaware of the Buddhist influence or the *Kathavatthu* but understood that the Nyayasutra marked an important transition in Indian logic. The Nyayasutras show the transition between mere dialogue and forms of debate to a concentration of logical form and inference. This is not to say the Nyayasturas abandoned debate. Instead they categorized the methods of debate and discussed proper procedure in each context. The three types of debate outlined are (1) The Good Debate, called *vada*, (2) The Tricky Debate, called *jalpa*, and (3) The Refutation-only Debate, which we talked about before, called *vitanda* (Ganeri 2004, 322).

The *vada*, builds upon *Kathavatthu* in that one puts forth a position, the opponent gives a counter-position and they defend their positions in a formal setting. The requirement of the use of Nyaya syllogism in this debate is the first time we see the Nyaya syllogism. This debate is to be conducted in a friendly and honest manner. The aim of the debate is not victory over your opponent, but an assessment of the arguments. In the *jalpa*, the aim is to defeat your opponent by any means necessary. We can think of this type of debate as political in the modern scene. Logical fallacies are often employed, such as *ad hominen* attacks. Other allowable exploits for the purpose of victory are quibbling, called *chala*, and false rejoinders, called *jati.* We previously discussed *vitanda*. Here there is no counter-thesis to be proven.

XIII. The Nyaya Syllogism and the Problem of *Jati*

We gave an example of the Nyaya syllogism in the first part of this paper without going too deeply into what the original sources had to say about the five parts of the syllogism. To briefly review, the syllogism consists of (1) The Premise (*pratijna*), (2) The Cause (*hetu*), (3) The Example (*udaharana*), (4) The Application (*upanaya*), (5) The Conclusion (*nigamana*).

According to the Nyayasutra 1.1.34-39, The Premise “is a statement of that which is to be proved.” Where this was seen as a redundancy in the syllogism when compared to the Aristotelian syllogism before, in the context of a debate, it would be awkward to omit this. The

Cause is “that which proves what is to be proven in virtue of a similarity with the example” (Ganeri 2004, 323). We still see this appeal to an analogy in the Nyayasutra. But as we explained in the first paper, the syllogism can only be true if it can be connected to something that is already known. Otherwise we would have to deal with vacuously true statements, which the Nyaya logicians rejected. The Example is “an illustration which, being similar to that which is to be proved, has its character.” This is the induction step the actual event that makes the syllogism conform to reality. The Application is “a drawing in together 'this is so' or 'this is not so,' depending on the example.” This connects that which is known to the current situation through inference. The Conclusion is “a restatement of the thesis as following from the statement of the reason.” This is the same as the conclusion in the Aristotelian syllogism.

The main contributor to Indian logic subsequent to the Nyaya was Vatsyayana (4th century CE). He interpreted the Nyayasutra and found certain patterns as to their theory of inference. Jonardon Ganeri reinterprets Vatsyayana's findings by categorizing three basic syllogisms given in the Nyayasutra. The syllogism is either based on (1) Similarities, (2) Dissimilarities, (3) Counter-proof. In modern notation,

By Similarities

1. Ga
2. Fa proves Ga, because b is similar to a
3. b has the 'character of a' because is is similar to a
4. a is the same as b with respect to G
5. Ga

By Dissimilarities

1. Ga
2. Fa proves Ga, because b is dissimilar
3. b does not have the 'character of a' because it is dissimilar to a.
4. a is not the same as b with respect to G.
5. Ga

Counter-proof

1. ~Ga
2. F'a proves Ga, because b is similar to a.
3. c has the 'character of a' because it is similar to a
4. a is the same as c with respect to G.
5. ~Ga (Ganeri 2004, 324)

One example of similarities taken up in the Nyayasutra is: “The soul is eternal because it is uncreated, like space.” Then the counter-proof would be: “The soul is non-eternal because it is perceptible, like a pot” (325). One may argue that analogy is not a strong enough ground for proof. The tactic of false rejoinder, or *jati*, that is allowed in a *jalpa* debate, is “an objection by means of similarity and dissimilarity” (NS 1.2.18). How one transfers characteristics from one object to the next matters. It is similar to the principal of mathematical induction where characteristics must transfer from one integer to the subsequent one. However, if we were to use the characteristic of a “black cloud” as an indication of rain, but failed to noticed that the necessary factor was the blackness and not the presence of a cloud, we would be committing *jati*.

Consider the Nyaya claim, “sound is eternal because it is audible”, and also the counter, “sound is non-eternal because it is audible” (Gokshale 76). There needs to be a restriction on audible. It cannot be both eternal and non-eternal. There is something wrong here. We should suspect *jati.*

This problem of *jati* concerned many Indian philosophers for many years. But it wasn't until the Buddhist logician Dinnaga (480-540 CE) that the general relationship between a characteristic and the property being proved was examined.

Dinnaga's text *Hetucakradamaru,* considered by many to be indecipherable even for those versed in ancient Sanskrit, remained a mystery for nearly 15 centuries (Chi 219). In this text he introduces three operators that delineate three modes (*trairupya*) of valid inferences used to determine correct thinking (*pramana*). The three modes are

(1) F occurs in *a* (*paksa*),
(2) F occurs in some homologue (*sapaksa*),
(3) F occurs in no heterologue (*vipaksa*) (Ganeri 326, Stcherbatsky 172)

Paksa (1) requires that the characteristic is present in the case under consideration. *Sapaksa* (2) requires that the characteristic occurs in a similar case. And *vipaksa* (3) requires that the characteristic is not present in a dissimilar case (219).

Dinnaga's first requirement for a proper debate was that both parties agree to the admission of the *paska* (Shcherbatskoi 20). From this admission there are two degrees of freedom for the possible combinations of *sapaksa* and *vipaksa* that contain all, none, or some of these terms, $3^2 = 9$ combinations. This diagram is called the Wheel of Reason, where + is all, - is none, and i is some for the terms in question (Ganeri 2004, 345).

vipaksa

sapaksa

++	*+-	i+
+-	--	i-
+i	**-i	ii

Of these nine cases only * and ** are cases of valid inferential sign. * is correct because F occurs in all homologue (*sapaksa*) and no heterologue (*vipaksa*), hence +-. Then ** is correct because F occurs in some homologue (*sapaksa*) and no heterologue (*vipaksa*), hence -i. It can be shown that none of the other combinations gives a valid inference based on the conditions of Dinnaga.

So by Dinnaga we can dismiss the inference about sound being eternal since it is audible. This is because there are no audible things other than sounds. In other words, there is no homologue of an eternal thing other than sound, which is also audible (Ganeri 2004, 347).

As was pointed out in the first part of this paper, the Indian logicians were the first to consider such a thing as an empty set. In the Wheel of Reason, we see this in the center, where neither the heterologue nor the homologue occurs. It is interesting to note how this may tie in to the Buddhist idea of *sunya*, or emptiness/nonbeing, as the essential feature of the universe.

XIV. Final Remark

In the first part of the chapter, we considered the possibility that the Aristotelian syllogism influenced the development of the Nyaya syllogism. In this second part of the chapter, we gave an account for the natural development of the Nyaya logic from within the culture. The evolution of the conception of debate and the nature of a valid argument created the environment for the Nyaya syllogism to arise naturally. We also see that the main interest of logicians after the syllogism was developed was in the inductive step, which is absent from the Aristotelian syllogism. The problem of inference is still being examined today since all of science depends on it.

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